PHILLIPS CURVE INSTABILITY
AND OPTIMAL MONETARY POLICY

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[PRELIMINARY]

Abstract. Instability in a Phillips curve relation originates from a theoretical model in which monopolistic firms face changing quadratic costs of price adjustment. Deriving the switching-Phillips curve explicitly from the firm’s optimization problem has benefits in terms of constructing a utility-based welfare criterion. Optimal monetary policy is computed under both ad-hoc and utility-based welfare criteria, where the utility-based criterion has a weight on the output gap that varies with the cost of price adjustment. The ad-hoc welfare criterion instructs monetary policy to change the systematic component of policy to offset inflation depending on the state determining the cost of price adjustment. In contrast, the utility-based welfare criterion instructs monetary policy to keep its systematic actions constant. The results imply that a central bank using an ad-hoc criterion will place too much weight on output stabilization in states with a ‘steep’ Phillips curve and not enough in states with a relatively flat Phillips curve.

1. Introduction

A Phillips curve relation with forward-looking elements is a popular specification describing inflation dynamics. Many variations exist, but most share the assumption that the parameters in the Phillips curve relation are time-invariant. Exceptions exist, particularly in frameworks accounting for model uncertainty, such as Svensson and Williams (2005) and Blake and Zampolli (2006), which develop methods to solve for optimal policies in forward-looking models when the parameters in the relations describing private sector behavior are subject to change. However, the private sector relations with changing parameters are not shown to derive explicitly from optimizing behavior. Instead of interpreting these changing parameters as representing model uncertainty, this paper posits changing cost of price adjustment as a source of structural instability and shows how they manifest in a ‘switching’ Phillips curve.

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Deriving the switching-Phillips curve explicitly from the firm’s optimization problem has benefits in terms of accurately understanding equilibrium outcomes and for constructing a utility-based welfare criterion to evaluate different policies. The utility-based criterion has a state-dependent weight on the output gap, whereas ad-hoc criterion commonly assume a constant weight. The primary result of the paper rests with the differences in optimal discretionary policy coming from minimizing the different welfare criteria subject to the switching-Phillips curve. The ad-hoc welfare criterion instructs monetary policy to change the systematic component of policy to offset inflation depending on the state, or regime. In contrast, the utility-based welfare criterion instructs monetary policy to keep its systematic actions constant across different states. The results imply that a central bank using an ad-hoc criterion will place too much weight on output stabilization in states with a ‘steep’ Phillips curve and not enough in states with a relatively flat Phillips curve.

The switching-Phillips curve relation derives explicitly from the optimal price-setting problem of a monopolistically competitive firm facing switching quadratic costs of price adjustment. The resulting relationship between inflation, expected inflation and a real marginal cost term resembles that derived under the assumption of fixed-costs of price adjustment, as in Rotemberg (1982), except the parameters switch in accordance with changes in the cost of price adjustment. Changes in the state governing the cost of price adjustment are exogenous, evolving according to a Markov-chain and are observed by both private agents and the central bank.

When the cost of price adjustment is subject to change, the mechanism causing the slope of the Phillips curve to change is clear. In states where the cost of adjusting is high, firms are less willing to pay the cost associated with adjusting prices and the coefficient on the marginal cost term is low. In states where the cost of adjusting is low, firms are more willing to pay the adjustment costs, increasing the slope of the Phillips curve. All pricing decisions made by the firm incorporate the probability of switching to different states in the future.

Although changes in parameters governing the parameters in the Phillips curve are exogenous, a potential underlying source of changes in the cost of price adjustment is the level and volatility of the inflation rate. For example, costs of price adjustment are likely to be lower in high and volatile inflationary environments than in low and stable inflationary environments. Changes in inflation will then stem from exogenous shocks other than changes in the term governing price adjustment. Under this view, lower costs of price adjustment are a symptom of high and volatile inflation, but not an underlying cause.

From an empirical standpoint, Ball, Mankiw, and Romer (1988) provide evidence, both cross-country and across time, that prices are more responsive to movements in aggregate demand when inflation is relatively high and volatile. Similarly, Caballero
and Engel (1993) present evidence that the degree of price flexibility does vary with economic conditions and prices were more flexible in the U.S. during the high and volatile inflation of the 1970s, a result consistent with firms facing a lower cost of relative price adjustment during this period. Similarly, Demery and Duck (2005) present evidence that the frequency of price adjustment increases in high inflation environments in the UK and Gagnon (2006) does the same for Mexico.

Optimal monetary policy in the presence of a switching-Phillips curve differs from previous work focusing on the implication of switching policy rules, such as Andolfatto and Gomme (2003), Leeper and Zha (2003), Davig and Leeper (2006), and Chung, Davig and Leeper (2006). These papers posit monetary rules that change regimes exogenously, while keeping parameters in the relations describing private sector behavior constant. For example, Davig and Leeper (2006) assesses the implications of a switching ‘simple’ monetary rule, where an exogenous Markov-chain governs the switching. Private sector parameters and structural relationships are invariant to the monetary policy rule in place, although the switching policy process does imply equilibrium relations with coefficients that switch in accordance with the monetary regime. In contrast, this paper posits parameters in the forward-looking Phillips curve are subject to change. Any resulting changes in the parameters describing optimal monetary policy then reflect an optimal response to the changing structure of the economy.

Solving for optimal monetary policy follows methods in Svensson and Williams (2005), where they minimize a loss function subject to private sector relations with time-varying parameters. The framework is useful in accounting for model uncertainty and solving for optimal policy when parameters change, regardless of the source of the change. In this paper, specifying the underlying source of the switching is beneficial in properly deriving a utility-based welfare metric to base optimal policy.

This paper is organized as follows: section 2 derives the switching-Phillips curve under the assumption of switching quadratic costs of price adjustment for a monopolistically competitive firm, section 3 solves for the optimal monetary policy under discretion using an ad-hoc welfare criterion, section 4 solves again the optimal discretionary policy, except using a utility-based criterion and section 5 concludes.

2. Switching Quadratic Costs of Price Adjustment

Although simply postulating a forward-looking Phillips curve relation with switching (i.e. state-dependent) coefficients may have appeal as a reduced-form empirical specification, it lacks a clear interpretation without being properly derived from an optimizing framework. Simply incorporating state-dependent coefficients into linearized structural relationships is useful for modeling model uncertainty, but confounds the microfoundations upon which many modern macroeconomic models are based.
This section derives a switching-Phillips curve relation based on the optimization problem of a monopolistically competitive firm that faces changing quadratic costs of price adjustment, based on Rotemberg (1982). In the context of the standard fixed-regime framework, the cost to firms of adjusting prices has implications for the slope of the Phillips curve. The slope-coefficient on the marginal cost term in the forward-looking Phillips curve decreases as the degree of cost adjustment rises. In other words, the more costly it is for firms to adjust their price, the less movements in marginal cost affect inflation. Permitting the variable determining the cost of price adjustment to change results in a switching-Phillips curve that has a state-dependent coefficient on the output gap, where this coefficient is a function of the state-dependent cost of price adjustment.

The Rotemberg (1982) approach of costly price adjustment is used instead of the Calvo (1983) mechanism because the distribution of prices at time $t$ under the Calvo mechanism is no longer a simple convex combination of the lagged aggregate price level and optimal relative price set at time $t$, since the average frequency of price adjustment evolves stochastically. Also, the firm’s first-order condition under the Rotemberg mechanism lends itself naturally to a recursive formulation in the presence of switching coefficients. Under the Calvo mechanism with a changing frequency of repricing, the firm’s first-order condition is an infinite sum embedding the changing coefficients and is not as easily mapped into a recursive form. A recursive formulation greatly simplifies the analysis in the presence of Markov-switching coefficients. In the standard fixed-regime setting, both approaches yield the same reduced-form forward-looking Phillips curve. Whether this is also true under regime-switching is not clear, although it will likely be the case that regimes with a high frequency of repricing will have a steeper Phillips-curve than in states with a low frequency of repricing.

2.1. Specification of Price Adjustment Costs. The fixed-regime approach imposes a cost on monopolistic intermediate-goods producing firms for adjusting their price, given by

$$\frac{\varphi}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 Y_t,$$

where $\varphi \geq 0$ is the magnitude of the price adjustment cost and $P_t(j)$ denotes the nominal price set by firm $j$ and $Y_t$ represents units of the final good produced by the representative final-goods producing firm.\(^1\) The assumption of quadratic adjustment costs implies that firms change their price every period in the presence of shocks, but will adjust only partially towards the price the firm would set in the absence of such adjustment costs. As with any type of quadratic adjustment cost, a firm prefers a sequence of small adjustments to very large adjustments in a given period.

\(^1\)See Ireland (2004) for a detailed, recent treatment of quadratic costs of price adjustment in a DSGE model.
Alternatively, these costs may vary according to a state, \( s_t \), such as

\[
\frac{\varphi(s_t)}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right) Y_t^2,
\]

(2)

where firms face a state-dependent cost of price adjustment. Embedding (2) in a dynamic stochastic general equilibrium model, as is done later in the paper, illustrates that a change in the state governing the cost of price adjustment (i.e. a change in \( s_t \)) does not generate any dynamics. If the economy is in its steady state and the economy does not experience an exogenous shock, except for the change in \( s_t \), the firm’s normal price adjusts equal to steady state inflation.

For \( s_t \in \{1,2\} \), the state evolves according to a two-state Markov chain with transition matrix

\[
\Pi = \begin{bmatrix}
p_{11} & 1 - p_{11} \\
1 - p_{22} & p_{22}
\end{bmatrix},
\]

(3)

with \( p_{mn} = \Pr[S_t = n|S_{t-1} = m] \) for \( m, n = 1,2 \).

In a model with Calvo (1983) price-setting, Cogley and Sbordone (2005) report estimates for the frequency of price adjustment across various U.S. subsamples. Their estimates of \( \alpha \), which denotes the fraction of firms that do not reoptimize their price, vary across different periods and point to prices being more flexible during periods with high and volatile inflation. For example, one set of estimates have \( \alpha = .566 \) in 1978 and \( \alpha = .734 \) in 2003. Though the differences are not statistically significant, they do suggest that price flexibility is greater amidst the high and volatile inflation of the 1970s.

2.2. The Intermediate Goods-Producing Firm’s Optimization Problem. Each of the monopolistically competitive intermediate-goods producing firms seek to maximize the expected present-value of profits,

\[
E_t \sum_{s=0}^{\infty} \beta^s \Delta_{t+s} \frac{D_{t+s}(j)}{P_{t+s}},
\]

(4)

where \( \Delta_{t+s} \) is the representative household’s stochastic discount factor, \( D_t(j) \) are nominal profits of firm \( j \in [0,1] \), and \( P_t \) is the nominal aggregate price level. Also, firm \( j \) produces good \( j \). For given \( s_t \), real profits are

\[
\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - \psi y_t(j) - \frac{\varphi(s_t)}{2} \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 Y_t,
\]

(5)

The assumption of two states, or regimes, is made for convenience and tractability, it can be replaced with an assumption concerning any finite number of states.

These are the ‘VAR mean’ estimates. The median estimates have \( \alpha = .567 \) in 1978 and \( \alpha = .682 \) in 2003.
where $\psi_t$ denotes real marginal cost and $y_t(j)$ is the production of intermediate goods by firm $j$.

There exists a final-goods producing firm that purchases the intermediate inputs at nominal prices $P_t(j)$ and combines them into a final good using the following constant-returns-to-scale technology

$$Y_t = \left[ \int_0^1 y(j)^{\theta-1} \, dj \right]^{\theta^{-1}}. \quad (6)$$

The profit-maximization problem for the final-goods producing firm yields a demand for each intermediate good given by

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} Y_t, \quad (7)$$

where $\theta > 1$ is the elasticity of substitution between goods. For a given $s_t$, substituting (5) − (7) into (4) and differentiating with respect to $P_t(j)$ yields the first-order condition

$$0 = (1 - \theta) \Delta_t \left( \frac{P_t(j)}{P_t} \right)^{-\theta} \left( \frac{Y_t}{P_t} \right) + \theta \Delta_t \psi_t \left( \frac{P_t(j)}{P_t} \right)^{-\theta-1} \left( \frac{Y_t}{P_t} \right) - \varphi(s_t) \Delta_t \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right) \left( \frac{Y_t}{\pi P_{t-1}(j)} \right) + $$

$$\beta \psi_t \left[ \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right] \left( \frac{Y_t}{\pi P_{t-1}(j)} \right) +$$

$$\beta E_t \left[ \varphi(s_{t+1}) \Delta_{t+1} \left( \frac{P_{t+1}(j)}{\pi P_t(j)} - 1 \right) \left( \frac{P_{t+1}(j) Y_{t+1}(j)}{\pi P_t(j)^2} \right) \right]. \quad (8)$$

The first-order condition can be written as a system, where each equation represents the first-order conditional give a particular regime. For $s_t = 1$, the conditional first-order condition after distributing the $\varphi(s_{t+1})$ term is

$$0 = (1 - \theta) \Delta_t \left( \frac{P_t(j)}{P_t} \right)^{-\theta} \left( \frac{Y_t}{P_t} \right) + \theta \Delta_t \psi_t \left( \frac{P_t(j)}{P_t} \right)^{-\theta-1} \left( \frac{Y_t}{P_t} \right) - \varphi(1) \Delta_t \left( \frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right) \left( \frac{Y_t}{\pi P_{t-1}(j)} \right) + $$

$$\beta p_{11} \varphi(1) E_t \left[ \Delta_{t+1} \left( \frac{P_{t+1}(1,j)}{\pi P_t(j)} - 1 \right) \left( \frac{P_{t+1}(1,j) Y_{t+1}(1)}{\pi P_t(j)^2} \right) \right] +$$

$$\beta (1 - p_{11}) \varphi(2) E_t \left[ \Delta_{t+1} \left( \frac{P_{t+1}(2,j)}{\pi P_t(j)} - 1 \right) \left( \frac{P_{t+1}(2,j) Y_{t+1}(2)}{\pi P_t(j)^2} \right) \right],$$

where $P_{t+1}(i,j)$ represents the nominal price for firm $j$ when $s_{t+1} = i$ and $Y_{t+1}(i)$ represents final output when $s_{t+1} = i$. An analogous first-order condition exists for $s_t = 2$, except $p_{11}$ is replaced with $(1 - p_{22})$ and $(1 - p_{11})$ is replaced with $p_{22}$.
In a symmetric equilibrium, every firm faces the same $\psi_t$ and $Y_t$, so the pricing decision is the same for all firms, implying $P_t(j) = P_t$. Also, steady state inflation and output are constant across states, where marginal costs obey the following steady state relation

$$\psi = \frac{\theta - 1}{\theta},$$  \hspace{1cm} (10)

and $\psi_t = \theta^{-1} (\theta - 1)$ also denotes marginal costs in the flexible-price case where $\varphi(1) = \varphi(2) = 0$.

Imposing symmetry and (10), for $s_t = 1$

$$\pi_t = p_{11} \beta E_t [\pi_{t+1}(1)] + (1 - p_{11}) \frac{\varphi_2}{\varphi_1} \beta E_t [\pi_{t+1}(2)] + \frac{\theta - 1}{\varphi_1} \psi_t,$$  \hspace{1cm} (11)

where $\varphi_i = \varphi(i)$ and for $s_t = 2$

$$\pi_t = p_{22} \beta E_t [\pi_{t+1}(2)] + (1 - p_{22}) \frac{\varphi_1}{\varphi_2} \beta E_t [\pi_{t+1}(1)] + \frac{\theta - 1}{\varphi_2} \psi_t,$$  \hspace{1cm} (12)

or more generally

$$\pi_t = \varphi_i^{-1} \beta E_t [\varphi(s_{t+1}) \pi_{t+1}] + \frac{\theta - 1}{\varphi_i} \psi_t,$$  \hspace{1cm} (13)

for $i = 1, 2$, which reduces to the fixed-regime specification when either $\varphi_i = \varphi$ for all $i$ or $p_{11} = p_{22} = 1$.\(^4\) Relations (11) and (12) illustrate how changing costs of price adjustment manifest themselves in the coefficient on marginal cost. Inflation, however, is a function of the expectation of the product of the future adjustment cost parameter and future inflation.

3. Switching Costs of Price Adjustment in a DSGE Model

A variant of the New Keynesian model arising from switching quadratic costs of price adjustment is given by

$$x_t = E_t x_{t+1} - \sigma^{-1} (i - E_t \pi_{t+1}) + u_t^D,$$
$$\pi_t = \varphi_i^{-1} \beta E_t [\varphi(s_{t+1}) \pi_{t+1}] + \kappa(s_t) x_t + u_t^S$$  \hspace{1cm} (14)\hspace{1cm} (15)

where $\kappa(s_t) = \varphi_i^{-1} (\theta - 1)$ and (14) is a log-linear approximation of the representative household’s consumption Euler equation. The switching-Phillips curve uses a measure of the output gap $x_t$ in place of the marginal cost term.\(^5\) The shock on (15) can be interpreted as a markup shock.

\(^4\)See Appendix A for detailed derivations of (11) and (12).
\(^5\)The straight substitution of $x_t = y_t - y_t^N$ for the marginal cost term is valid under a utility function of the form $u(C_t, N_t) = \log C_t - \kappa N_t$, where $y_t^N$ is the flexible-price level of output and $C_t$ is a composite consumption good equal to aggregate output ($C_t = Y_t$).
The exogenous disturbances are autoregressive and mutually uncorrelated,
\[ u^D_t = \rho_D u^D_{t-1} + \varepsilon^D_t, \]
\[ u^S_t = \rho_S u^S_{t-1} + \varepsilon^S_t, \]
where \(|\rho_D| < 1, |\rho_S| < 1, \varepsilon^D_t \sim N(0, \sigma^D_0), \varepsilon^S_t \sim N(0, \sigma^S_0)\) and \(E[\varepsilon^D_t \varepsilon^S_s] = 0\) for all \(t\) and \(s\). Assuming monetary policy is set according to \(i_t = \alpha \pi_t\), solutions take the form
\[ \pi_t = a^D(s_t) u^D_t + a^S(s_t) u^S_t, \]
\[ x_t = b^D(s_t) u^D_t + b^S(s_t) u^S_t, \]
where the coefficients solutions \(a^K(i)\) and \(b^K(i)\) are solved using the method of undetermined coefficients for \(i = 1, 2\) and \(K = S, D\).

A simple case arises when \(\rho_D = \rho_D = 0\), where the solution for \(s_t = i\) is
\[ a^D(i) = \frac{\sigma \kappa_i}{\sigma + \alpha \kappa_i}, \quad a^S(i) = \frac{\sigma}{\sigma + \alpha \kappa_i}, \]
\[ b^D(i) = \frac{\sigma}{\sigma + \alpha \kappa_i}, \quad b^S(i) = -\frac{\alpha}{\sigma + \alpha \kappa_i}. \]

Since there is no serial correlation in the shocks and no internal propagation mechanism, the impact of switching \(\kappa\) is contemporaneous, where the solutions match their fixed-regime counterparts. A state with a high cost of price adjustment implies a small value for \(\kappa\), resulting in output gap movements having a small impact on inflation. As \(\kappa\) declines, the impact of demand shocks on inflation also declines, but the impact of supply shocks on inflation rises.

If shocks are serially correlated, the intuition is similar to that in fixed-regimes, but convenient analytic expressions are not available. Numerical simulations calibrate values for \(\kappa_i\) for \(i \in \{1, 2\}\) using estimates of the Calvo parameter \(\alpha\) from Cogley and Sbordone (2005), where \(\alpha\) represents the fraction of firms unable to reset their price in a given period. Although their estimates derive from the Calvo mechanism of price adjustment, Keen and Wang (2005) provide the following mapping from the Calvo parameter to the Rotemberg cost of adjustment parameter in the case of a basic New Keynesian model without strategic complementarities
\[ \varphi = [(1 - \alpha)(1 - \alpha \beta)]^{-1} [(\theta - 1) \alpha]. \]

Using the Cogley and Sbordone (2005) values of \(\alpha\) from 1978, set to \(\alpha_1 = .566\), and 2003, set to \(\alpha_2 = .734\), the state-dependent Rotemberg cost of adjustment parameters are calibrated as
\[ \varphi_i = [(1 - \alpha_i)(1 - \alpha_i \beta)]^{-1} [(\theta - 1) \alpha_i]. \]

The value of \(\theta\) is set to be 10, implying a markup of 11% and is consistent with the values in Cogley and Sbordone (2005), yielding \(\kappa_1 = .34\) and \(\kappa_2 = .10.\) The other parameters are \(\rho_D = \rho_S = .75\) and \(p_{11} = p_{22} = .9.\)

\(^6\)Cogley and Sbordone (2005) report markups ranging over time from 8.6% to 11.6%.
Figure 1 reports the response to a demand shock conditional on the two different states. In the state with a high cost of price adjustment, which implies a low value for $\kappa$, the impact demand shocks have on inflation through their affect on output is relatively small. If the cost of price adjustment is low, so $\kappa$ is high, firms are more willing to pay the lower price adjustment cost, resulting in output movements having a greater transmission through to price inflation.

Figure 2 reports the responses to supply shocks, where the impact on inflation declines as $\kappa$ increases. Lower price adjustment costs imply that firms are more willing to adjust their price in response to movements in output. Since supply shocks move inflation and output in opposite directions, an adverse supply shock directly increases inflation, but is offset to some extent by the downward movement in output. The degree to which the decline in output attenuates the affect of a supply shock on inflation depends on the price of cost adjustment. A high cost, implying low $\kappa$, diminishes the degree to which marginal cost movements offset the supply shock, whereas a low cost of price adjustment reduces the extent to which movements in marginal cost offset a supply shock.

4. Optimal Discretionary Policy with an Ad-hoc Loss

Short-run inflation dynamics have an important impact on the appropriate conduct of monetary policy. Consequently, changing parameters in the Phillips relation governing inflation dynamics should have important implications for optimal monetary policy. This section uses methods in Svensson and Williams (2005) to solve for optimal discretionary policy when the switching-Phillips curve takes the form derived in the previous section.

The central bank’s ad-hoc loss function is

$$L_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left( \pi_{t+i}^2 + \lambda x_{t+i}^2 \right),$$  \hspace{1cm} (22)$$

where $\lambda$ is the relative weight on output deviations. Woodford (2003) derives a loss function with the same form as (22) using a second-order Taylor series expansion to the representative household’s expected utility function. An advantage of this approach is that it yields a utility-based value for $\lambda$ that depends on structural parameters of the model, one of which is the slope-coefficient on the marginal cost term in the Phillips curve. Given this parameter is subject to change, the current assumption that $\lambda$ is constant is most-likely to be misleading concerning optimal monetary policy. The next section derives the relevant utility-based welfare criterion when the term governing the cost of price adjustment is subject to change. However, using the ad-hoc loss above is useful as a starting benchmark.
The optimal discretionary policy minimizes (22) subject to
\[ \pi_t = \varphi_t^{-1} \beta E_t \left[ \varphi_t (s_{t+1}) \pi_{t+1} \right] + \kappa (s_t) x_t + u_t^S, \] (23)
under the assumption that policy actions do not affect private agents' expectations. Since the optimization problem is static, the central bank only needs to be concerned with setting policy based on the current state and does not need to take into account how states evolve going forward. A first-order condition exists for each state and is given by
\[ x_t = -\frac{\kappa (s_t)}{\lambda} \pi_t, \] (24)
indicating that the central bank should optimally vary how aggressively it acts to offset aggregate supply disturbances depending on the current state. States where costs of price adjustment are relatively low, as in \( s_t = 1 \), stabilizing inflation is less costly than when the cost of price adjustment is high. So with \( \kappa (1) > \kappa (2) \), the central bank will set policy to contract aggregate demand more aggressively when \( s_t = 1 \) than when \( s_t = 2 \).

In terms of conditional expectations, the first-order conditions can be written as
\[ \lambda p_{11} E_t x_{1t+1} + (1 - p_{11}) E_t x_{2t+1} + \kappa_1 p_{11} E_t \pi_{1t+1} + \kappa_2 (1 - p_{11}) E_t \pi_{2t+1} = 0, \]
\[ \lambda (1 - p_{22}) E_t x_{1t+1} + \lambda p_{22} E_t x_{2t+1} + \kappa_1 (1 - p_{22}) E_t \pi_{1t+1} + \kappa_2 p_{22} E_t \pi_{2t+1} = 0. \]
Solving this system in terms of expected conditional inflation yields
\[ E_t \pi_{1t+1} = -\frac{\lambda}{\kappa_2} E_t x_{1t+1}, \] (25)
\[ E_t \pi_{2t+1} = -\frac{\lambda}{\kappa_2} E_t x_{2t+1}, \] (26)
which restate the original first-order conditions in terms of expectations. Substituting (25) – (26) into the Phillips curve relation yields
\[ \frac{\lambda}{\kappa_1} x_1 = p_{11} \frac{\lambda}{\kappa_1} \beta E_t x_{1t+1} + (1 - p_{11}) \frac{\lambda}{\kappa_2} \varphi_2 \beta E_t x_{2t+1} - \kappa_1 x_t - u_t^S, \] (27)
\[ \frac{\lambda}{\kappa_2} x_2 = p_{22} \frac{\lambda}{\kappa_2} \beta E_t x_{2t+1} + (1 - p_{22}) \frac{\lambda}{\kappa_1} \varphi_1 \beta E_t x_{1t+1} - \kappa_2 x_t - u_t^S. \] (28)
Dynamics for the above system can be solved using methods in Davig and Leeper (2006), which requires postulating state-contingent decision rules on the minimum set of state variables. In this case, the model is purely forward-looking, so the only state variables are \( s_t \) and \( u_t^S \), so decision rules have the form
\[ x_t (s_t) = a (s_t) u_t^S. \] (29)
Inflation is then given by
\[ \pi_t = -\frac{\lambda a (s_t)}{\kappa (s_t)} u_t^S. \] (30)
Substituting the decision rules with unknown $a(1)$ and $a(2)$ into (27) − (28) yields

$$\frac{\lambda}{\kappa_1}a_1\varepsilon_t = p_{11}\frac{\lambda}{\kappa_1}\beta a_1\rho\varepsilon_t + (1 - p_{11})\frac{\lambda}{\kappa_2}\varphi_1\beta a_2\rho\varepsilon_t - \kappa_1a_1\varepsilon_t - u_s^t, \quad (31)$$

$$\frac{\lambda}{\kappa_2}a_2\varepsilon_t = p_{22}\frac{\lambda}{\kappa_2}\beta a_2\rho\varepsilon_t + (1 - p_{22})\frac{\lambda}{\kappa_1}\varphi_2\beta a_1\rho\varepsilon_t - \kappa_2a_2\varepsilon_t - u_s^t, \quad (32)$$

implying the following relationships between parameters

$$\begin{bmatrix}
\left(\frac{\lambda}{\kappa_1} - p_{11}\frac{\lambda}{\kappa_1}\beta \rho + \kappa_1\right) \\
- (1 - p_{22})\frac{\lambda}{\kappa_1}\varphi_1\beta \rho \\
- (1 - p_{22})\frac{\lambda}{\kappa_2}\varphi_2\beta \rho \\
(\frac{\lambda}{\kappa_2} - p_{22}\frac{\lambda}{\kappa_2}\beta \rho + \kappa_2)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2
\end{bmatrix} = 
\begin{bmatrix}
-1 \\
-1
\end{bmatrix}.$$

The optimal policy under discretion with the ad-hoc welfare criterion implies that policy should vary the response to aggregate supply disturbances depending on the current state. Figure 3 shows impulse responses differ across regimes for $\lambda = .05$. Output movements are very similar across regimes and in comparison to the fixed-regime response for $s_t = 1$. For $s_t = 1$, policy is able to exert a greater influence over inflation, so the decline in output is relatively effective at stabilizing inflation. For $s_t = 2$, inflation is less volatile under its fixed-regime counterpart due to the expectation of more aggressive policy in the future when the regime changes, thereby reducing the impact adverse supply shocks have on contemporaneous inflation relative to the fixed-regime.

Although the optimal policy description is under discretion, the central bank has committed to behave in a certain way in each regime. The more aggressive policy in the state with lower costs of price adjustment work to control expectations of future inflation, mitigating the impact of shocks on inflation in the regime with higher costs of price adjustment. In the U.S., the Volker disinflation represents an episode where rather large output losses were tolerated to reduce inflation. To the extent this episode remains embedded in expectations, the optimal discretionary solution suggests how this episode has benefitted subsequent policymakers. If private expectations anticipate a regime with very aggressive monetary policy, these actions control expected inflation and consequently, current inflation.

Although the interpretation of the switching-Phillips curve rests in this paper on changing costs of price adjustment, the implications for optimal monetary policy are the same regardless of the source of the instability in the Phillips curve, as long as the resulting reduced form of the switching-Phillips curve is the same. Also, the implications for optimal monetary policy are the same given the ad-hoc assumption concerning the functional form of the loss function.

Taylor (1979) demonstrates that markup shocks, or shocks to aggregate supply, force upon policymakers a trade-off between inflation and output volatility. The position of the trade-off frontier, or Taylor curve, depends on the variance of the
underlying shock and structural parameters of the model. The weight policy makers place on output gap stabilization determines the point on the curve that minimizes the ad-hoc loss function. With changing structural parameters, there exist conditional Taylor curves that depend on the current regime.

The monetary authority controls inflation by adjusting the output gap, so the extent to which inflation responds to output gap movements depends on the slope-coefficient on the output gap. To completely stabilize inflation, the monetary authority will need to move output relatively more in states with a low slope-coefficient. Figure 4 highlights this intuition by reporting the Taylor curves conditional on different states. For a given $\lambda$, inflation and output volatility in the $\kappa_1$ regime are relatively lower compared to the regime with the smaller slope-coefficient $\kappa_2$ on the output gap. Under a discretionary monetary policy with $\lambda = 0$, the central bank does not respond to output gap fluctuations and the switching-Phillips curve implies zero inflation volatility in both states, but different levels of output volatility if $\kappa_1 \neq \kappa_2$. If $\lambda = 0$, then $a_i = \kappa_i^{-1}$, indicating the output gap responds inversely to the slope-coefficient on the output gap in the Phillips curve. As $\kappa \to \infty$, the economy approaches its flexible-price limit, where the output gap is no longer a relevant concept and all movements in output arise from factors determining the natural rate of output. As $\kappa \to 0$, the economy approaches its fixed-price limit and the elasticity of the output gap to mark-up shocks rises. As $\lambda \to \infty$, the central bank cares exclusively about output stabilization, pushing output volatility to zero.

Figure 4 may appear paradoxical, as the inflation-output volatility frontier for $\kappa_1$ lies inside the frontier with the smaller $\kappa_2$. The apparent paradox arises because these values are taken from Lubik and Schorfheide (2004), where the larger slope-coefficient $\kappa_1$ is from the pre-Volker U.S. subsample. Evidence exists, such as McConnell and Perez-Quiros (2000) and Stock and Watson (2003), that the pre-Volker period was more volatile than the post-1982 period, seeming to suggest a reversal of the relative position of the two frontiers. However, the frontiers represent the volatility trade-off under optimal discretionary policy, which is unlikely to be an accurate characterization of U.S. monetary policy in the 1970s. Substantial empirical evidence suggests monetary policy was systematically less aggressive in the 1970s than afterward, for example, see Clarida, Gali, and Gertler (2000). The optimal policy under discretion advises exactly the opposite, that in states with a large slope-coefficient on the output gap, as in the 1970s, monetary policy should react systematically more aggressively. Due to the likely non-optimal monetary policy in the 1970s, the economy was operating well away from its optimal frontier during long periods in the pre-Volker era.
5. Utility-Based Welfare Criterion

A loss function in squared deviations of the output gap and inflation from their steady state values is a common specification, such as Clarida, Gali, and Gertler (1999). Woodford (2003), however, shows how a second-order approximation to the expected utility of the consumer under the assumption of staggered price-setting as in Calvo (1983) gives rise to a loss function of this form, where the weight on the output gap term is a function of the frequency of price adjustment. Eusepi (2005) derives the utility-based welfare function for price adjustment subject to quadratic costs, as in Rotemberg (1982), and shows how the weight on the output gap term depends on the parameter governing the cost of price adjustment. In a setting where this cost can change, this section shows how the weight on the output gap also changes in accordance with the cost of price adjustment and how this affects the optimal policy under discretion.

The appendix derives the following approximated period utility of the representative household

\[ L_t = -\Omega(s_t) \left[ \pi_t^2 + \lambda(s_t) x_t^2 \right], \]  
\( (33) \)

where \(-\Omega(s_t) = -0.5Y^{-\sigma}\varphi(s_t)\) scales the loss according to the cost of price adjustment and

\[ \lambda(s_t) = \frac{\eta + \sigma}{\varphi(s_t)}, \]  
\( (34) \)

indicating that the weight on output gap deviations depends on the regime governing the cost of price adjustment. If the utility function has log consumption and is linear in labor, then (34) is simply \(\lambda(s_t) = \varphi(s_t)^{-1}\). Thus, the utility-based welfare criteria is a loss function featuring a state-dependent weight on the output gap term. The above loss function is a utility-based version of the implied loss function in Owyang and Ramey (2004), where they estimate an adaptive expectations model where policy makers preferences exogenously switch between ‘hawk’ and ‘dove’ regimes.

In a state with a relatively low cost of price adjustment, deviations in inflation create a small loss, so the weight on the output gap is relatively high. Conversely, in a state with a high cost of price adjustment, deviations in inflation are costly, so the central bank should place less emphasis on output stabilization. This intuition is similar to that from the utility-based welfare criteria derived under the Calvo mechanism of price adjustment, as in Woodford (2003). When the price adjustment is infrequent, losses arise from price dispersion, so the central bank should place low weight on output stabilization relative to the case when price adjustment occurs more frequently.

Minimizing the central bank’s utility-based loss function subject to the switching-Phillips curve (13) under the assumption that policy actions do not affect private agents’ expectations results in the first-order conditions for inflation and the output
Combining them yields
\[ x_t = -\frac{\eta + \sigma}{\theta - 1} \pi_t, \] (37)
indicating the central bank should not optimally vary how aggressively it acts to offset aggregate supply disturbances. That is, the optimal targeting rule is a constant relation between output and inflation, independent of the state. This result differs from the optimal discretionary policy under an ad-hoc loss, where the optimal discretionary policy instructs the central bank to switch policies in accordance with the structure of the economy, as in the previous section and in Svensson and Williams (2005) and Zampolli.

In states with relatively high costs of price adjustment, the weight attached to output gap stabilization is relatively low, reflecting the high costs to firms of adjusting prices. Also, in this state, the slope-coefficient on the output gap is lower, so output gap movements are less effective at stabilizing inflation. Under an ad-hoc loss, a low slope-coefficient directs policy to reduce the systematic output gap response to inflation deviations precisely because such movements are less effective at stabilizing inflation. However, it is in states with a low-slope coefficient, or high costs of price adjustment, when inflation volatility is more costly to firms. The utility-based welfare criterion reflects this higher cost of inflation volatility by down-weighting the emphasis on output gap stabilization. The opposing forces of a lower slope-coefficient on the output gap, directing policy to reduce output gap movements to stabilize inflation, along with a lower weight on the output gap, directing policy to increase output gap movements to stabilize inflation, exactly cancel.

Dynamics from the optimal discretionary policy under the utility-based welfare metric are governed by
\[
\begin{align*}
\delta x_{1t} &= \delta \beta E_t x_{t+1} + \kappa_1 x_t + u_t^S, \\
\delta x_{2t} &= \delta \beta E_t x_{t+1} + \kappa_2 x_t + u_t^S.
\end{align*}
\] (38) (39)
Equilibrium dynamics are of the form
\[ x_t(s_t) = a(s_t) \varepsilon_t, \] (40)
where substitution then yields
\[
\begin{align*}
\delta a_{1t} &= p_{11} \delta \beta a_{1t} \rho \varepsilon_t + (1 - p_{11}) \frac{\phi_2}{\phi_1} \beta a_{2t} \rho \varepsilon_t + \kappa_1 a_{1t} \varepsilon_t + \varepsilon_t, \\
\delta a_{2t} &= p_{22} \delta \beta a_{2t} \rho \varepsilon_t + (1 - p_{22}) \frac{\phi_1}{\phi_2} \beta a_{1t} \rho \varepsilon_t + \kappa_2 a_{2t} \varepsilon_t + \varepsilon_t,
\end{align*}
\] (41) (42)
implying the following relationship between parameters
Figure 5 restates the conditional Taylor curves and marks the optimal inflation-output volatility combinations for both the ad-hoc and utility-based welfare criteria. The ×s mark the optimal combinations for the utility-based metric and indicate that in the state with a relatively high cost of adjusting prices, the ad-hoc criteria places too much emphasis on stabilizing output relative to the utility-based criterion. The reason is that when the cost of adjusting prices is high, less weight should be placed on stabilizing output, so the greater output volatility and lower inflation volatility reflects this decreased weight on output stabilization. In the regime with a lower cost of price adjustment, the utility-based criterion indicates more weight should be placed on the output stabilization objective. This is apparent in the lower contour representing the regime with lower costs of price adjustment, where the ad-hoc combination places too little weight on output stabilization.

6. Conclusion

This paper has shown that changing costs of price adjustment can generate instability in a forward-looking Phillips curve relation. In particular, the coefficients on both the expected inflation and marginal cost, or output gap, terms are subject to change in coordination with changes in the state governing the cost of adjusting prices. In addition, Phillips curve instability has implications for optimal monetary policy. Under an ad-hoc welfare criterion, the optimal policy adjusts the systematic component of policy along with changes in the state. However, since the microfoundations of the switching behavioral relations are explicitly stated, it is possible to derive a utility-based welfare metric, which has a state-dependent weight on the output gap term. The weight depends inversely on the cost of price adjustment, so in the low cost state, relatively more weight is placed on output stabilization. The implication of this analysis is that the optimal policy should not vary the systematic component of policy, so the ad-hoc criterion recommends placing too much emphasize on output stabilization when costs to adjusting prices are high and too little emphasize when the costs of adjusting prices are low.
References


Appendix A. Deriving the Switching Phillips Curve

Using (8), the firm’s optimal pricing condition for \( s_t = 1 \), after imposing \( P_t (j) = P_t \), is given by

\[
0 = (1 - \theta) \Delta_t \left( \frac{Y_t}{P_t} \right) + \theta \Delta_t \psi_t \left( \frac{Y_t}{P_t} \right) - \varphi \left( 1 \right) \Delta_t \left( \frac{P_t}{\pi P_{t-1}} - 1 \right) \left( \frac{Y_t}{P_t} \right) + \beta p_{11} \psi \left( 1 \right) E_t \left[ \Delta_{t+1} (1) \left( \frac{P_{t+1} (1)}{\pi P_t} - 1 \right) \left( \frac{P_{t+1} (1) Y_{t+1} (1)}{\pi P_t^2} \right) + \beta (1 - p_{11}) \varphi \left( 2 \right) E_t \left[ \Delta_{t+1} (2) \left( \frac{P_{t+1} (2)}{\pi P_t} - 1 \right) \left( \frac{P_{t+1} (2) Y_{t+1} (2)}{\pi P_t^2} \right) \right] \right],
\]

where substituting in \( P_t / P_{t-1} = \pi_t \) yields

\[
0 = (1 - \theta) \Delta_t + \theta \Delta (1 + \Delta_t) \psi_t - \varphi (1) \Delta_t \left( \frac{\pi_t}{\pi} - 1 \right) \left( \frac{\pi_t}{\pi} \right) + \beta p_{11} \psi \left( 1 \right) E_t \left[ \Delta_{t+1} (1) \left( \frac{\pi_{t+1} (1)}{\pi} - 1 \right) \left( \frac{\pi_{t+1} (1) Y_{t+1} (1)}{\pi y_t} \right) + \beta (1 - p_{11}) \varphi \left( 2 \right) E_t \left[ \Delta_{t+1} (2) \left( \frac{\pi_{t+1} (2)}{\pi} - 1 \right) \left( \frac{\pi_{t+1} (2) Y_{t+1} (2)}{\pi y_t} \right) \right] \right].
\]

Log-linearizing around the constant steady state yields

\[
0 = (1 - \theta) \Delta \left( 1 + \hat{\Delta} \right) + \theta \Delta \left( 1 + \hat{\Delta} \right) \psi \left( 1 + \hat{\psi} \right) - \varphi \left( 1 \right) \Delta \left( 1 + \hat{\Delta} \right) \hat{\pi}_t \left( 1 + \hat{\pi}_t \right) + \beta p_{11} \psi \left( 1 \right) E_t \left[ \Delta \left( 1 + \hat{\Delta}_{t+1} (1) \right) \hat{\pi}_{t+1} (1) \left( 1 + \hat{\pi}_{t+1} (1) \right) \left( 1 + \hat{Y}_{t+1} (1) \right) \left( 1 - Y_t \right) \right] + \beta (1 - p_{11}) \varphi \left( 2 \right) E_t \left[ \Delta \left( 1 + \hat{\Delta}_{t+1} (2) \right) \hat{\pi}_{t+1} (2) \left( 1 + \hat{\pi}_{t+1} (2) \right) \left( 1 + \hat{Y}_{t+1} (2) \right) \left( 1 - Y_t \right) \right],
\]

which simplifies, after eliminating higher-order terms and using \( \psi = \theta^{-1} (\theta - 1) \), to

\[
\hat{\pi}_t = \beta p_{11} E_t \left[ \hat{\pi}_{1, t+1} \right] + (1 - p_{11}) \beta \frac{\varphi_2}{\varphi_1} E_t \left[ \hat{\pi}_{2, t+2} \right] + \frac{(\theta - 1)}{\varphi_1} \psi_t,
\]

which can be rewritten as (13).

Appendix B. Solving the NK Model with Switching \( \kappa \)

The posited solutions (18) – (19) can be substituted into the structural equations (14) – (15) to yield systems that relate the structural parameters to the solution coefficients. For supply shocks, the system is

\[
\begin{bmatrix}
\alpha - p_{11} \rho_u & - (1 - p_{11}) \rho_u & 1 - p_{11} \rho_u & - (1 - p_{11}) \rho_u \\
- (1 - p_{22}) \rho_u & \alpha - p_{22} \rho_u & - (1 - p_{22}) \rho_u & 1 - p_{22} \rho_u \\
1 - \beta p_{11} \rho_u & - \beta (1 - p_{11}) \frac{\varphi_2}{\varphi_1} \rho_u & - \kappa_1 & 0 \\
- \beta (1 - p_{22}) \frac{\varphi_2}{\varphi_1} \rho_u & 1 - \beta p_{22} \rho_u & 0 & - \kappa_2
\end{bmatrix}
\begin{bmatrix}
a^S (1) \\
a^S (2) \\
b^S (1) \\
b^S (2)
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}
\]
and for demand shocks
\[
\begin{bmatrix}
\alpha - p_{11} \rho_g & - (1 - p_{11}) \rho_g & 1 - p_{11} \rho_g & - (1 - p_{11}) \rho_g \\
- (1 - p_{22}) \rho_g & \alpha - p_{22} \rho_g & - (1 - p_{22}) \rho_g & 1 - p_{22} \rho_g \\
1 - \beta p_{11} \rho_g & -\beta (1 - p_{11}) \frac{\varphi_2}{\varphi_1} \rho_g & - \kappa_1 & 0 \\
-\beta (1 - p_{22}) \frac{\varphi_2}{\varphi_2} \rho_g & 1 - \beta p_{22} \rho_g & 0 & - \kappa_2
\end{bmatrix}
\begin{bmatrix}
a^D (1) \\
a^D (2) \\
b^D (1) \\
b^D (2)
\end{bmatrix}
= \begin{bmatrix}
1 \\
1 \\
0 \\
0
\end{bmatrix}.
\]

**Appendix C. Deriving the Utility-Based Welfare Criterion Under Switching Costs of Price Adjustment**

The representative household’s period utility function is
\[
U(C_t, N_t) = C_t^{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \tag{43}
\]
where \(C_t\) is the composite good and \(N_t\) is time spent working. Firm level production function is
\[
y_t(j) = n_t(j). \tag{44}
\]
The aggregate resource constraint is
\[
Y_t = C_t + \frac{\varphi(s_t)}{2} (\pi_t - 1)^2, \tag{45}
\]
where steady state inflation is set to zero. Substituting (45) into (43) yields
\[
U(Y_t, \pi_t, s_t) = \frac{(Y_t - \frac{\varphi(s_t)}{2} (\pi_t - 1)^2)^{1-\sigma}}{1-\sigma} - \frac{Y_t^{1+\eta}}{1+\eta}. \tag{46}
\]
The second-order approximation to the first term of the representative agent’s period utility function is given by
\[
\frac{(Y_t - \frac{\varphi(s_t)}{2} (\pi_t - 1)^2)^{1-\sigma}}{1-\sigma} \approx \frac{Y_t^{1-\sigma} + Y^{-\sigma} \tilde{Y}_t - \frac{1}{2} \sigma Y^{-\sigma - 1} \tilde{Y}_t^2 - \varphi(s_t) \frac{\varphi(s_t)}{2} Y^{-\sigma} \tilde{\pi}_t^2}{1-\sigma} \tag{47}
\]
where, after using the following approximations,
\[
\tilde{Y}_t \approx Y \left( Y_t + \frac{1}{2} Y_t^2 \right) \tag{48}
\]
\[
\tilde{\pi}_t \approx \left( \pi_t + \frac{1}{2} \pi_t^2 \right) \tag{49}
\]
yields
\[
\left( Y_t - \frac{\varphi(s_t)}{2} \right) \left( \pi_t - 1 \right)^2 \left( 1 - \sigma \right) \approx Y_t^{1-\sigma} + Y^{-\sigma} Y \left( Y_t + \frac{1}{2} Y_t^2 \right) - \frac{1}{2} \sigma Y^{-\sigma - 1} Y^2 \left( Y_t + \frac{1}{2} Y_t^2 \right)^2 - \frac{\varphi(s_t)}{2} Y^{-\sigma} \left( \pi_t + \frac{1}{2} \pi_t^2 \right)^2.
\] (50)

Collecting terms that are independent of policy \((t.i.p.)\) yields
\[
\left( Y_t - \frac{\varphi(s_t)}{2} \right) \left( \pi_t - 1 \right)^2 \left( 1 - \sigma \right) \approx \frac{1}{Y_t} Y_t^2 + \frac{1}{2} \left( Y_t - \sigma \right) Y_t^2 - \frac{1}{2} \frac{\varphi(s_t)}{Y_t} Y_t^2 + t.i.p.
\] (51)

The second-order approximation to the second argument of the utility function is
\[
\frac{Y_t^{1+\eta}}{1+\eta} \approx Y_t^{1+\eta} + \frac{1}{2} Y_t^{1+\eta} (1 + \eta) Y_t^2 + t.i.p.
\]

Combining both components of the utility function yields
\[
U \left( Y_t, \pi_t, s_t \right) \approx Y_t^{1-\sigma} Y_t + \frac{1}{2} Y_t^{1-\sigma} Y_t^2 - \frac{1}{2} \sigma Y_t^{1-\sigma} Y_t^2 - \frac{1}{2} \frac{\varphi(s_t)}{Y_t} Y_t^2 - Y_t^{1+\eta} \left( Y_t + \frac{1}{2} (1 + \eta) Y_t^2 \right). \]

A constant employment subsidy \(\gamma\) exists that is proportional to the households labor income, which offsets the inefficiently low level of production in the steady state arising from monopolistic distortions. Under perfectly flexible prices, firms set their relative price equal to a markup \(\mu > 1\) that exceeds their marginal cost of production. The employment subsidy given to households, financed by lump-sum taxes on households, results in an efficient steady state level of production. Monetary policy then focuses on stabilization policies, versus policies to undo the monopolistic distortions. In the deterministic steady state, the monopolistic firm is not adjusting its price, so the changing parameter governing the costs of price adjustment does not create any distortions.

In the absence of fiscal subsidies,
\[
\frac{W_t}{P_t} = \frac{1}{\mu} = \frac{N_t^\eta}{C_t^{1-\sigma}}
\]
where \(\Phi\) represents the magnitude of the distortion from monopolistic competition.

\[
Y^\eta = (1 - \Phi) Y^{1-\sigma}.
\]
Substitution yields

\[ U(Y_t, \pi_t, s_t) = Y^{1-\sigma} Y_t + \frac{1}{2} Y^{1-\sigma} Y_t^2 - \frac{1}{2} \sigma Y^{1-\sigma} Y_t^2 - \frac{1}{2} \frac{\varphi(s_t)}{\pi_t^2} = (1 - \Phi) \left( Y_t + \frac{1}{2} \gamma Y_t^2 \right), \]

\[ = \frac{1}{2} Y^{1-\sigma} \left( (-1 + \sigma + (1 - \Phi) \gamma) Y_t^2 + 2 \Phi Y_t \right) - \frac{1}{2} \frac{\varphi(s_t)}{\pi_t^2} Y_t, \]

\[ = -\frac{1}{2} Y^{1-\sigma} (\sigma + \gamma - 1) \left( Y_t^2 + 2 \Phi (\sigma + \gamma - 1) Y_t \right) - \frac{1}{2} \frac{\varphi(s_t)}{\pi_t^2} + \frac{1}{2} Y^{1-\sigma} \Phi \gamma Y_t^2, \]

\[ = -\frac{1}{2} Y^{1-\sigma} (\sigma + \gamma - 1) (x_t - x^*)^2 - \frac{1}{2} \frac{\varphi(s_t)}{\pi_t^2} + \frac{1}{2} Y^{1-\sigma} \Phi \gamma Y_t^2. \]

The fiscal subsidy to remove the monopolistic distortion results in

\[ (1 + s) \frac{W_t}{P_t} = \frac{N_t}{\zeta_t} = 1 \]

\[ \frac{1 + s}{C_t^{\sigma}} = \mu \]

so \( \Phi = 0 \), and

\[ U(Y_t, \pi_t, s_t) = -\frac{1}{2} Y^{1-\sigma} (\sigma + \gamma - 1) x_t^2 - \frac{1}{2} \frac{\varphi(s_t)}{\pi_t^2}, \]

\[ U(Y_t, \pi_t, s_t) = -\frac{1}{2} \frac{\varphi(s_t)}{\pi_t^2} \left( \frac{\pi_t^2 + \gamma - 1 + \sigma}{\varphi(s_t)} x_t^2 \right). \]
FIGURE 1. Aggregate demand shock with switching quadratic costs of price adjustment.
Inflation (% deviation from ss)

Output Gap (% of natural rate)

\( \kappa_1 = 0.33712 \)
\( \kappa_2 = 0.099058 \)

Figure 2. Aggregate supply shock with switching quadratic costs of price adjustment.
Figure 3. Aggregate supply shock under optimal discretionary policy.
Figure 4. Inflation-output volatility tradeoff.
Figure 5. Inflation-output volatility tradeoff.