Multi-jurisdictional Income Tax Competition and the Provision of Local Public Goods

(Preliminary and Incomplete)

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Abstract
In this paper, an income tax competition model is considered, in which two local governments collect income taxes for both redistributional purposes, as well as to fund the provision of local public goods. The local governments seek to maximize the average utility of their residents, while a finite population of perfectly mobile, heterogeneous agents make labor/leisure and residency decisions, so as to maximize individual utility. This is a problem of multiple principals and multiple agents, with an added requirement of budget balancedness. We will attempt to use both the mechanism design problem as well as the menu design problem to prove existence of an optimal income tax-public goods mechanism, as well as to characterize the solutions.

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1. Introduction

Forthcoming

2. Framework

The model in this paper consists of two ex-ante identical jurisdictions and a finite number, $B$, of heterogeneous agents. Let $W$ denote a finite set of $n$ agent types, with elements $w_i$, $i = 1, \ldots, n$. An agent’s type, $w_i$, represents their ability. We will assume that labor markets are perfectly competitive, so that a type $w_i$ agent’s wage rate is exactly $w_i$. Without loss of generality, we will assume that $w_n > w_{n-1} > \ldots > w_2 > w_1$. That is, ability (or wage) is increasing with type index $i$.

Let $B_{w_i}$ denote the number of type $w_i$ agents, so that $\sum_{i=1}^{n} B_{w_i} = B$. Since the type space is discrete, we can assign to $W$ a probability mass function $P(\cdot)$ so that $P(w_i) = \frac{B_{w_i}}{\sum_{j=1}^{n} B_{w_j}}$.

The following will employ notation consistent with that of Berliant and Page (2001 and 2006) as much as possible. Let $Y = [0, m]$, $m > 0$ denote the set of feasible income levels, and $T = [s, m]$, $s \leq 0$ the set of feasible tax liability. We will assume that these are identical for both jurisdictions. We will define the set of feasible and nonbankrupting income and tax liability pairs, $K$, as follows,

$$K := \{(y, \tau) \in Y \times T : y \geq \tau\},$$

where $y \in Y$ denotes income level, and $\tau \in T$ denotes the corresponding tax liability. Note that $(K, d_e)$ is a compact metric space, where $d_e$ denotes the Euclidean metric in $\mathbb{R}^2$.

Since different types earn different wages, and therefore have different earning potentials, we let $m(w_i)$ denote the maximal income attainable by a type $w_i$ agent. We assume that, ex-ante, individuals do not prefer one jurisdiction over another, and so for every $w_i \in W$, $m(w_i)$ is constant across jurisdictions. This means that every agent’s job opportunities are identical across jurisdictions. The only restrictions we will place on $m(\cdot) : w \to Y$ are

$$0 < m(w_i) \leq m, \forall i = 1, \ldots, n,$$

and
This second restriction is a direct consequence of the assumption that \( w_1 < w_2 < \ldots < w_n \).

Define the feasible income levels for a type \( w \) agent as follows:

\[
Y(w_i) := \{ y \in Y : 0 \leq y \leq m(w_i) \}.
\]

Again, for each \( w_i \in W \), \( Y(w_i) \) is identical across jurisdictions, and we have \( Y(w_1) \subset Y(w_2) \subset \ldots \subset Y(w_n) \).

Define the set of feasible, nonbankrupting tax liabilities corresponding to an income level \( y \) as follows:

\[
T(y) := \{ \tau \in T : s \leq \tau \leq y \}.
\]

Denote the set of feasible, nonbankrupting income and tax liability pairs for a particular type of agent, \( w_i \), as follows:

\[
K(w_i) = \{ (y, \tau) \in K : y \in Y(w_i) \}.
\]

For simplicity, we will consider the provision of a single local public good in each jurisdiction. Let \( z_i \) denote the level of the public good provided by jurisdiction \( i \), and assume that \( z_1, z_2 \in Z \), where \( Z \) is a compact subset of \( \mathbb{R}_+ \).

Let \( h(z) \) denote the cost of providing level \( z \) of the local public good, in terms of the consumption good. Again we will assume that \( h(\cdot) : Z \to \mathbb{R}_+ \) is the same for both jurisdictions, and that \( h(0) = 0 \). This last assumption allows our model to incorporate the case of a purely redistributive income tax.

**Public Sector Mechanisms:**

A public sector mechanism is a pair of triples, \( \{(y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\} \), where \( y_k(\cdot) : W \to T \) are direct income functions, \( t_k(\cdot) : Y \to T \) are indirect tax functions, and \( z_i \in Z \) are public good levels. The distinction that a function is direct means that it is a mapping from the type space into the relevant range. Indirect means that the domain of the function is not the type space.

For example, for any type \( w_i \in W \), \( y_1(w_i) \) represents the income level in-
tended for a type \( w_i \) agent, if that agent chooses to reside in jurisdiction 1, and 
\( t_1(y_1(w_i)) \) denotes the corresponding tax liability. However, if an agent of type 
\( w_i \) instead chooses to reside in jurisdiction 2, the income level intended for them 
will be \( y_2(w_i) \), and \( t_2(y_2(w_i)) \) will be the corresponding tax liability. This is an 
important distinction that will be addressed again later in this paper.

Let

\[
S(Y(\cdot)) := \{y(\cdot) : y(w_i) \in Y(w_i), \forall w_i \in W\} \tag{7}
\]

denote the set of all feasible direct income functions, and let

\[
S(T(\cdot)) := \{t(\cdot) : t(y) \in T(y), \forall y \in Y\} \tag{8}
\]

denote the set of all feasible indirect tax functions.

**Agents’ Utility Functions:**

For any agent type \( w_i \in W \), agents’ utility is a function of their income, 
their tax liability, and the level of the public good in the jurisdictions in which 
they choose to reside. That is, we will have

\[
u(w_i, \cdot, \cdot, \cdot) : K(w_i) \times Z \to \mathbb{R}. \tag{9}
\]

We will make the following assumptions about the utility function:

(A1) **(Continuity):** For each \( w_i \in W \), \( u(w_i, \cdot, \cdot, \cdot) \) is continuous on 
\( K(w) \times Z \).

(A2) **(Boundedness):** There exists some \( b \in \mathbb{R}_+ \), with \( b < \infty \) such 
that for all \( w_i \in W \) and all 
\( (y(\cdot), t(\cdot), z) \in S(Y(\cdot)) \times S(T(\cdot)) \times Z, \)

\[ | u(w_i, y(w_i), t(y(w_i)), z) | \leq b. \]

(A3) **(Monotonicity in Taxes):** For each \( (w_i, y, z) \in W \times Y(w_i) \times Z, \)

\( u(w_i, y, \cdot, z) \) is strictly decreasing on \( T(y) \).

(A4) **(Essentiality of Leisure):** For all \( w_i \in W, \)

\( u(w_i, 0, 0, z) > u(w_i, m(w_i), \tau, z) \) for all 
\( (\tau, z) \in T(m(w_i)) \times Z. \)
(A5) **Monotonicity in Type**: For any \( w_i, w_j \in W \) such that \( j > i \),
and for any fixed \((y, \tau, z) \in K \times Z\), \( u(w_j, y, \tau, z) > u(w_i, y, \tau, z) \).

This last assumption is more intuitive when the problem is viewed as one over consumption and labor/leisure, instead of one over ability, income, and taxes.

For any \((y, \tau) \in K\), consumption, denoted by \( c \), is defined by

\[
c = y - \tau.
\]

That is, consumption is equal to after-tax income. For any agents of types \( w_i, w_j \in W \), the agents’ quantity of labor provided for some income level \( y \), denoted by \( l_i \) and \( l_j \), respectively, are as follows:

\[
l_i = \frac{y}{w_i},
\]

and

\[
l_j = \frac{y}{w_j}.
\]

So, for any fixed \((y, \tau, z) \in K \times Z\), the consumption levels of both type \( w_i \) and type \( w_j \) individuals are the same \( (c_i = y - \tau = c_j) \), and the levels of the public good enjoyed by both individuals are the same \((z)\), but the lower type individual, \( w_i \), works more \((w_i < w_j \Rightarrow l_i = \frac{y}{w_i} > \frac{y}{w_j} = l_j)\) Thus, assumption (5) simply says that, all else equal, individuals would prefer to work less.

**Cost Function**:

We will make the following assumption on the cost function \( h(z) \),

\[
(6) \quad h(\cdot) : Z \to \mathbb{R}_+ \text{ is lower semicontinuous.}
\]

**Competitive Tax Design Problem with Local Public Goods**

This is a problem with multiple, competing principals (the jurisdictions) and multiple, heterogeneous agents (the citizens), with hidden information in the form of agent types (abilities or wages). Neither jurisdiction can observe agents’ types (or their hours worked). They can however observe agents’ income levels and can use those to assign tax liabilities.

The goal of each agent is to maximize their own utility. They do this by not
only choosing how much to work (which they also do in the single jurisdiction case), but also by choosing where to live. That is, after seeing the public sector mechanism \((y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\) chosen by the jurisdictions, each agent then decides where to live (ie. in jurisdiction 1 or jurisdiction 2) and how much income to earn (ie. how much to work). Note that we are assuming that all individuals are initially residence-free. By doing this, no individual has any particular ties to one jurisdiction over another. This is equivalent to assuming that all agents are fully mobile, with zero cost of moving.

We will further assume that the number of agents, \(B\), is sufficiently large so that when each individual makes income and residency decisions, their individual decisions do not have a significant impact on the levels of the public good provided in either jurisdiction.

We will assume for now, that the goal of each jurisdiction is to maximize the average utility of its residents. They do so by taking the income tax and public good levels of the other jurisdiction as given, because both \((y_1(\cdot), t_1(\cdot), z_1)\) and \((y_2(\cdot), t_2(\cdot), z_2)\) will be taken into consideration when agents decide where to live and how much to work. That is, each jurisdiction takes into consideration any mobility that would result from its choices. In this sense, we are employing a Nash equilibrium solution concept.

Before we can state the tax design problem, we must introduce some additional notation. Let \(B^k_{w_j}\) denote the number of type \(w_j\) individuals in jurisdiction \(k\), so that the fraction of individuals in jurisdiction \(k\) who are of type \(w_j\) is \(\frac{B^k_{w_j}}{\sum_{i=1}^n B^k_{w_i}}\). Note that if \(B^k_{w_j} = 0\), then no individuals of type \(w_j\) reside in jurisdiction \(k\). If \(B^k_{w_j} = B_{w_j}\), then all individuals of type \(w_j\) reside in jurisdiction \(k\), and \(B^1_{w_j} + B^2_{w_j} = B_{w_j}\) for all \(j = 1, \ldots, n\).

We are now ready to state the competitive tax design problem with local public goods. Jurisdictions \(k = 1, 2\) each solve,

\[
\max_{(y_k(\cdot), t_k(\cdot), z_k)} \sum_{j=1}^n u(w_j, y_k(w_j), t_k(y_k(w_j)), z_k) \frac{B^k_{w_j}}{\sum_{i=1}^n B^k_{w_i}}
\]

subject to

(1) (Feasibility) For every \(w_j \in W\),

\((y_k(\cdot), t_k(\cdot), z_k) \in S(Y(\cdot)) \times S(T(\cdot)) \times Z\)
(2) (Intrajurisdictional Incentive Compatibility) For every $w_j \in W$ such that $B_{w_j}^k > 0$, and every $y \in Y(w_j)$,

$$u(w_j, y_k(w_j), t_k(y_k(w_j)), z_k) \geq u(w_j, y, t_k(y), z_k)$$ (15)

(3) (Interjurisdictional Incentive Compatibility) For every $w_j \in W$ such that $B_{w_j}^k > 0$ and every $y \in Y(w_j)$,

$$u(w_j, y_k(w_j), t_k(y_k(w_j)), z_k) \geq u(w,y,t_{-k}(y),z_{-k})$$ (16)

(where $-k$ denotes the "other" jurisdiction)

(4) (Budget Balancedness)

$$\sum_{j=1}^{n} t_k(y_k(w_j))B_{w_j}^k = h(z_k)$$ (17)

taking $(y_{-k}(), t_{-k}(), z_{-k})$ as given.

Note that due to the competitive, multi-jurisdictional component to this problem, there are two types of incentive compatibility constraints, as compared with the one type in the single jurisdiction optimal income tax literature. This is because not only must agents of a given type who choose to reside in a given jurisdiction select the income level intended for them, but they must actually choose to live in that jurisdiction as well. That is, constraint (2) implies that for agents of a given type who choose to reside in jurisdiction $k$, they must be at least as well off choosing the income level that was intended for them (in jurisdiction $k$) than any other income level that is feasible for agents of their type. Note that constraint (2) need not hold for every agent type in every jurisdiction, but rather only for the agent types that actually choose to reside in a particular jurisdiction. That is, if for some $w_j \in W$ and some jurisdiction $k$, $B_{w_j}^k = 0$, then constraint (2) need not hold. This will become an important distinction later in this paper.

Constraint (3) implies that agents of a given type who choose to live in a jurisdiction would be at least as well off choosing the income level that was intended for them in that jurisdiction, than if they chose any feasible income level for them in the other jurisdiction.

Note that even though there are finitely many agent types, there are infinitely
may income levels available to each agent, i.e. $Y(w_j)$, because we cannot appeal to the Revelation Principle in the multiple principal, multiple agent setting.

Constraint (4), the budget balancedness constraint, requires that any feasible public sector mechanism $\{(y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\}$ generates revenues for each jurisdiction that exactly equal the costs of providing public goods $z_1$ and $z_2$, respectively. (see Berliant and Page (2006) for an explanation as to why these, and not simply financing constraints, are necessary)

Note that there are no individual rationality constraints in this problem. Although the agents do have a choice over two jurisdictions, all agents must decide to live in one of the two. That is, there is no outside option.

3. Initial Observations

Any solution to the competitive tax design problem, $\{(y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\}$, must satisfy the following:

[1] For any $w_i \in W$ such that $B_{w_i}^1 > 1$, if $B_{w_i}^1 > 0$ and $B_{w_i}^2 > 0$,

$$u(w_i, y_1(w_i), t_1(y_1(w_i)), z_1) = u(w_i, y_2(w_i), t_2(y_2(w_i)), z_2).$$

(18)

The reason here is obvious. If (1) did not hold, then the interjurisdictional incentive compatibility constraint would be violated.

Note that this does not necessarily imply that $(y_1(w_i), t_1(y_1(w_i))) = (y_2(w_i), t_2(y_2(w_i)))$, as it would if there were no public good and this were only a purely redistributal income tax.

Let

$$(y^*(w_i), t^*(y^*(w_i)), z^*) = \begin{cases} (y_1(w_i), t_1(y_1(w_i)), z_1) & \text{if } u(w_i, y_1(w_i), t_1(y_1(w_i)), z_1) \\ (y_2(w_i), t_2(y_2(w_i)), z_2) & \text{otherwise} \end{cases}$$

so that $(y^*(w_i), t^*(y^*(w_i)), z^*)$ represents the income level, tax liability, and public good level for a type $w_i$ agent in the jurisdiction in which they actually choose to live.

[2] Realized utility must be strictly increasing in type, i.e. for any $j > i$,
\[ u(w_j, y^*(w_j), t^*(y^*(w_j)), z^*) > u(w_i, y^*(w_i), t^*(y^*(w_i)), z^*) \].

**Proof.** Suppose not. That is, suppose that
\[ u(w_j, y^*(w_j), t^*(y^*(w_j)), z^*) \leq u(w_i, y^*(w_i), t^*(y^*(w_i)), z^*) \]. We know that
\[ Y(w_i) \subseteq Y(w_j) \], so \((y, \tau) \in K(w_i) \Rightarrow (y, \tau) \in K(w_j)\). In other words, a type \(w_j\) agent could choose \((y^*(w_j), t^*(y^*(w_j)), z^*)\) if they wanted to. By assumption (5) on the utility function, we know that
\[ u(w_j, y^*(w_j), t^*(y^*(w_j)), z^*) \geq u(w_i, y^*(w_i), t^*(y^*(w_i)), z^*) \],
which implies that \(u(w_j, y^*(w_j), t^*(y^*(w_j)), z^*) > u(w_i, y^*(w_i), t^*(y^*(w_i)), z^*)\).

\[ \Box \]

[3] Within a jurisdiction, \(k\), for any two types \(w_i, w_j\) such that \(B^k_{w_i} > 0\) and \(B^k_{w_j} > 0\), \(y_k(w_j) = y_k(w_i)\) if and only if \(c_k(w_j) = c_k(w_i)\), where
\[ c_k(w_j) = y_k(w_j) - t_k(\gamma_k(w_j)) \] and \(c_k(w_i) = y_k(w_i) - t_k(\gamma_k(w_i))\).

**Proof.** \((\Rightarrow)\) This direction is trivial.

\((\Leftarrow)\) Without loss of generality, assume that \(w_j > w_i\). Then by assumption, \(B^k_{w_j} > 0, B^k_{w_i} > 0\), and \(y_k(w_j) - t_k(\gamma_k(w_j)) = y_k(w_i) - t_k(\gamma_k(w_i))\). If \(y_k(w_j) \neq y_k(w_i)\), then either

\[ (1) y_k(w_j) > y_k(w_i) \]

or

\[ (2) y_k(w_i) > y_k(w_j). \]

If \(y_k(w_j) > y_k(w_i)\), then the type \(w_j\) agent would instead claim to be a type \(w_i\) agent. (This is feasible because \(Y(w_i) \subseteq Y(w_j)\)). This way they would enjoy the same level of consumption as before, but work less. By assumption (5) on the utility function, we know that a type \(w_j\) agent would choose to do this, and so the intra-jurisdictional incentive compatibility constraint for type \(w_j\) agents would be violated. If \(y_k(w_i) > y_k(w_j)\), then the type \(w_i\) agent would instead claim to be a type \(w_j\) agent. Since \(Y(w_i) = [0, m(w_i)], y_k(w_i) \in Y(w_i)\), and \(y_k(w_i) > y_k(w_j)\), we know that a type \(w_i\) agent could feasibly mis-report his type as \(w_j\). Since this would afford him the same level of consumption and more leisure, we again know by assumption (5) on the utility function that a type \(w_i\) agent would indeed prefer to misreport his type. Thus, the intra-jurisdictional incentive compatibility constraint for type \(w_i\) agents would be violated. \(\Box\)
The following two "observations" require the utility function to also satisfy the single crossing property. We will first state this property in \((y,c)\)-space, as this seems more intuitive, and will then transform the property to an equivalent statement in \((y,\tau)\)-space.

**A6)** (Single-crossing property in \((y,c)\)-space): In \((y,c)\)-space, indifference curves are upward sloping, with the slope along an indifference curve increasing with \(y\). The slope of the indifference curve through a given point \((y,c)\) decreases as \(w\) increases.

Since \(c\) represents after-tax income, interpreting the indifference curves for a particular type in \((y,c)\)-space is essentially equivalent to looking at indifference curves in \((l,c)\)-space, where \(l\) denotes labor. The reason that \((y,c)\)-space is used instead, is that once we want to compare indifference curves across types, the two spaces are notably different. This is because each \(y \in Y\) corresponds to a different amount of labor for each agent type.

The fact that indifference curves are upward-sloping in \((y,c)\)-space implies that in order to induce an agent to work more, they must be compensated with higher consumption.

The property that the slope along the indifference curves increases with \(y\) implies that the greater the level of income (or hours worked) for an individual of a particular type, the greater the increase in consumption (take-home pay) that would be needed to induce the agent to work a little more. In other words, leisure becomes increasingly valuable as agents enjoy less of it.

The fact that the slope of the indifference curve through a given point, \((y,c)\), increases with type is a direct consequence of the meaning of agent types. That is, for a particular level of income, \(y\), the amount of labor that is required to produce that income level is decreasing with type. Higher types do not need the marginal return to income in terms of consumption to be as high, because the increase in income requires less effort (labor hours) for them.
Figure 1 depicts a type $w_j$ agent’s indifference curves in $y, c$-space. The arrows denote the direction of increasing utility. If the type $w_j$ agent worked all of time and received the maximum allowable subsidy, $s$, their consumption (after-tax income) would be $m(w_j) - s$, hence the upper bound on $c$. Because $K(w_j)$
restricts us to non-bankrupting income-tax liability pairs, \( c \) is also bounded below by \( c = 0 \). Similarly, income is bounded by \( y \in [0, m(w_j)] \).

Figure 2 depicts indifference curves for two different types of agents. The black indifference curves are those of some type \( w_i \) agent, and the blue indifference curves are those of some type \( w_j \) agent. Since the blue indifference curves are relatively flatter than the black indifference curves, it must be the case that \( w_j > w_i \).

(In \((y, \tau)\)-space, indifference curves are concave, with the slope along an indifference curve decreasing with \( y \). The slope of the indifference curve through a given point \((y, \tau)\) increases with \( w_i \).)

![Figure 3](image_url)

**Figure 3:**

Figure 3 depicts a type \( w_j \) agent’s indifference curves in \( y, \tau \)-space. The arrows denote the direction of increasing utility. Both income and taxes are bounded above by \( m(w_j) \), the maximum amount an agent of type \( w_j \) can earn. Income is bounded below by zero, and taxes are bounded below by \( s \), the maximum allowable subsidy level.

[4] Within a jurisdiction, \( k \), for any two types \( w_j > w_i \) such that \( B^k_{w_i} > 0 \) and \( B^k_{w_j} > 0 \), \( C_k(w_j) \geq C_k(w_i) \).
However, it cannot be the case that both (y_k(w_i), y_k(w_i) - t_k(\gamma_k(w_i))) denote the income and consumption of a type w_i agent in jurisdiction k. Since w_j > w_i by assumption, 
(y_k(w_j), t_k(\gamma_k(w_i))) \in K(w_j), and so either 
(y_k(w_j), t_k(\gamma_k(w_i))) = (y_k(w_i), t_k(\gamma_k(w_i))), in which case c_k(w_j) = c_k(w_i), or 
u(w_j, y_k(w_j), t_k(\gamma_k(w_i)), z_k) > u(w_j, y_k(w_i), t_k(\gamma_k(w_i))), i.e. (y_k(w_j), c_k(w_j)) lies on an indifference curve for type w_j agents that is strictly above their indifference curve that goes through the point (y_k(w_i), c_k(w_i)) in the (y, c)-plane. However, it cannot be the case that both (y_k(w_j), t_k(\gamma_k(w_i))) \in K(w_i) and 
u(w_i, y_k(w_j), t_k(\gamma_k(w_i)), z_k) > u(w_i, y_k(w_i), t_k(\gamma_k(w_i)), z_k), because then type w_i agents’ intrajurisdictional incentive compatibility constraint in jurisdiction k would be violated. This argument is easiest to depict graphically.

FIGURES 5 and 6

[5] Within a jurisdiction, k, for any two types w_j > w_i such that \(B^k_{w_i} > 0\) and \(B^k_{w_j} > 0\), \(y_k(w_j) > y_k(w_i)\).

Proof. Suppose not. That is, suppose that \(y_k(w_i) > y_k(w_j)\). We know from (4) above, that consumption is increasing in type, so that 
y_k(w_j) - t_k(\gamma_k(w_j)) \geq y_k(w_i) - t_k(\gamma_k(w_i)). Furthermore, since 
y_k(w_i) > y_k(w_j), income y_k(w_j) is feasible for a type w_i agent. This means that a type w_i agent could consume more and work less if they claimed to be of type w_j. It must then be the case that \(y_k(w_j) \geq y_k(w_i)\) 

Definition 1 (Implementation and Financing). The income tax-public goods mechanism \{(y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\} is said to implement income tax functions \(t_1(\cdot)\) and \(t_2(\cdot)\) and exactly finance public goods \(z_1\) and \(z_2\) if and only if \{(y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\} satisfies constraints (1)-(4) of the competitive tax design problem, for \(k = 1, 2\).

Definition 2 (Efficiency). An income tax-public goods mechanism \{(y_1(\cdot), t_1(\cdot), z_1), (y_2(\cdot), t_2(\cdot), z_2)\} satisfying constraints (1)-(4) of the competitive tax design problem for \(k = 1, 2\) is efficient if an only if there does not exist
another income tax-public goods mechanism \[ \{(y_1(\cdot), t_1(\cdot), z_1^1), (y_2(\cdot), t_2(\cdot), z_2^2)\} \]
also satisfying the above mentioned constraints and such that
\[
 u(w_i, y^{\ast}(w_i), t^{\ast}(y^{\ast}(w_i)), z^{\ast}) \geq u(w_i, y^{\ast}(w_i), t^{\ast}(y^{\ast}(w_i)), z^{\ast})
\]
for all \( w_i \in W \), and
\[
 u(w_i, y^{\ast}(w_i), t^{\ast}(y^{\ast}(w_i)), z^{\ast}) > u(w_i, y^{\ast}(w_i), t^{\ast}(y^{\ast}(w_i)), z^{\ast})
\]
for some \( w_i \in W \).

**Definition 3** (Nash equilibrium). An income tax-public goods mechanism
\[ \{(y_1(\cdot), t_1(\cdot), z_1^1), (y_2(\cdot), t_2(\cdot), z_2^2)\} \] is a Nash equilibrium of the competitive tax
design problem if \( (y_1^{\ast}(\cdot), t_1^{\ast}(\cdot), z_1^1) \) solves the constrained maximization problem
for jurisdiction 1, taking \( (y_2^{\ast}(\cdot), t_2^{\ast}(\cdot), z_2^2) \) as given, and vice versa.

**Optimal Delegation Problem over Menus:**

**Definition 4** (menu). a set of income and tax liability pairs

Let \( C_1 \) and \( C_2 \) denote menus selected by jurisdictions 1 and 2, respectively.
In the delegated choice problem over menus, both jurisdictions will select a
menu and a level of the public good, \( \{(C_1, z_1), (C_2, z_2)\} \), and then the choice
over elements of the menus will be delegated to the agents. Each agent will then
select a single income-tax liability pair from one of the two menus.

Additional notation and definitions:

Let \( P_f(K) \) denote the collection of all nonempty, closed subsets of \( K \). By
Berge (1963), \( (P_f(K), d_h) \) is a compact metric space, where \( d_h \) denotes the
Hausdorff metric.

Any feasible pair of menus \( C_1, C_2 \in P_f(K) \) must satisfy \( \text{proj}_y(C_i) = Y \). This
is because neither jurisdiction can restrict agents’ income choices (although they
can restrict after-tax income).

As in Berliant and Page (2001,2006), we will define the following:

\[
\Lambda := \{ C \in P_f(K) : \text{proj}_y(C) = Y \}.
\]  
(19)
So, jurisdictions must select $C_i \in \Lambda$, $i = 1, 2$. Note that $\Lambda$ is nonempty and closed with respect to the Hausdorff metric (see Berliant and Page (2001)), which implies that $(\Lambda, d_h)$ is a compact metric space.

**The Delegated Choice Problem:**

Given $(C_1, z_1), (C_2, z_2) \in \Lambda \times z$, each agent solves

$$
\max_{k=1,2} \left[ \max_{(y_k, \tau_k) \in C_k \cap K(w_j)} u(w_j, y_k, \tau_k, z_k) \right].
$$

(20)

**Proposition 1.** For all $w_j \in W$ and all $(C_1, z_1), (C_2, z_2) \in \Lambda \times Z$, the delegated choice problem has a solution.

**Proof.** Since $C_k \in \Lambda$, both $C_k$ and $K(w_j)$ are nonempty, closed, and bounded. Thus, $C_k \cap K(w_j)$ is nonempty, closed, and bounded. By assumption, $u(w_j, \cdot, \cdot, \cdot)$ is continuous on $K(w_j) \times Z$. Then by Weierstrass’s Theorem, $\max_{(y_k, \tau_k) \in C_k \cap K(w_j)} u(w_j, y_k, \tau_k, z_k)$ has a solution for $k = 1$ and has a solution for $k = 2$. Finally, since any finite set has a maximum, the Delegated choice problem has a solution. $\square$

Let

$$
u^{\wedge k}(w_j, C_k, z_k) = \max_{(y_k, \tau_k) \in C_k \cap K(w_j)} u(w_j, y_k, \tau_k, z_k).$$

(21)

By Berliant and Page (2001), $u^{\wedge k}(w_j, \cdot, \cdot), k = 1, 2$, are continuous on $\Lambda \times Z$ for each $w_j \in W$, with respect to the product measure.

Let

$$
u^{\wedge \wedge}(w_j, C_1, z_1, C_2, z_2) = \max_{k=1,2} u^{\wedge k}(w_j, C_k, z_k).$$

(22)

**Proposition 2.** For each $w_j \in W$, $u^{\wedge \wedge}(w_j, \cdot, \cdot, \cdot, \cdot)$ is continuous on $\Lambda \times Z \times \Lambda \times Z$, with respect to the product measure.

**Proof.** First we will show that for any fixed $(\bar{C}_2, \bar{z}_2) \in \Lambda \times Z$, and any $w_j \in W$, $u^{\wedge \wedge}(w_j, \cdot, \cdot, \bar{C}_2, \bar{z}_2)$ is continuous on $\Lambda \times Z$. Let $\bar{u}^{\wedge 2} = u^{\wedge 2}(w_j, \bar{C}_2, \bar{z}_2)$. Then we have that for $(C_1, z_1) \in \Lambda \times Z$,
There exists a $d$ such that $u|_{\Lambda}$ shows that for all $(C_1, z_1) \in \Lambda \times Z$ and any $\epsilon > 0$, there exists a $\delta > 0$ such that $d_\pi((C_1, z_1), (C_1', z_1')) < \delta \Rightarrow |u|_{\Lambda}((w_j, C_1, z_1)) - u|_{\Lambda}((w_j, C_1', z_1'))| < \epsilon$. The problem can be divided into the following four cases:

**Case 1:** $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) > \tilde{u}^\Lambda_2$ and for all $(C_1', z_1')$ such that $d_\pi((C_1, z_1), (C_1', z_1')) < \delta$, $u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2) > \tilde{u}^\Lambda_2$. Then $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) = u|_{\Lambda}((w_j, C_1, z_1))$ and $u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2) = u|_{\Lambda}((w_j, C_1', z_1')) \Rightarrow |u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) - u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2)| < \epsilon$.

**Case 2:** $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) = \tilde{u}^\Lambda_2$ and for all $(C_1', z_1')$ such that $d_\pi((C_1, z_1), (C_1', z_1')) < \delta$, $u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2) = \tilde{u}^\Lambda_2$, $\Rightarrow |u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) - u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2)| = 0 < \epsilon$.

**Case 3:** $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) > \tilde{u}^\Lambda_2$ and for some $(C_1', z_1') \in \Lambda \times Z$, such that $d_\pi((C_1, z_1), (C_1', z_1')) < \delta$, $u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2) = \tilde{u}^\Lambda_2$, $\Rightarrow u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) = u|_{\Lambda}((w_j, C_1, z_1))$ and $u|_{\Lambda}((w_j, C_1', Z_1')) < u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2)$, since $u|_{\Lambda}((w_j, C_1, z_1)) - u|_{\Lambda}((w_j, C_1', z_1')) < \epsilon$ by continuity of $u|_{\Lambda}$. This gives us $u|_{\Lambda}((w_j, C_1, z_1)) - u|_{\Lambda}((w_j, C_1', z_1')) < \epsilon < \epsilon. \Rightarrow u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2)| < \epsilon$.

**Case 4:** $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) = \tilde{u}^\Lambda_2$ and for some $(C_1', z_1') \in \Lambda \times Z$, such that $d_\pi((C_1, z_1), (C_1', z_1')) < \delta$, $u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2) > \tilde{u}^\Lambda_2$. This implies that $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) \leq \tilde{u}^\Lambda_2$ and $u|_{\Lambda}((w_j, C_1', z_1'), \tilde{C}_2, z_2) > \tilde{u}^\Lambda_2$. Again by continuity of $u|_{\Lambda}$, we have that $u|_{\Lambda}((w_j, C_1, z_1)) - u|_{\Lambda}((w_j, C_1', z_1')) < \epsilon$, which implies that $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2) - u|_{\Lambda}((w_j, C_1', z_1), \tilde{C}_2, z_2)| < \epsilon$.

Thus, $u|_{\Lambda}((w_j, C_1, z_1), \tilde{C}_2, z_2)$ is continuous on $\Lambda \times Z$. A symmetric argument shows that $u|_{\Lambda}((w_j, C_1, z_1), \cdot, \cdot)$ is continuous on $\Lambda \times Z$.

Next we wish to show that for any $(C_1, z_1, C_2, z_2) \in \Lambda \times Z \times \Lambda \times Z$, all $w_j \in W$, and any $\epsilon > 0$, for all $(C_1', z_1', C_2', z_2') \in \Lambda \times Z \times \Lambda \times Z$, there exists a $\delta > 0$ such that $d_\pi((C_1, z_1), (C_1', z_1')) < \delta \Rightarrow |u|_{\Lambda}((w_j, C_1, z_1), C_2, z_2) - u|_{\Lambda}((w_j, C_1', z_1'), C_2', z_2')| < \epsilon$.

We know that for any $(C_1, z_1, C_2, z_2) \in \Lambda \times Z \times \Lambda \times Z$, any $w_j \in W$, and any $\epsilon > 0$, there exist $\delta_1$ and $\delta_2$ such that $d_\pi((C_1, z_1), (C_2, z_2)) < \delta_1 \Rightarrow |u|_{\Lambda}((w_j, C_1, z_1)) - u|_{\Lambda}((w_j, C_1, z_1))| < \epsilon$, and $d_\pi((C_2, z_2), (C_2', z_2')) < \delta_2 \Rightarrow |u|_{\Lambda}((w_j, C_2', z_2')) - u|_{\Lambda}((w_j, C_2, z_2))| < \epsilon$. 

Without loss of generality, assume \( u^{\wedge \exists}(w_j, C_1, z_1, C_2, z_2) = u^{\wedge \exists}(w_j, C_1, z_1) \).

Then

(1) If for all \((C'_1, z'_1, C'_2, z'_2) \in \Lambda \times Z \times \Lambda \times Z\) such that
\[
d_{\pi}((C'_1, z'_1, C'_2, z'_2), (C_1, z_1, C_2, z_2)) < \min\{\delta_1, \delta_2\},
\]
\[
u^{\wedge \exists}(w_j, C'_1, z'_1, C'_2, z'_2) = u^{\wedge \exists}(w_j, C'_1, z'_1),
\]
then
\[
| u^{\wedge \exists}(w_j, C'_1, z'_1, C'_2, z'_2) - u^{\wedge \exists}(w_j, C_1, z_1, C_2, z_2) | < \epsilon
\]
by the above continuity result.

(2) If for some \((C'_1, z'_1, C'_2, z'_2) \in \Lambda \times Z \times \Lambda \times Z\) such that
\[
d_{\pi}((C'_1, z'_1, C'_2, z'_2), (C_1, z_1, C_2, z_2)) < \min\{\delta_1, \delta_2\},
\]
\[
u^{\wedge \exists}(w_j, C'_1, z'_1, C'_2, z'_2) = u^{\wedge z}(w_j, C'_2, z'_2),
\]
then we have the following inequalities:

(i) \( u^{\wedge \exists}(w_j, C_1, z_1, C_2, z_2) = u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge z}(w_j, C_2, z_2) \),

and

(ii) \( u^{\wedge \exists}(w_j, C'_1, z'_1, C'_2, z'_2) = u^{\wedge \exists}(w_j, C'_2, z'_2) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \).

By the above continuity results, we also know that

(iii) \( | u^{\wedge \exists}(w_j, C_1, z_1) - u^{\wedge \exists}(w_j, C'_1, z'_1) | < \epsilon \),

and

(iv) \( | u^{\wedge z}(w_j, C_2, z_2) - u^{\wedge z}(w_j, C'_2, z'_2) | < \epsilon \).

All that remains to be shown is that
\[
| u^{\wedge \exists}(w_j, C_1, z_1) - u^{\wedge z}(w_j, C'_2, z'_2) | < \epsilon.
\]
(i) and (ii) \( \Rightarrow \) one of the following must hold

(a) \( u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge z}(w_j, C_2, z_2) \geq u^{\wedge z}(w_j, C'_2, z'_2) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \)

(b) \( u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge z}(w_j, C'_2, z'_2) \geq u^{\wedge z}(w_j, C_2, z_2) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \)

(c) \( u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge z}(w_j, C'_2, z'_2) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \geq u^{\wedge z}(w_j, C_2, z_2) \)

(d) \( u^{\wedge z}(w_j, C'_2, z'_2) \geq u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge z}(w_j, C_2, z_2) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \)

(e) \( u^{\wedge z}(w_j, C'_2, z'_2) \geq u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \geq u^{\wedge z}(w_j, C_2, z_2) \)

(f) \( u^{\wedge \exists}(w_j, C'_1, z'_1) \geq u^{\wedge \exists}(w_j, C_1, z_1) \geq u^{\wedge \exists}(w_j, C'_1, z'_1) \geq u^{\wedge \exists}(w_j, C_2, z_2) \)

For cases (a), (b), and (c), condition (iii) gives us the desired result, and for cases (d), (e), and (f), condition (iv) gives us the desired result. \( \square \)
For $k = 1, 2$, let

$$\Phi_k(w_j, C_k, z_k) = \{(y_k, \tau_k) \in C_k \cap K(w_j) : u(w_j, y_k, \tau_k, z_k) = u^\wedge_k(w_j, C_k, z_k)\}.$$  

(23)

$w_j \rightarrow \Phi_k(w_j, C_k, z_k)$ is the best response mapping for an agent of type $w_j$ in jurisdiction $k$.

Let

$$\Phi(w_j, C_1, z_1, C_2, z_2) = \{(\{y_1, \tau_1\} \in C_1 \cap K(w_j) : u(w_j, y_1, \tau_1, z_1) = u^\wedge(w_j, C_1, z_1, C_2, z_2)\} \cup \{(y_2, \tau_2) \in C_2 \cap K(w_j) : u(w_j, y_2, \tau_2, z_2) = u^\wedge(w_j, C_1, z_1, C_2, z_2)\}.$$  

That is, $w_j \rightarrow \Phi(w_j, C_1, z_1, C_2, z_2)$ is the best response mapping of an agent of type $w_j$ in our two jurisdiction economy.

We have

$$\Phi(w_j, C_1, z_1, C_2, z_2) = \begin{cases} 
\Phi_1(w_j, C_1, z_1) & \text{if } u^\wedge_1(w_j, C_1, z_1) > u^\wedge_2(w_j, C_2, z_2) \\
\Phi_2(w_j, C_2, z_2) & \text{if } u^\wedge_2(w_j, C_2, z_2) > u^\wedge_1(w_j, C_1, z_1) \\
\Phi_1(w_j, C_1, z_1) \cup \Phi_2(w_j, C_2, z_2) & \text{if } u^\wedge_2(w_j, C_2, z_2) = u^\wedge_1(w_j, C_1, z_1).
\end{cases}$$

It is easy to show that since both $\Phi_1(w_j, C_1, z_1)$ and $\Phi_2(w_j, C_2, z_2)$ are nonempty and compact (Berliant and Page (2001)), $\Phi(w_j, C_1, z_1, C_2, z_2)$ is also nonempty and compact for each $(w_j, C_1, z_1, C_2, z_2) \in W \times \Lambda \times Z \times \Lambda \times Z$.

As shown in Berliant and Page (2001), by the Kuratowski, Ryll-Nardzewski Theorem (Himmelberg (1975)), the following results hold.

1. Given any pair $(C_1, z_1) \in \Lambda \times Z$, there exists a measurable function $w_j \rightarrow (y_1(w_j), \tau_1(w_j))$ such that $(y_1(w_j), \tau_1(w_j)) \in \Phi_1(w_j, C_1, z_1)$ for all $w_j \in W$, where $\tau(w)$ is a direct tax function, and

2. Given any pair $(C_2, z_2) \in \Lambda \times Z$, there exists a measurable function $w_j \rightarrow (y_2(w_j), \tau_2(w_j))$ such that $(y_2(w_j), \tau_2(w_j)) \in \Phi_2(w_j, C_2, z_2)$ for all $w_j \in W$.

That is, for all $w_j \in W$, and any $(C_1, z_1) \in \Lambda \times Z$ and $(C_2, z_2) \in \Lambda \times Z$, there exist measurable functions $w_j \rightarrow (y_1(w_j), \tau_1(w_j))$ and $w_j \rightarrow (y_2(w_j), \tau_2(w_j))$ such that
\[ u(w_j), y_1(w_j), \tau_1(w_j), z_1 = u^{w_1}(w_j, C_1, z_1) = \max_{(y_1, \tau_1) \in C_1 \cap K(w_j)} u(w_j, y_1, \tau_1, z_1). \tag{24} \]

and

\[ u(w_j, y_2(w_j), \tau_2(w_j), z_2 = u^{w_2}(w_j, C_2, z_2) = \max_{(y_2, \tau_2) \in C_2 \cap K(w_j)} u(w_j, y_2, \tau_2, z_2). \tag{25} \]

Unlike in the single jurisdiction case of Berliant and Page (2001, 2006), for the multi-jurisdictional case we do not necessarily require that 
\[(y_1(w_j), \tau_1(w_j)) \in \Phi_1(w_j, C_1, z_1) \text{ for all } w_j \in W, \] but rather just for \( w_j \in W \) such that \( B^1_{w_j} > 0. \) Similarly, it need not be the case that 
\[(y_2(w_j), \tau_2(w_j)) \in \Phi_2(w_j, C_2, z_2) \text{ for all } w_j \in W, \] but just for \( w_j \in W \) such that \( B^2_{w_j} > 0. \)

The question now becomes whether or not we may restrict attention to 
\[\{(y_1(\cdot), \tau_1(\cdot)), (y_2(\cdot), \tau_2(\cdot))\} \text{ such that } (y_1(\cdot), \tau_1(\cdot)) \in \Phi_1(\cdot, C_1, z_1) \text{ and }\]
\[(y_2(\cdot), \tau_2(\cdot)) \in \Phi_2(\cdot, C_2, z_2) \text{ for all } w_j \in W.\] Any such \(\{(y_1(\cdot), \tau_1(\cdot)), (y_2(\cdot), \tau_2(\cdot))\}\) is an incentive compatible, direct public sector mechanism corresponding to 
\[\{(C_1, z_1), (C_2, z_2)\}.\]

By restricting to the above-mentioned direct public sector mechanisms, we loose those mechanisms such that for at least one agent type, \( w_i \in W, \) either \((y_1(w_i), \tau_1(w_i)) \notin \Phi_1(w_i, C_1, z_1) \) or \((y_2(w_i), \tau_2(w_i)) \notin \Phi_2(w_i, C_2, z_2). \) Note, this is a strict "or", as the two cannot both hold.

Without loss of generality, assume that \((y_1(w_i), \tau_1(w_i)) \notin \Phi_1(w_i, C_1, z_1).\) Then for incentive compatibility to still hold, it must be the case that \(B^1_{w_i} = 0.\)

We now claim that there is a strategically equivalent mechanism that does satisfy \(\Phi_1(w_i, C_1, z_1).\)

**Proposition 3.** For any incentive compatible, direct public sector mechanism corresponding to menu-public good pairs \((C_1, z_1), (C_2, z_2) \in \Lambda \times Z, \) there exists a strategically equivalent direct public sector mechanism,
\[\{(y'_1(\cdot), \tau'_1(\cdot)), (y'_2(\cdot), \tau'_2(\cdot))\}, \text{ satisfying } (y'_1(w_j), \tau'_1(w_j)) \in \Phi_1(w_j, C_1, z_1) \text{ for all } w_j \in W \text{ and } (y'_2(w_j), \tau'_2(w_j)) \in \Phi_2(w_j, C_2, z_2) \text{ for all } w_j \in W.\]
Proof. Forthcoming

Definition 5 (Budget balancedness). Menu-public good pairs 
\( \{(C_1, z_1), (C_2, z_2)\} \in \Lambda \times Z, \Lambda \times Z \) are said to be budget balancing if the best response mappings, \( w_j \rightarrow \Phi_1(w_j, C_1, z_1) \) and \( w_j \rightarrow \Phi_2(w_j, C_2, z_2) \) have measurable selections, \( (y_1(\cdot), \tau_1(\cdot)) \) and \( (y_2(\cdot), \tau_2(\cdot)) \), respectively, such that
\[
\sum_{j=1}^{n} \tau_1(w_j) B_{w_j}^1 = h(z_1) \quad \text{and} \quad \sum_{j=1}^{n} \tau_2(w_j) B_{w_j}^2 = h(z_2).
\]

Let \( R \) denote the set of budget balancing, menu-public good pairs \( \{(C_1, z_1), (C_2, z_2)\} \).

Competitive Menu Design Problem:

Each jurisdiction maximizes (separately) the average utility of its residents, taking the contract of the other jurisdiction as given. That is, each jurisdiction solves
\[
\max_{(C_k, z_k) \in R} \sum_{j=1}^{n} u^{w_j}(w_j, C_1, z_1, C_2, z_2) \frac{B_{w_j}^k}{\sum_{i=1}^{n} B_{w_i}^k} \tag{26}
\]
taking \( (C_{-k}, z_{-k}) \) as given.
References:


