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Unifying Inflation and Dark Energy Using an Interacting Holographic Model

Abby E. Besemer

DEPARTMENT OF PHYSICS
INDIANA UNIVERSITY
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Supervisor: ______________________________
Michael Berger

Co-signer: ______________________________
Richard Van Kooten

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Abby Besemer*
Advisor: Michael Berger†
Physics Department, Indiana University
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Abstract

The universe has gone through at least two very different periods of accelerated expansion. The earliest stage was a rapid exponential expansion known as inflation while the acceleration we are experiencing at the current epoch is driven by dark energy. Because the energy scale of dark energy is many orders of magnitude smaller than that of inflation, the relationship between the two periods of acceleration is unknown. The introduction of an interaction between dark energy and matter and the holographic principle offers a possible way to unify these two eras of expansion using a model based on a simple physical principle. Here we present a possible expansion history for the universe using a model of interacting holographic dark energy.

*abesemer@indiana.edu
†berger@indiana.edu

Introduction

The theorized exponential expansion of the early Universe called Inflation is considered part of the standard big bang cosmology. It was proposed to smooth out any inhomogeneities, anisotropies and curvature in the Universe. Until the late 1990's it was believed that matter's gravitational attraction has since been slowing down the expansion. Then in 1998, the HST observed the redshift-distance relation of Type Ia supernovae and revealed that the Universe has actually begun to accelerate again [1]. It is believed that this acceleration is driven by a mysterious cosmological component called dark energy. The matter energy density naturally decreases as the Universe expands and at some point in the not so distant past, dark energy began to dominate the total energy density of the Universe and thereby initiated the second accelerated expansion.

Models of interacting dark energy have been employed in the past to offer a possible solution to the cosmic coincidence problem of why the matter ($\Omega_m \approx .3$) and dark energy ($\Omega_\Lambda \approx .7$) densities are so close at the present epoch [2, 3]. In these models there exists a mechanism that converts dark energy to matter so that the energy densities become comparable at late times, thus solving the cosmic coincidence problem. For our purposes, an interaction provides a way to unite the expansion history of the Universe using a simple physical principle. Even though we do not specify the physical mechanism for the interaction, it provides an interesting way to explain the observed cosmological quantities at a phenomenological level [2]. The nature of such an interaction is something that can be investigated once the nature of dark energy itself is better known.

This model also uses a holographic principle in order to account for the small scale of the dark energy density ($\rho_0 \approx 3 \times 10^{-3} eV^4$ where the subscript "0" indicates the value at the present time) [4]. This principle is motivated by the fact that the present value for the dark energy density seems to be consistant with taking the geometric average with the Planck mass M_{pl} and the size of the Universe given by the Hubble parameter H_0 [4, 2]. Essentially, imposing holography on dark energy allows the density to be expressed in terms of some length scale associated with a horizon.

The cosmology of interacting dark energy is described by the Freidmann equations. The equilibrium points of these equations determine how the Universe evolves. This paper presents a possible expansion history assuming that the Universe consists of dark energy, matter, negligible radiation, and non-zero curvature. We know that currently the Universe is flat within a margin of 2% [5], but this may not have always been so. Without curvature, the equilibria of the Friedmann equations describe a fixed point that the Universe will either evolve away from (repelling fixed point) or towards (attracting fixed point). By allowing curvature to evolve similar to the other cosmological components, it is possible for these equilibria to become saddle points representing a transitional state which is the key to tying inflation in to the rest of the expansion history. Here we show that an interaction and holographic condition for dark energy allows the Universe to start out in a dark energy dominated state, experience a transitional period of inflation and matter creation, and then evolve towards zero curvature where the dark energy density begins to take over again causing a second period of acceleration.

Evolution Equations

The relative expansion of the Universe can be described by a scale factor $a(t)$ which relates the comoving distance l_t at time t to the distance l_p is at the present time t_p through the relation $a(t) = l_t/l_p$. The evolution of the scale factor can be determined from Friedmann equations for a

Friedmann-Lemaître-Robertson-Walker (FLRW) Universe:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{1}$$
$$\dot{H} = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(1+3\omega\right)\rho,$$

where H is the Hubble parameter and ω is the native equation of state that descibes the ratio of the pressure and density P/ρ. k is a contstant that defines the spatial curvature and k equals 1, -1, and 0 for a closed, open, and flat Universe respectively [6]. Here the constants c and $\hbar$ have been defined as 1 and the dot refers to the time deriviative. The pair of equations in (1) lead to the following continuity equations for dark energy and matter:

$$\dot{\rho}_\Lambda + 3H(1+\omega_\Lambda)\rho_\Lambda = 0,$$
$$\dot{\rho}_m + 3H(1+\omega_m)\rho_m = 0. \tag{2}$$

In the presence of an interaction that converts the dark energy into matter, an interaction term Q appears on the right hand side of the continuity equations in order to ensure the conservation of the energy momentum tensor so that

$$\dot{\rho}_\Lambda + 3H(1+\omega_\Lambda)\rho_\Lambda = -Q,$$
$$\dot{\rho}_m + 3H(1+\omega_m)\rho_m = Q. \tag{3}$$

However, the interaction can be combined with the native equations of state in order to make a comparison with the ΛCDM model easier [4]. The effective equations of state can then be defined as

$$\omega_\Lambda^{eff} = \omega_\Lambda + \frac{\Gamma}{3H} \quad \text{and} \quad \omega_m^{eff} = -\frac{1}{r}\frac{\Gamma}{3H}. \tag{4}$$

Here the rate $\Gamma = Q/\rho_\Lambda$ and the ratio $r = \rho_m/\rho_\Lambda$ have been defined for simplification. Additionally, one usually takes $\omega_m = 0$ assuming matter is pressureless. The continuity equations then become

$$\dot{\rho}_\Lambda + 3H(1+\omega_\Lambda^{eff})\rho_\Lambda = 0, \tag{5}$$
$$\dot{\rho}_m + 3H(1+\omega_m^{eff})\rho_m = 0.$$

When the equations are written in this way it is easier to see that when there is no interaction, $Q = 0$, then the right-hand side of the differential equations is zero and the effective equations of state reduce to the native ones.

It is often more convenient to refer to the energy densities in terms of the energy density parameter Ω_i which is the ratio of the energy density and the critical energy density required to have a flat universe $\Omega_i \equiv \rho_i/\rho_{critical}$. The energy density parameter for each component is then given by

$$\Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3M_{pl}^2H^2}, \quad \Omega_m = \frac{8\pi\rho_m}{3M_{pl}^2H^2}, \quad \Omega_k = \frac{k}{a^2H^2}. \tag{6}$$

These are defined in this way so that the Friedmann equation in Eq (1) gives the relation

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \tag{7}$$

The Ω_k term can be thought of as the energy density parameter of curvature [6]. Even though it does not represent a real energy density, it "contributes to the expansion rate analogously to the honest density parameters" [6]. Using these definitions, we can then rewrite the continuity equations in a more suggestive form using the energy density parameters. The ratio r and its time derivative can then be written in terms of the density parameters

$$r \equiv \frac{\rho_m}{\rho_\Lambda} = \frac{1+\Omega_k-\Omega_\Lambda}{\Omega_\Lambda} \quad \text{and} \quad \dot{r} \equiv 3H\left(\omega_\Lambda - \omega_m + \frac{1+r}{r}\frac{\Gamma}{3H}\right) = 3Hr(\omega_\Lambda^{eff} - \omega_m^{eff}). \tag{8}$$

From these, we can then write the differential equations describing the time evolution of the density parameters as

$$\begin{aligned} \frac{d\Omega_\Lambda}{dx} &= -3\Omega_\Lambda(1-\Omega_\Lambda)(\omega_\Lambda^{eff} - \omega_m^{eff}) + \Omega_\Lambda\Omega_k(1+3\omega_m^{eff}), \\ \frac{d\Omega_k}{dx} &= 3\Omega_k\Omega_\Lambda(\omega_\Lambda^{eff} - \omega_m^{eff}) + \Omega_k(1+\Omega_k)(1+3\omega_m^{eff}), \\ \frac{d\Omega_m}{dx} &= 3\Omega_m\Omega_\Lambda(\omega_\Lambda^{eff} - \omega_m^{eff}) + \Omega_m\Omega_k(1+3\omega_m^{eff}), \end{aligned} \tag{9}$$

where $\dot{\Omega}_i = H\frac{\partial\Omega_i}{\partial x}$ and $x = \ln(a/a_0)$. However, because of the relation in Eq. (7), the third differential equation is redundant.

Then by combining the Friedmann equations (1) and the evolution equations (9) for the density parameters we can derive the evolution of the Hubble parameter

$$\frac{1}{H}\frac{dH}{dx} = \frac{\dot{H}}{H^2} = -\frac{3}{2}\left(1+\omega_\Lambda\Omega_\Lambda\right) - \frac{1}{2}\Omega_k. \tag{10}$$

Physical Assumptions

In order to determine the evolution of the density parameters, two physical assumptions are required [4]. One specification that can be made is a choice of an interaction Γ which involves defining how the ratio of rates Γ/H depends on Ω_m, Ω_k, and Ω_Λ. The second specification is an assumption about the native equations of state of dark energy ω_Λ. Lastly, an assumption about the holographic principle for dark energy can be made. This is commonly done by specifying the dark energy density ρ_Λ in terms of a length scale L_Λ through

$$\rho_\Lambda = \frac{3c^2 M_{pl}^2}{8\pi {L_\Lambda}^2}. \tag{11}$$

Common choices for length scales include the Hubble horizon, Particle horizon, and Future horizon. In the ΛCDM model, a cosmological constant implies that the dark energy density is constant with time and therefore requires that $\dot{L_\Lambda}/L_\Lambda = 0$. The assumptions of an interaction, native equation of state, and holographic principle are not independent [4], but are related through the expression

$$\frac{\Gamma}{3H} = (-1-\omega_\Lambda) + \frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda}. \tag{12}$$

Therefore, the specification of two physical assumptions determines the third.

The evolution of the Hubble parameter can then be rewritten as a function of the above terms

$$\frac{1}{H}\frac{dH}{dx} = \frac{\dot{H}}{H^2} = -\frac{3}{2}\left(1-\Omega_\Lambda\right) - \frac{\Omega_\Lambda}{H}\frac{\dot{L}_\Lambda}{L_\Lambda} + \frac{\Gamma}{2H}\Omega_\Lambda. \tag{13}$$

Equilibria Solutions

The equilibrium points of the energy density evolution equations, Eqs. (9), occur when the right-hand sides are zero. The first kind of equilibria is a fixed point in (Ω_Λ, Ω_k) space. By setting the differential equations in Eqs. (9) equal to zero and solving the linear combination, it can be seen that these fixed points occur at (Ω_Λ, Ω_k) = (0,0), (1,0), (0,-1) unless the effective equations of state, Eqs (4) are defined in such a way that they have singularities at these points. The second kind of equilibrium occurs when one density parameter is specified and a constraint is put on the effective equation of state. The existance of these points depend strongly on the two physical assumptions (holographic principle, interaction, or native equation of state) made, but one can see that these may occur when $\Omega_\Lambda = 0$ and $\omega_m^{eff} = -1/3$ or when $\Omega_k = 0$ and $\omega_m^{eff} = \omega_\Lambda^{eff}$. Lastly, a constraint on just the effective equations of state can be made and it can be seen that the right-hand side of the evolution equations are zero when $\omega_m^{eff} = \omega_\Lambda^{eff} = -1/3$.

Without curvature, these equilibria describe a fixed point that the Universe will either evolve away from (repelling fixed point) or towards (attracting fixed point). By allowing curvature to evolve similar to the other the cosmological components as the Universe expands, it is possible for these equilibria to become saddle points representing a transitional state. The behavior near these points is important because it does not depend on the choice of initial conditions. The nature of these fixed points can be determined from the eigenvalues of the Jacobian matrix A composed of the first order partial derivitives of the evolution Eqs. (9) [7, 8, 9]. If the evolution equations are generally given by

$$\frac{d\mathbf{\Omega}}{dx} = \mathbf{f}(\Omega_\Lambda, \Omega_k), \tag{14}$$

then the eigenvalues can be determined from the 2-by-2 matrix

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial \Omega_\Lambda} & \frac{\partial f_2}{\partial \Omega_\Lambda} \\ \frac{\partial f_1}{\partial \Omega_k} & \frac{\partial f_2}{\partial \Omega_k} \end{pmatrix}. \tag{15}$$

If the resulting eigenvalues are both negative at a certain fixed point, then that equilibria point is an attractor and the Universe will eventually evolve towards this state. If both of the eigenvalues are positive, then this equilibria point will be a repeller and the Universe will evolve away from this point. Lastly, if one eigenvalue is positive and one is negative then the fixed point will be a saddle point representing a transitional period.

In the case of no interaction $\Gamma = 0$ and constant dark energy $\dot{L}/L = 0$ one retains the familiar behavior of the ΛCDM model where the Universe is driven from a small amount of vacuum energy $\Omega_\Lambda \approx 0$ towards a de Sitter Universe at the attractive fixed point at (1,0) [4].

Possible Expansion Histories

We have found an interesting expansion history that exhibits an evolution that is bound to the triangular region where Ω_Λ and Ω_m are always positive and vary between 0 and 1 and where Ω_k is always negative and varies between -1 and 0. This solution can be seen in Figure 1. Because of the relation between the density parameters, Eq. (7), the line $1 + \Omega_k - \Omega_\Lambda = 0$ corresponds to $\Omega_m = 0$. This line can be thought of as the matter axis and since the density of matter is physically required to be greater than zero, the Universe must evolve in the shaded region above this line. The origin then corresponds to when $\Omega_m = 1$.

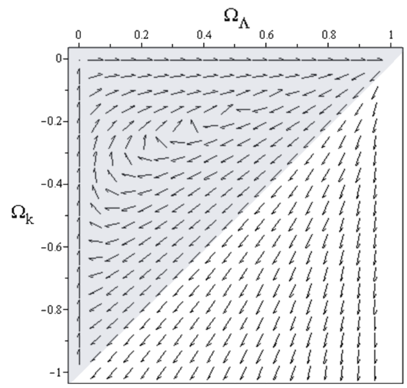


Figure 1: A vector flow diagram of the evolution of the density parameters

As previously mentioned, the behavior of the density parameters requires the specification of two physical assumptions. In order to obtain this triangular region in the vector field, certain constraints must be put on our choices for the physical assumptions. There are two requirements in order to have the vertical flow lines on the $\Omega_\Lambda = 0$ line point in the positive direction. Firstly, in order to have

$$\frac{\partial \Omega_k}{\partial x} > 0 \tag{16}$$

on the $\Omega_\Lambda = 0$ line, the differential equation for Ω_k requires that

$$\omega_\Lambda \propto \frac{1}{\Omega_\Lambda}. \tag{17}$$

Additionally, in order to have

$$\frac{\partial \Omega_\Lambda}{\partial x} \propto \Omega_\Lambda \tag{18}$$

on the $\Omega_\Lambda = 0$ line, the differential equation for Ω_Λ requires that

$$\frac{\Gamma}{3H} \propto \frac{1}{\Omega_\Lambda}. \tag{19}$$

The other constraint on the interaction comes from the requirement that

$$\frac{\partial \Omega_m}{\partial x} \propto \Omega_m \tag{20}$$

on the $\Omega_m = 0$ line. This then requires

$$\frac{\Gamma}{3H} \propto \Omega_m. \tag{21}$$

For the evolution seen in Figure 1, the interaction was chosen to be

$$\frac{\Gamma}{3H} = \frac{(1 + \Omega_k - \Omega_\Lambda)(1 - c_i \Omega_\Lambda)}{2\Omega_\Lambda} \tag{22}$$

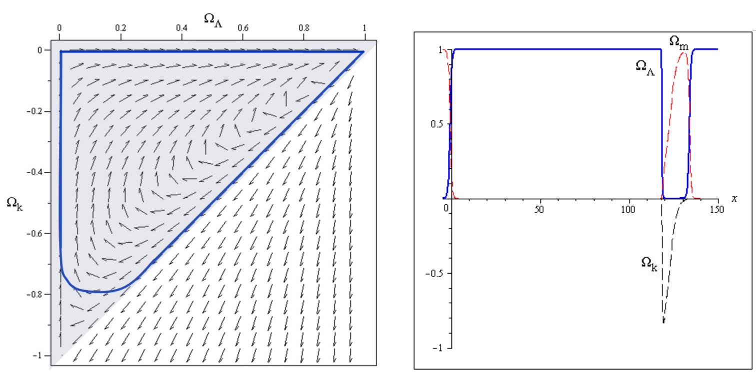
where $c_i = 0.4$, and the equation of state of dark energy was chosen to be

$$\omega_\Lambda = \frac{-\left(1+\Omega_\Lambda\right)\left(1+\Omega_k-{\Omega_k}^2\left(\frac{5\Omega_\Lambda}{1-\Omega_\Lambda}\right)-\Omega_\Lambda\right)}{2\Omega_\Lambda(1-\Omega_\Lambda)} \tag{23}$$

and the lengthscale was chosen such that

$$\frac{2}{3H}\frac{\dot{L}_\Lambda}{L_\Lambda} = \frac{3(2(\Omega_\Lambda-1)^3+25{\Omega_k}^2(1+\Omega_\Lambda)-2\Omega_k(6-7\Omega_\Lambda+{\Omega_\Lambda}^2))}{20(\Omega_\Lambda-1)^2}. \tag{24}$$

This solution has two saddle points at $(\Omega_\Lambda, \Omega_k) = (0, 0)$ and $(0, -1)$. Additionally, the region near $(1, 0)$ exhibits saddle like behavior even though the differential equations are undefined at this point. Now, we assume that the Universe should start out in a dark energy dominated state with little or no matter, which corresponds to the region near $(1, 0)$. Additionally, observational data suggests that we currently live in a period where the total energy density is composed of about 70% dark energy and about 30% matter and the curvature is almost perfectly flat [5]. Therefore, this requires the initial conditions be chosen such that the Universe will evolve through these two regions such as the ones chosen in Figure 1. The blue line then represents the evolution of the the density parameters given those particular initial conditions.

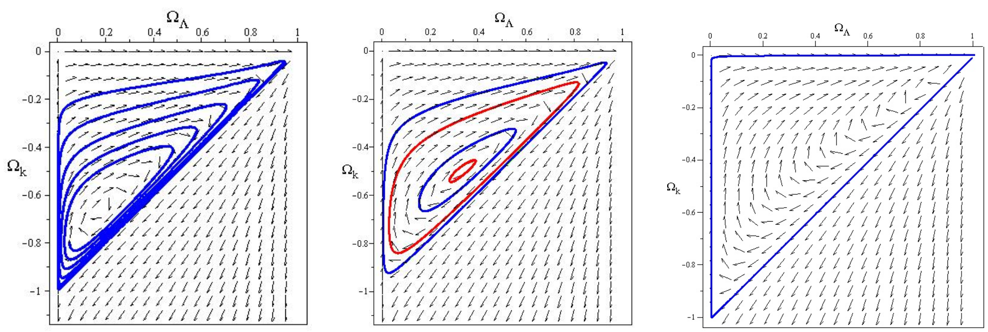


(a) (b)

Figure 2: (a) The blue line represents the evolution of the the density parameters given initial conditions of $(\Omega_\Lambda, \Omega_k) = (0.8, -1.1 \times 10^{-5})$. With these initial conditions, the evolution passes through a period where $(\Omega_\Lambda, \Omega_k, \Omega_m) \approx (.7, 0, .3)$ which agrees with the observational data for the present day values for the density parameters. (b) The evolution of the density parameters as a function for x, where $x = \ln(a/a_0)$.

In Figure 2, the Universe starts out in a dark energy dominated state with little or no matter. Then, because the interaction, Eq. (22), is proportional to the amount of matter, the interaction will remain weak and the Universe will evolve along the diagonal $\Omega_m = 0$ line. As the dark energy density decreases, the interaction gets stronger and begins creating matter while the curvature is driven towards zero. Then as the Universe evolves along the $\Omega_k = 0$ line from $(0, 0)$ towards $(1, 0)$, the interaction gets weaker and the matter density then begins to decrease as the Universe expands, like it would naturally in the absence of an interaction. Dark energy will then begin to dominate the total energy density as the Universe approaches the point $(1, 0)$ and it will pass through a period where $\Omega_m \approx 0.274$, $\Omega_\Lambda \approx 0.724$, and $\Omega_k \approx -0.0011$. These values for the density parameters are all within the uncertainty of the values that we observe today where $\Omega_m = 0.281^{+0.016}_{-0.015}$, and $\Omega_\Lambda = 0.724^{+0.015}_{-0.016}$, and $\Omega_k = 0.0046^{+0.0068}_{-0.0067}$ [5]. Then, in this particular solution, there is an additional attracting fixed point in the interior of the triangle. This will cause the Universe to spiral towards this point where the density parameters will become nearly constant.

Four different types of interior points can occur within the triangle by making slightly different choices for the physical assumptions. This interior fixed point corresponds to when the effective equations of state for matter and dark energy are equal, $\omega_\Lambda^{eff} = \omega_m^{eff} = -1/3$. If both the eigenvalues are positive at this point, the interior point will be a repeller and the Universe will keep circulating around the triangle edges of the triangle and will evolve closer and closer to the density parameter axes with each evolution. If both the eigenvalues are negative, then the interior point will be an attractor and the Universe spiral towards this point regardless of the initial conditions chosen. An example of an attracting equilibrium point can be seen in Figure 3 (a). If the interior point has two zero eigenvalues then it is neither an attractor or a repeller. In this case, the Universe evolves along cyclic paths and the behavior seems to suggest the conservation of some unknown quantity. An example of this type of solution can be see in Figure 3 (b). Lastly, if there aren't any interior equilibrium points, then the solution will only evolve around the triangle once and will continue to approach the point $(\Omega_\Lambda, \Omega_k) = (1, 0)$ where dark energy dominates the total energy density of the Universe. An example of this type of solution can be see in Figure 3 (c). Whether any of these evolutions would be more desirable than the others is still unclear. However, the simplicity of an interior fixed point with zero eigenvalues makes it a slightly more attractive result. Also, an interior repelling point would allow the Universe to go through multiple increasing periods of inflation.



(a) (b) (c)

Figure 3: The behavior of various expansion histories is determined by the existence of an additional equilibrium point corresponding to when $\omega_\Lambda^{eff} = \omega_m^{eff} = -1/3$. (a) This solution has an interior attracting fixed point which causes the Universe to spiral towards this point. (b) This solution has an interior fixed point that is neither attracting or repelling causing a cyclic evolution. (c) This solution does not have an interior fixed point and will only evolve around the triangle once.

We have shown that an interacting holographic model is able to produce an expansion history that passes through a period where the matter, dark energy, and curvature parameters are very close to the values that we observe today. But in order to see if this model can unify the accelerated expansions caused by dark energy and inflation, we have to look at the evolution of the Hubble parameter. During the inflation phase, the Universe was dominated by some sort of scalar field that had a large energy density that varied slowly with time. The Friedmann equations then require the Hubble parameter H to be nearly constant during inflation as the Universe expanded like $a(t) = e^{Ht}$. The differential equations describing the evolution of the Hubble parameter can be seen in Eq. (10). Now, since the initial conditions require the Universe to start near the upper right-hand corner we can design the universe to inflate at the point (1, 0). Near that point, the right hand side of Eq. (13) reduces to $-3/2(1+\omega_\Lambda)$. Therefore, if $\omega_\Lambda = -1$ at this point, the right hand side of Eq. (13) will be zero, H will be constant, and the universe will inflate as long as it passes close enough to this point.

During inflation the universe must expand by a factor of about 60 e-foldings, or $a(t_2)/a(t_1) \approx e^{60}$, in order to smooth out any inhomogeneities and anisotropies in the cosmic microwave background [12]. Therefore, because $x = \ln(a/a_0)$, H is required to be constant for at least $x = 60$ in order to have an inflation of 60 e-foldings. In order to see how the solution is behaving near the point (1,0) we can use perturbation theory and set $\Omega_\Lambda = 1 - \epsilon_\Lambda$ and $\Omega_k = \epsilon_k$. The differential equations that describe the behavior of ϵ_Λ and ϵ_k are given by

$$\begin{aligned} \frac{\partial \epsilon_\Lambda}{\partial x} &= -a\epsilon_\Lambda + (\text{higher order terms}) \\ \frac{\partial \epsilon_k}{\partial x} &= -b\epsilon_k - \frac{{\epsilon_k}^2}{\epsilon_\Lambda} + (\text{higher order terms}). \end{aligned} \tag{25}$$

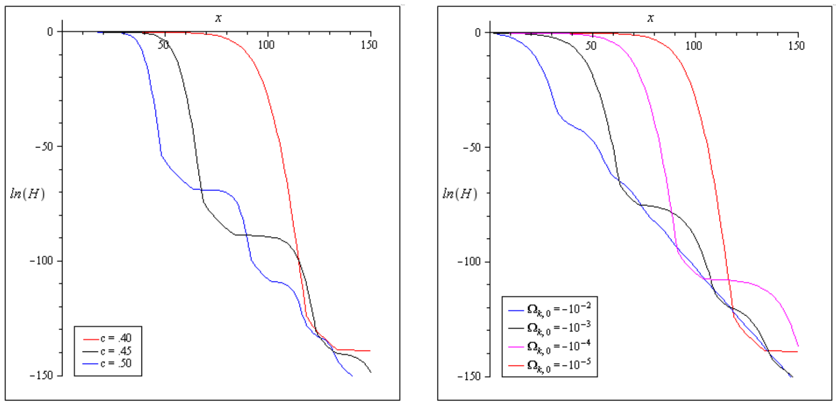
Therefore, to the first order, ϵ_Λ and ϵ_k can be approximated by

$$\epsilon_\Lambda = \epsilon_{\Lambda,0} e^{-ax} \tag{26}$$
$$\epsilon_k = \epsilon_{k,0} e^{-bx} \tag{27}$$

where the coefficients a and b determine how rapidly the Universe is approaching the point (1,0). From Eqs. 25 we can see that the Universe will start to turn around and leave the point (1,0) when the second order term ${\epsilon_k}^2/\epsilon_\Lambda$ becomes comparable to $b\epsilon_k$. This happens when

$$x_t = \frac{1}{a-b} \ln\left(\frac{\epsilon_{k,0}}{\epsilon_{\Lambda,0}}\right) \tag{28}$$

where x_t denotes the value x when the turnaround occurs. From this equation, we can see that there are four ways to control the amount of time that the Universe stays near (1,0) and hence control the amount of inflation. One way to increase the amount of inflation is to increase the power of $\epsilon_{\Lambda,0}$ and/or decrease the power of $\epsilon_{k,0}$. However, the physical assumptions that must be chosen to make these adjustments make it difficult to retain the triangular behavior in the vector field diagram. Another way to increase the amount of inflation is to make the values of the coefficients a and b close to one another. It turns out that the values of a can be made arbitrarily close to b by simply adjusting the coefficient c_i in the interaction in Eq. (22). The coefficient c_i also happens to determine the nature of the point in the interior of the triangle. When $c_i \leq 1/3$ the interior fixed point disappears and the Universe will then be attracted to the point (1,0) and will not turn around just as in Figure 3 (c). However, as long as $c_i > 1/3$, for these choices of physics assumptions, there will be an interior fixed point and the Universe will pass near (1,0) without being attracted to it. Additionally, the closer c_i gets to 1/3, the closer the values of a and b become and the more attractive the point (1,0) becomes. The evolution of the Hubble parameter, H, as a function of x for various choices of c_i can be seen in Figure 4 (a). As expected, the closer c_i gets to 1/3 the longer the Universe will inflate. The last way to control the amount of inflation is to adjust the initial conditions. The smaller value we choose for $\Omega_{k,0}$, the closer it will start to the edge of the



(a) (b)

Figure 4: The evolution of the Hubble parameter for various choices of (a) interaction coefficients c_i and (c) initial conditions. The red curve in both plots have a region where $\ln(H)$ is nearly constant for $x = 60$ corresponding to an inflation of about 60 e-foldings.

triangle and the longer it will stick around the point (1,0). Figure 4 (b) shows the evolution of the Hubble parameter for various choices of $\Omega_{k,0}$. As expected, smaller values $\Omega_{k,0}$ cause the Universe to inflate longer. The red curve in both Figure 4 (a) and (b) represents the desired inflation of at least 60 e-foldings.

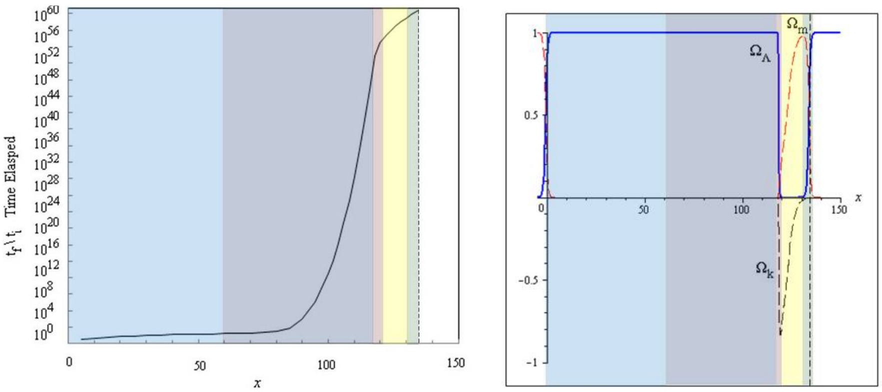
In order to determine what time corresponds to the various phases of the evolution, we can solve for the time elapsed, $t_e = t_f/t_i$, as a function of x. This can be calculated by using the definition of the Hubble parameter where

$$H(x) = \frac{\dot{a}}{a} = \frac{d}{dt}\ln(a) = \frac{dx}{dt} \tag{29}$$

and then integrating the Hubble parameter with respect to x

$$\begin{aligned} \int_{t_i}^{t_f} dt &= \int_{x_i}^{x_f} \frac{1}{H(x)} dx \\ t_e &= \int_{x_i}^{x_f} \frac{1}{H(x)} dx. \end{aligned} \tag{30}$$

For the evolution seen in Figure 2, the time elapsed, $t_e = t_f/t_i$, is plotted as a function of x in Figure 5. Because our equations may not be valid before the Planck time, we will assume that the evolution must start after this time and assume that $t_i = t_{pl} \approx 10^{-43}$s. The light blue region in Figure 5 corresponds to the period of time when the Universe is near $(\Omega_\Lambda, \Omega_k) = (1, 0)$ and inflating. This happens very rapidly and lasts until about 6.9×10^{-42}s. The purple region represents the period of time when the Universe starts to turn away from the point (1,0) but is not inflating. Most of the time elapses during this part of the expansion history and lasts until about 60 yrs after the big bang. The red region represents when $\Omega_m = 0$ and this period lasts until about 2×10^4 yrs. Then the yellow region corresponds to when the interaction starts to become stronger and the curvature is driven back up towards zero along the $\Omega_\Lambda = 0$ line. The Universe then reaches the origin at about 5.2×10^8 yrs. Finally, the green region represents the time when $\Omega_k = 0$. The dark energy density starts to dominate the total energy density as the Universe moves



(a) (b)

Figure 5: The time elapsed, $t_e = t_f/t_i$, as a function of x. The light blue corresponds to inflation, the purple corresponds to when the Universe is very near to the point (1,0) but not inflating, the red corresponds to when $\Omega_m = 0$, the yellow corresponds to when $\Omega_\Lambda = 0$, the green corresponds to when $\Omega_k = 0$ until it reaches the dotted line which represents the present day values for the density parameters.

along this line. The dotted line then corresponds to the time when the density parameters become comparable to the present day values. This occurs at an age of 12×10^9 yrs which is slightly less than $13.75 \pm 0.17 \times 10^9$ yrs, the current age of the Universe [13].

However, at the expense of fine-tuning, we can easily increase the lifetime to be exactly the age of the universe at the point where $\Omega_m \approx 0.274$, $\Omega_\Lambda \approx 0.724$, and $\Omega_k \approx -0.0011$ by simply adjusting the coefficient, c_e, in the equation of state

$$\omega_\Lambda = \frac{-\left(1+\Omega_\Lambda\right)\left(1+\Omega_k - {\Omega_k}^2\left(\frac{c_e\Omega_\Lambda}{1-\Omega_\Lambda}\right) - \Omega_\Lambda\right)}{2\Omega_\Lambda(1-\Omega_\Lambda)}. \tag{31}$$

In the case that $c_e = 0$, ω_Λ would be proportional to $1 + \Omega_k - \Omega_\Lambda$ or Ω_m. Then, as the Universe travels along the $\Omega_k = 0$ line, $\Omega_k \approx 0$ and ω_Λ simplifies to

$$\begin{aligned} \omega_\Lambda &\approx \frac{-\left(1+\Omega_\Lambda\right)\left(1-\Omega_\Lambda\right)}{2\Omega_\Lambda(1-\Omega_\Lambda)} \\ &\approx \frac{-\left(1+\Omega_\Lambda\right)}{2\Omega_\Lambda}. \end{aligned} \tag{32}$$

Then as $\Omega_\Lambda \to 1$, we have $\omega_\Lambda \to -1$ and the Universe will inflate near (1,0). However, when the Universe starts to turn away from this point, Ω_k starts to increase and the Ω_m term stops cancelling with the $(1-\Omega_\Lambda)$ term and ω_Λ can no longer be approximated as -1 which is required to have inflation. In the case that $c_e = 0$, ω_Λ will start to deviate from -1 and stop inflating even though it remains close to the point (1,0). Because of this, a lot of time is spent near this point and causes the whole evolution to last longer then the age of the Universe. The addition of the ${\Omega_k}^2$ term changes the way the numerator cancels with the denominator near the point (1,0) and, consequently, larger values of c_e will decrease the amount of time spent near (1,0) while not inflating. Plots of the equation of state with various choices for c_e can be seen in Figure 6. Larger values of c_e will shorten the entire lifetime of the evolution, by decreasing the amount of time spent

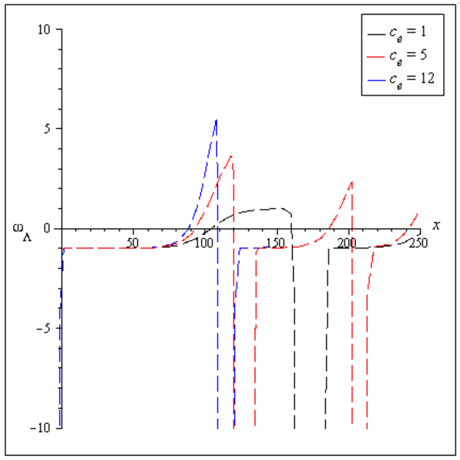


Figure 6: The equation of state with various choices for c_e. Larger values of c_e will shorten the lifetime of the evolution. The spike corresponds to the x at which the Universe starts evolving away from the point (1,0).

near (1,0). Therefore, the coefficient c_e in the ${\Omega_k}^2$ term allows us to easily control the total lifetime of the Universe in this model.

Despite the fact that the extra ${\Omega_k}^2$ term can shorten the amount of time spent near the point (1,0), the Universe still doesn't start to leave this point until at least 60 yrs after the big bang and matter isn't produced until about 2×10^4 yrs. This is not consistent with Big Bang nucleosynthesis which requires Hydrogen ^{1}H, deuterium ^{2}H, the helium isotopes ^{3}H and ^{4}H, and the lithium isotope ^{7}Li to be produced between 3-20 minutes after the big bang [14]. Therefore, in order for BBN to be consistent with this model, we would have to shorten the time that the Universe spends in the dark energy dominated state near (1,0) and matter production would have to begin within about 3 minutes. Ideally, the equation of state should be designed such that $\omega_\Lambda \approx -1$ for at least $x = 60$ and then once the ω_Λ is no longer comparable to -1, the Universe should leave the point (1,0) and start traveling along the $\Omega_m = 0$ line until about 3 minutes after the Big Bang. While this type of behavior for ω_Λ is easy to express with respect to x, it is difficult to design an equation in terms of the density parameters.

Summary and Conclusions

We have found that by introducing an interaction between dark energy and matter and imposing holography on dark energy it is possible to reproduce various key points of the expansion history of the Universe. In this paper we have found a solution where the Universe starts out in a dark energy dominated state, experiences a transitional period of inflation and matter creation, and then evolves towards zero curvature where the dark energy then begins to take over the total energy density causing a second period of acceleration. This model is capable of producing values for the energy densities that are consistent with observation. One possible expansion history we have found passes through a period where $\Omega_m \approx 0.274$, $\Omega_\Lambda \approx 0.724$, and $\Omega_k \approx -0.0011$. These values for the density parameters are consistant with the values that we observe today where $\Omega_m = 0.281^{+0.016}_{-0.015}$, and $\Omega_\Lambda = 0.724^{+0.015}_{-0.016}$, and $\Omega_k = 0.0046^{+0.0068}_{-0.0067}$ [5]. Additionally, we have also shown that the physical assumptions can be chosen such that this occurs exactly at the time corresponding to the present day age of the Universe.

In this paper we have also found that an interacting holographic model is also capable of unifying the accelerated expansions caused by dark energy and inflation as long as the initial conditions are chosen such that the solution passes through the region where the Hubble parameter is nearly constant. The interaction can then be chosen such that the Hubble parameter will remain constant for a period of at least $x = 60$ and the Universe will inflate by a factor of about 60 e-foldings, or $a(t_2)/a(t_1) \approx e^{60}$, which is required to smooth out any inhomogeneities and anisotropies in the cosmic microwave background [12].

To further improve this model and to be consistent with Big Bang nucleosynthesis, the equation of state, ω_Λ, needs to be chosen such that inflation, as well as the start of matter production, will occur within the first 3 minutes of the Universe. Additionally, in order to eliminate some of the fine-tuning in this model, we ultimately want to investigate the main features of our choices for the physical assumptions and come up with simpler equations that still produce the same general behavior. Other future work includes investigating if any of the length scales we have found correspond to any known horizons or possibly ones that are yet unknown. In this model the radiation energy density was assumed to be negligible. In the future this assumption needs to be tested by exploring whether or not the addition of radiation energy density to the evolution equations has a significant effect on the expansion history. Lastly, this model should be compared with additional observational data in order to constrain the results further.

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