Migration, Remittances and Human Capital Formation

Hilcias E. Morán
E-mail: hmoransa@indiana.edu

Indiana University at Bloomington
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ABSTRACT

The paper highlights a difference between migrant and non-migrant agents’ decisions in a two-sector growth model with exogenous migration and endogenous remittances. The purpose of this paper is to study the effects of migration and remittances on human capital formation or equivalently effects on economic growth in the source countries. The central question of this paper is: under which conditions migration and remittances may contribute to accelerate human capital formation and hence economic growth or alternatively may slow down human capital accumulation in the source countries. From combining optimal remittances function with the conditions under which migration and remittances might have a positive or negative effect on economic growth, we find that migrants’ degree of altruism toward non-migrants, labor income-ratio between migrant workers and non-migrant and migration rate are critical parameters to assess migration (remittances) effects on economic growth. Finally the findings of calibrating the model for 8 Latin American countries suggest mixed effects of migration and remittances on economic growth, which depend, in some way, on the assumptions about how local labor markets respond to international migration.

Keywords: Economic growth, human capital, migration, remittances

1. Introduction (incomplete)

Almost 1 of every 10 persons living in more developed regions is a migrant. It is estimated that around 180 millions people or 3% of the world’s population are living in countries in which they were not born. In addition, by the end of the 1970s, remittances for all developing countries represented only around 0.5 while in 2004 it reached around 2.0 as percentage of GDP. Likewise, remittances represent one-third of the total financial flows to developing countries and the most important characteristic of remittances inflow is that remittances have steadily grown and have become the major international financial source for developing countries.

The purpose of this paper is to study the effects of migration and remittances on human capital formation or equivalently effects on economic growth in the source countries. We
investigate the central questions of this paper: under which conditions migration and remittances may contribute to accelerate human capital formation and hence economic growth in the source countries, or alternatively may slow down human capital accumulation in the source countries.

In this paper we construct a two-sector growth model with two types of agents (migrants and non-migrants), which combines recent theoretical microeconomic literature and macroeconomic literature on migration and remittances. When we allow remittance as a complement of labor income, migration with remittances enhances human capital formation through higher human capital investment\(^1\). Migration and remittances effect on economic growth not only depend on labor-income share and the own size of migration rate, but also on labor income ratio in the host country relative to labor income in the source country and, crucially, from migrants’ degree of altruism. In this model, where remittances are altruistically motivated and migration is exogenously determined, there are three critical parameters to assess the effect of migration and remittances on human capital formation and consequently on economic growth in the source countries. The first critical parameter is the migrants’ degree of altruism, which affect welfare in the source countries by means of its effect on the amount of remittances sent by migrants to the source country. Higher migrants’ degree of altruism implies higher amount of remittances per-migrant and therefore higher disposal income for non-migrant agent. Then higher disposal income means higher consumption, higher saving and higher human capital investment in the source country. Thus, higher migrants’ degree of altruism implies higher human capital formation (i.e. higher economic growth) and hence this means that non-migrants will be better-off.

The second important aspect of analysis in this framework is the migration rate. The migration rate affects the source country through two different channels. First, higher migration rate implies higher remittances received by the source countries. On other

\(^1\) A different approach to analyze migration and human capital formation is in Vidal (1998), who finds that a positive probability of emigration to a higher return to skill country provides incentive to invest in human capital. His approach contrasts with the presented in this paper, while the author argues that investment in human capital is motivated by the possibility to move to a higher wage country, we assume that migration affect human capital formation through departing workers and through remittances.
hand, migration rate negatively affects human capital accumulation in the source country through of its impact on local labor market, which signifies loss in output. In general, we could expect that migration rate negatively affect human capital formation in the source country when the labor income ratio between migrant workers and non-migrant workers is relative low (that is, closer to one) or migration rate is relatively high, (that means in this framework $\theta_2 \equiv 0.38$)\(^2\). Finally, the third critical factor in our model is the labor income-ratio in the host country relative to the income in the source country. This ratio, to the best of our understanding plays an important role because higher income ratio induce to higher remittances relative to the labor income in the source countries and also higher income ratio implies that a lower opportunity cost in terms of output provoked by departing workers. Thus, the differences in income between host country and source countries may magnify the effect of received remittances in the source country and diminish the negative effect of migration rate. In short, we might expect that the probability that migration and remittances positively affect a source country is higher when income-ratio is higher between the migrant workers in the host country and the non-migrant workers in the source country.

There are several contributions of this paper. First we find that optimal remittances function is an increasing function of migrants’ wages, is decreasing function of non-migrants’ wages and an increasing function of migration rate\(^3\)\(^4\). One of the main findings in this simple model with migration and remittances, which establishes the optimal condition under which migration and remittances have a positive effect on human capital.

\[
\Delta Q / Q = \left(\alpha L \sigma_\varepsilon \varphi + \alpha L \sigma / 2 + \beta m \right),
\]
where $m$ is the migration rate, $\alpha_L$ is the income share of labor, $\varepsilon = (1 - \alpha L) / \sigma$ where $\sigma$ is the elasticity of substitution from a CES production function and $R = \beta m$ denotes remittances as a function of population living abroad. He finds that if $m$ is not exceedingly large, that is, $m < \beta / \varepsilon \alpha L \equiv 0.43$, the welfare impact of additional out-migration is positive.

\(^3\)The relationship between remittances and migrants’ income and non-migrants’ income is similar to that derived by Lucas and Stark (1985), by McCormick and Wahba (2000), by Osili (2006) and by Amuedo-Dorantes and Pozo (2006).

\(^4\)Faini (2002, 2003) using a cross-country regression reports, where the ratio of remittances to GDP (or, alternatively, to the home country population) is the endogenous variable and the set of regressors includes the stock of migrants, the income per capita in the source country and the skilled composition of migrant workers, reports three facts that might support our theoretical findings. First, remittances are an increasing function of the stock of migrants. Second, remittances decline with income per capita in the source country. Third, remittances decline as the share of migrants with a tertiary education goes up.
capital formation in the host country. Given the rate of migration, these conditions
determine the borderline between positive and negative effect of migration and
remittances on economic growth in the source country. In particular, for a given
migration rate is valid to say that higher income ratio between migrant workers in the
host country and non-migrant workers in the source country requires a lower degree of
altruism to obtain a positive effect of migration and remittances on human capital
accumulation or equivalently on economic growth (i.e. for low-income countries the
altruism threshold is lower than middle-income countries). Likewise, in this paper we
find a set of theoretical consistent and testable conditions that link migrants’ degree of
altruism, migration rate, labor income in the host country, labor income in the source
country and labor share. Thus, those conditions might produce new lights for empirical
analysis to assess the relationship between migration, remittances and economic growth.
In addition, by evaluating different scenarios on how local labor markets react to
international migration, we provide new elements to asses how robust are the effects on
migration and remittances on economic growth in the source countries.

Because the potential implications of the findings, however, is important to note that at
the outset that the discussion ignores some very important issues related with migration
and remittances. For instance, by focusing in the impact of exogenous migration and
remittances in the source country, we ignore how those factors affect the host country and
how the migration decisions are made. Likewise, we ignore some of the short-run effects
of remittances in the source countries (i.e. economic activity, relative prices, on foreign
exchange, etc.). Moreover, we disregard the probability of migrants return home with a
higher human capital and we ignore the migrants’ saving effect in the source country.

2. A simple model with migration and remittances

In this section we construct a two-sector growth model of a small open economy with
perfect capital mobility inhabited by two kinds of agents belonging to each generation:
non-migrants (agent 1) and migrants (agent 2)\(^5\). In an attempt to highlight the effects of

\(^5\) Small open economy assumption is useful to simplify computations without affect the main findings of
this paper.
migration and remittances on human capital formation and therefore on economic growth in the source country, we make several simplifying assumptions. We consider an overlapping generation model in which agents live two periods. While migrants represent the fraction of total population who work in the host country, non-migrants represent the fraction of population who work in the source country. We assume that at the beginning of the first period of their life all individuals are endowed of the human capital necessary for being productive and they face an exogenous migration process, which occurs as a lottery without moving costs\textsuperscript{6}. Therefore, just starting the first period of their life a fraction of population migrates to a higher wage country while the rest of population does not. Migrants move to a higher wage country, where is assumed that immigrants represent only a small fraction of total population and hence they are unable to affect real wages in the host country\textsuperscript{7}. Similarly, we assume that migrants from a particular source country are homogeneous in terms of human capital while across source countries migrants are heterogeneous\textsuperscript{8}. Likewise, we assume that migrants do not carry physical capital from the source country to the host country (e.g. land). Whereas non-migrants only are altruistic toward their children, migrants are altruistic toward their children and toward non-migrant agents, whose wages are lower than migrants’ wages by assumption. In this economy there is a single good which can be consumed or invested, and the investment can be in physical capital and human capital. Prices are assumed the same across the source and the host country\textsuperscript{9}. Labor supply is assumed inelastic, there is no government and there is no population growth and the size of each generation is normalized to one. This section is organized as follows: first, we describe the firms’ technology used to produce the single good and human capital production function.

\textsuperscript{6} Alternative approaches to zero moving costs are Ilahi and Jafarey (1999), where by considering an extended family approach with migration costs and remittances, the authors assume that migration costs are financed by informal family loan, and Carrington, et. al. (1996), where they analyze a dynamic model of labor migration in which moving costs decrease with the number of migrants living in the host country. This second approach is used by Rapoport and Docquier (2006) to analyze remittances and economic inequality. In addition, Djajić (1989) considers a moving approach in which cost of moving to work in another country tend to rise with the age of the moving individual.

\textsuperscript{7} Here, we assume that the host country use a superior technology relative to the source country.

\textsuperscript{8} This assumption comes from the empirical observation. Specially, migrants across country in the United States have different skill levels and therefore different earnings. Thus, this assumption allows us to be more precise when we calibrate the model for specific source countries.

\textsuperscript{9} Djajić (1989) and Dutsmann (1997, 1999) consider international migration models where is assumed that relative prices are higher in the host country relative to prices in the source country.
Next, we describe the preferences and the maximization problem for migrants and non-migrants agents. Then, we solve for the optimal decision and finally we solve for human capital growth rate considering the local labor market responses to international migration\textsuperscript{10}.

2.1 Technology

Firms in the source countries use physical and human capital, and face a competitive output and inputs markets and maximize profits. These firms produce one single tradable good which can be exported or consumed domestically. While all firms in each source country are identical, firms across source countries use different technology. The technologies exhibit constant return to scale in both factors, are strictly increasing in both arguments and are strictly concave. Since we assume that the population, \( N \) is constant in the source countries, changes in human capital reflect changes only the net investment in human capital and changes in migration rate. Thus, the production function of a representative firm in the source country \( i \) take the form of Cobb-Douglas technology, which is given by the following equation:

\[
\tilde{Y}_{it} = A_i K_{it}^{\alpha_i} \tilde{H}_{it}^{(1-\alpha_i)}, \quad \text{for } i = 1, 2, \ldots, S
\]

Equations (1) is the production function in the source country \( i \) and production function in the host country, respectively. \( S \) represents an arbitrary number of source countries and \( K_{it}, \tilde{H}_{it} \) and \( A_i \) represent physical capital, human capital employed in the source country \( i \) and a constant factor productivity in the source country \( i \), respectively. Let define migration rate \( \theta_i^j \in [0,1] \), then equation (1) can be rewritten as

\[
\tilde{Y}_i = A_i (1-\theta_i^j)^{1-\alpha} K^{\alpha'} H^{(1-\alpha')} \quad \text{for } i = 1, 2, \ldots, S, \text{ which is the standard definition of human capital (that is, “effective” labor without migration)}\textsuperscript{11}.
\]

Since firms face competitive markets, the solution of firms’

\textsuperscript{10} We consider two scenarios, which can be seen as extreme cases. The first one, as a situation where workers that migrates are easily substituted either by new entrant workers to the labor market or by unemployed (or underemployed) workers (that is, there is no costs in terms of labor force or brain-drain). And the second scenario can be seen as equilibrium where there is brain-drain, in which workers are not substituted by home workers.

\textsuperscript{11} The discussion about the relationship between local labor markets and international migration is presented in section 2.4.
optimization problem is such that the return of each factor is equal to its marginal product; that is, the firm’s profit is equal to zero. Given the price of the single good and the prices of production factors, firms choose physical and human capital so as the first order conditions for the firms’ maximization problem are given by the following equations:

$$\alpha_i \frac{\bar{Y}_u}{K_u} = 1 + r_i, \quad \text{for } i = 1, 2, \ldots, S \quad (2)$$

$$\frac{(1 - \alpha_i)\bar{Y}_u}{H_u} = \bar{w}_u, \quad \text{for } i = 1, 2, \ldots, S \quad (3)$$

In equations (2) $r_i$ is the world real interest rate, which is given by international capital markets and in equations (3), $\bar{w}_u$ represents the real wage per efficiency unit of labor for non-migrant workers in the source country $i$. Likewise, just for calibration purposes let assume that immigrants from a specific source country work in specific sectors or areas with a particular technology in the host country (i.e. immigrants from Dominican Republic in the U.S. work at service sector in New York or Guatemalan immigrants in the U.S. work at agricultural sector in California) such that the earnings are different for migrants from different source countries. Thus, the Cobb-Douglas technology used by firms in the host country in which migrants from a individual source country work is:

$$\bar{Y}_u^* = A_i^* \theta_2^{(i-\alpha^*)} K_u^* H_u^* \theta_1^{(i-\alpha^*)} \theta_2^{(i-\alpha^*)}$$

Therefore, the first order conditions for a representative firm which employs migrants from the country $i$ are shown below

$$\alpha_i \frac{\bar{Y}_u^*}{K_u^*} = 1 + r_i, \quad \text{for } i = 1, 2, \ldots, S \quad (2')$$

$$\frac{(1 - \alpha_i)\bar{Y}_u^*}{H_u^*} = \bar{w}_u^*, \quad \text{for } i = 1, 2, \ldots, S \quad (3')$$

Where, $H_u^*$, $K_u^*$ and $\bar{w}_u^*$ denote migrants’ human capital from country $i$, capital stock used by firm that employs migrants form the source country $i$, and real wages per efficiency unit of labor for migrants from the source country $i$. Moreover, since we assume that firms in the host country use a superior technology relative to firms in the source countries, we have that $A_i^* > A_i, \forall i$. 


Similarly, human capital $h_{i,t}$ of a young individual in generation $t+1$ is produced using the technology,
\begin{equation}
\begin{aligned}
h_{i,t+1} = B_i \tilde{e}_{it}^{\gamma} h_{it}^{1-\gamma}, & \quad \text{for } i = 1, 2, \cdots, S \quad B_i > 0, \\
\end{aligned}
\end{equation}
\begin{equation}
\begin{aligned}
h_{i,t+1}^* = B_i e_{it}^{\gamma} h_{it}^{1-\gamma}, & \quad \text{for } i = 1, 2, \cdots, S \quad B^* > 0.
\end{aligned}
\end{equation}

Since there is no government in this economy in equations (4) and (5) like in Lucas (1988) and Glomm and Ravikumar (1992) with private education, it does not play any role in the provision of education. Hence, the human capital of each individual in generation $t+1$ in the source country $i$ is a function of private investment in education $\tilde{e}_{it}$ adjusted by migration rate and the parents human capital $h_{it}$ and a constant parameter $B_i$. Similarly, in the host country the human capital of an individual from country $i$ is a function of his human capital investment in the host country $e_{it}$, of the human capital inherited from country $i$ and an arbitrary constant $B^*$, which is common for all individuals.

### 2.2 Migrants and non-migrants maximization problem

In this small open economy there are two representative agents. Whereas non-migrants from country $i$ take remittances as given to solve their maximization problem, migrants from country $i$ take as given the non-migrants’ consumption to decide about altruistically motivated remittances. Hence, we use a non-cooperative solution, in which both migrants and non-migrants agents maximize their utility individually taking the other agent’s actions as given. Since non-migrants only work when young, they distribute the first period’s total income (labor income plus remittances) to consumption, investment in human capital of their offspring and saving. Thus, the budget constraint of a non-migrant agent from country $i$ for the period $t$ is
\begin{equation}
\begin{aligned}
c_{it,t} + e_{it} + s_{it} = \tilde{w}_{it} h_{it} + a_{it}, & \quad \text{for } i = 1, 2, \cdots, S
\end{aligned}
\end{equation}

where, $\tilde{w}_{it}$ represents the real wage per efficiency unit of labor in the source country $i$ and $h_{it}$ represents the human capital of an individual belonging to generation $t$, $e_{it}$ is non-migrants’ current consumption, $e_{it}$ denotes investment in education and $s_{it}$ is saving.
required to finance their own consumption when old \( c_{n,t+1} \). Therefore, the budget constraint of an old non-migrant agent in country \( i \) is
\[
c_{n,t+1} = (1 + r_{t+1})s_{n} \quad \text{for } i = 1, 2, \cdots, S \tag{7}
\]
Young migrants from country \( i \) allocate their labor income to current consumption in the host country \( c_{i,t}^* \), saving \( s_{i}^* \), investment in human capital of their children in the host country \( e_{i,t}^* \), and sending altruistically motivated remittances \( a_{i} \) to non-migrant agent in country \( i \). Hence, the budget constraint of a young migrant agent at period \( t \) is
\[
c_{i,t}^* + e_{i,t}^* + s_{i,t}^* + a_{i} = \bar{w}_{i}^* h_{i,t}^* \quad \text{for } i = 1, 2, \cdots, S. \tag{8}
\]
In equation (8) \( \bar{w}_{i}^* \) and \( h_{i,t}^* \) denote the real wages per efficiency unit of labor in the host country of migrants from country \( i \) and migrants’ human capital level inherited from the source country \( i \), respectively. All migrants return home when old carrying their saving\(^{12}\). Thus, the budget constraint for an old migrant agent from country \( i \) is
\[
c_{i,t+1}^S = (1 + r_{t+1})s_{i}^* \quad \text{for } i = 1, 2, \cdots, S, \tag{9}
\]
where \( c_{i,t+1}^S \) represents migrants’ consumption in the source country when old. Agents take as given good prices, wages, human and physical capital returns, their initial wealth and maximize their utility functions, \( u(c) \), which are assumed the same functional form for both migrants and non-migrants agents. In addition, we assume a utility function that is strictly increasing and concave, twice differentiable and satisfy the Inada conditions. Thus, the migrants’ maximization problem from country \( i \) take the following form

\[
\text{P1.A.} \quad \text{Max } \ln c_{i,t}^* + \delta^i \ln c_{i,t} + \beta \ln(c_{i,t+1}^S) + \eta^* \ln h_{i,t}^* \quad \text{for } i = 1, 2, \cdots, S \tag{10}
\]
s.t.
\[
c_{i,t}^* + e_{i,t}^* + a_{i} + \frac{c_{i,t+1}^S}{(1 + r_{t+1})} = \bar{w}_{i}^* h_{i,t}^* \tag{11}
\]
\[
h_{i,t+1}^* = B^* e_{i}^{* \gamma} h_{i,t}^{* 1-\gamma} \quad \text{for } B^* > 0, \tag{5'}
\]

\(^{12}\) Due to the small open assumption with perfect capital mobility, we do not give a relevant role to the saving carrying by migrants when they return home. However, in a future extension of this paper where we might drop the assumption of perfect capital mobility, it could allow only a fraction of migrants to return home, which may be a more realistic assumption.
Migrants derive utility from consumption when young in the host country $c_{it,j}$, from consumption when old in the source country $i$ $c_{it,j+1}$, from human capital of their children $h_{it,j+1}$, and from non-migrants’ consumption when young $c_{it,j}$. Likewise, $\beta \in (0,1)$ is a constant discount factor, $\delta_i^* > 0$ and $\eta^* > 0$ represent the migrants’ degree of altruism toward non-migrants and toward their children, respectively. Equation (11) is the intertemporal migrants’ budget constraint, which comes from combining migrants’ budget constraints when young and old. Equation (5’) represents migrants’ human capital production function. Moreover, conditions (12) say that migrants’ consumption and human capital investments are constrained to be strictly positive and remittances cannot be negative. In order to complete the specification of this model, the non-migrants’ maximization problem is as follows

$$\max_{\{c_{it,j}, c_{it,j+1}, e_{it}\}} \ln(c_{it,j}) + \beta \ln(c_{it,j+1}) + \eta \ln h_{it+1}, \quad \text{for } i = 1,2,\ldots,S$$ \tag{13}

s.t.

$$c_{it,j} + e_{it} + \frac{c_{it,j+1}}{(1 + r_{j+1})} = \tilde{w}_it + a_{it}$$ \tag{14}

$$h_{it,j+1} = B_i e_{it}^{\gamma} h_{it}^{1-\gamma}, \quad B_i > 0,$$ \tag{4’}

$$c_{it,j}, c_{it,j+1}, e_{it} > 0, \quad \text{given } a_{it} \geq 0.$$ \tag{15}

From non-migrants’ utility function, we see that non-migrants derive utility from consumption when young in the source country $i$ $c_{it,j}$, from consumption when old $c_{it,j+1}$, and from human capital of their children $h_{it,j+1}$. Equation (14) is the intertemporal non-migrants’ budget constraint in the source country $i$ and conditions (15) state that non-migrants’ consumption when young and old, and human capital investments are constrained to be strictly positive and remittances cannot be negative and taken as given.

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13 Migrants’ degree of altruism can vary across source countries, which might be supported by the remittances data across countries.
2.3 Optimal remittances and their relationship with optimal consumption, saving and human capital investment in the source country

By computing the first order conditions of the migrants’ maximization problem (P1.A) with respect to $c_{it}$, which denotes consumption in the host country when migrants are young, respecting to $c_{it+1}$, that represents consumption in the source country when migrants are old, respecting to $a_{it}$, which is altruistic motivated remittances and after some manipulations, and respecting to non-migrants consumption $c_{it}$, we get the following equations

$$
c_{it}^* = \left( \frac{1}{1 + \beta + \delta + \eta \gamma} \right) \left[ \hat{w}_{it} \hat{h}_{it} + \left( \hat{w}_{it} \hat{h}_{it} - s_{it} - e_{it} \right) \right]
$$

(16)

$$
e_{it}^* = \left( \frac{\eta \gamma^*}{1 + \beta + \delta + \eta \gamma} \right) \left[ \hat{w}_{it} \hat{h}_{it} + \left( \hat{w}_{it} \hat{h}_{it} - s_{it} - e_{it} \right) \right]
$$

(17)

$$
a_{it} = \left( \frac{1}{1 + \beta + \delta + \eta \gamma} \right) \left[ \delta \hat{w}_{it} \hat{h}_{it}^* - \left( 1 + \beta + \eta \gamma \right) \left( \hat{w}_{it} \hat{h}_{it} - s_{it} - e_{it} \right) \right]
$$

(18)

$$
s_{it}^* = \left( \frac{\beta}{1 + \beta + \delta + \eta \gamma} \right) \left[ \hat{w}_{it} \hat{h}_{it}^* - \left( \hat{w}_{it} \hat{h}_{it} - s_{it} - e_{it} \right) \right]
$$

(19)

Thus, equation (16) denotes migrants’ optimal consumption in the host country, equation (17) is the migrants’ optimal investment in human capital in the host country, equation (18) indicates migrants’ optimal remittances sent to non-migrant agent, and equation (19) is optimal migrant’s saving. To verify those solutions, observe that the RHS term inside the brackets in equations (16)-(19) comes from migrants’ budget constraint when young $\hat{w}_{it} - s_{it} - e_{it} = c_{it} - a_{it}$. Thus, by using $c_{it} - a_{it}$ instead of $\hat{w}_{it} - s_{it} - e_{it}$ in equation (18) and solving for $a_{it}$ and then plugging that result in equation (16), we can return to the migrants’ first order condition with respect to $c_{it}$: $1/c_{it}^* = \delta_j / c_{it}$. This first order condition says that migrants’ marginal utility of consumption is equal to non-migrants’ marginal utility weighted by migrants’ degree of altruism from the source country $i$.

Now, by computing the first order conditions with respect to $c_{it}, c_{it+1}$ and $e_{it}$ from non-migrant’s maximization problem (P1.B) and after some algebraic manipulations, we get
From this set of equations, we can see that when remittances increases (decreases) unambiguously increases (decreases) consumption, human capital investment and saving of non-migrant agent in the source country.

Then, plugging (21) and (22) into (18) and solving for remittances, we find remittances as function of migrants’ labor income and non-migrants labor income,

\[
a_{it} = \phi^i \left[ \delta^i (1 + \beta + \eta \gamma^*) \tilde{w}_{it}^* h_{it}^* - (1 + \beta + \eta^* \gamma^*) \tilde{w}_{it} h_{it} \right]
\]

where \( \phi^i \equiv 1 / [(1 + \beta + \eta^* \gamma^*) + \delta^i (1 + \beta + \eta \gamma)] \). Since remittances cannot be negative, then the term inside brackets in (20) must be greater than zero. In particular, equation (23) implies that \( a_{it} = \max \{ \delta^i (1 + \beta + \eta \gamma^*) \tilde{w}_{it}^* h_{it}^* - (1 + \beta + \eta^* \gamma^*) \tilde{w}_{it} h_{it}, 0 \} \). Let \( a_{it} = a(w_i, w_i^*, \delta_i, \eta^*) \), then equation (23) means that \( \partial a / \partial \tilde{w}_i < 0, \ \partial a / \partial \tilde{w}_i^* > 0, \ \partial a / \partial \delta_i > 0, \partial a / \partial \eta^* < 0 \) and \( \partial a / \partial \eta > 0 \). That is, remittances are an increasing function of migrants’ wages, migrants’ degree of altruism toward non-migrants, and non-migrants’ degree of altruism toward their kids. Moreover, remittances are a decreasing function of non-migrants’ wages and degree of altruism toward migrants’ children are higher. Finally, from (23) and (3) and (3’), we obviously see that remittances also will be higher when higher is the migration rate.

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14 Notice that \( \delta^i \), the degree of altruism toward non-migrants consumption must satisfy

\[
\delta^i > \frac{(1 + \beta + \eta^* \gamma^*) \tilde{w}_{it}^* h_{it}^*}{(1 + \beta + \eta^* \gamma^*) \tilde{w}_{it} h_{2i}^*}
\]


16 Faini (2002, 2003) using a cross-country regression reports, where the ratio of remittances to GDP (or, alternatively, to the home country population) is the endogenous variable and the set of regressors includes the stock of migrants, the income per capita in the source country and the skilled composition of migrant
Finally, by plugging (23) into (20)-(22), we get non-migrants’ consumption, human capital investment and saving as a function of non-migrants’ labor income, migrants’ labor income and migrants’ degree of altruism toward non-migrants.

\[
e^{it}_{it} = \left( \frac{\delta^{it}}{(1 + \beta + \eta^* \gamma^*) + \delta^{it}(1 + \beta + \eta \gamma)} \right) \left( \bar{w}^* h_{it} + \bar{w}^* h_{it}^* \right) \tag{24}
\]

\[
e^{it}_{it} = \left( \frac{\delta^{it} \eta \gamma}{(1 + \beta + \eta^* \gamma^*) + \delta^{it}(1 + \beta + \eta \gamma)} \right) \left( \bar{w}^* h_{it} + \bar{w}^* h_{it}^* \right) \tag{25}
\]

\[
e^{it}_{it} = \left( \frac{\delta^{it} \beta}{(1 + \beta + \eta^* \gamma^*) + \delta^{it}(1 + \beta + \eta \gamma)} \right) \left( \bar{w}^* h_{it} + \bar{w}^* h_{it}^* \right) \tag{26}
\]

Thus, non-migrants’ consumption, human capital investment and saving are increasing function of non-migrants’ and migrants’ labor income and of migrants’ degree of altruism. Expressions (24)-(26) show three novels results to asses the impact of migration and remittances on consumption, human capital investment and saving in the source countries. First, the migrants’ degree of altruism toward non-migrants is a crucial parameter to evaluate the effects of migration and remittances in the source country, which positively affect consumption, human capital investment and saving. A second novel element in those solutions, is that optimal consumption, human capital investment and saving in the source countries are all a function of both migrants’ labor income and non-migrants labor income. As we shall see later, the labor income relationship between the host country and the source country is also a critical factor in this analysis. Finally, from those expressions and from equations (3) and (3’), we clearly see that optimal $c^{it}$, $e^{it}$ and $s^{it}$ depends on migration rate as well. How migration rate affect non-migrants optimal decisions will be, in general, an ambiguous effect. This effect is analyzed in the next section. In short, to the best of our knowledge, we have consistently gotten non-migrants’ optimal solutions that integrate aspects of the analysis of migration and remittances that had not been analyzed in previous studies.

workers, reports three facts that might support our theoretical findings. First, remittances are an increasing function of the stock of migrants. Second, remittances decline with income per capita in the source country. Third, remittances decline as the share of migrants with a tertiary education goes up.
2.4 Migration, labor market response and stationary growth rate

How the local labor market responds to migration depend on particular characteristics of each labor market in the source countries. To sketch the phenomenon we follow Lucas (2005), who says “virtually all of the evidence indicates that tighter labor markets at home discourage departure”. Lucas (2005) analyzes labor supply responses, wage and unemployment responses, and capital intensity and output responses. He concludes:

“In the end, economic theory offers few unambiguous hypothesized effects upon local markets. Emigration probably does reduce labor supply overall, and more specifically reduces the availability of the departing labor categories, even in the longer run. Whether this results in increased wages or diminished unemployment in the market for workers, similar to those who are leaving, depends upon institutional barriers to wage flexibility in that particular market, upon the prevalence of surplus labor of this type, the role of international trade in the relevant product markets, ability of others to rapidly acquire skills or relocate residence to take up vacated positions, and the passage of time.”

Thus, previous to define equilibrium and to find the human capital growth rate in the source country, we analyze the relationship between migration and labor markets. When we allow remittance as a complement of labor income, migration with remittances enhances human capital formation through higher human capital investment\(^{17}\). On other hand, the specific features of labor market might play a relevant role to explain human capital accumulation in the source country as well. For instance, we would not expect the same effect of migration and remittances on human capital formation in a labor market where departing workers are easily replaced with no loss in output, and those labor markets where migrant workers are difficult or almost impossible to replace\(^{18}\). In fact, these two different cases indeed might represent two limit cases, the fully replaced migrant workers where there is no cost in terms of output (that is, an “optimistic view”)

\(^{17}\) A different approach to analyze migration and human capital formation is in Vidal (1998), who finds that a positive probability of emigration to a higher return to skill country provides incentive to invest in human capital. His approach contrasts with the presented in this paper, while the author argues that investment in human capital is motivated by the possibility to move a higher wage country, we assume that migration affect human capital formation through departing workers and through remittances.

\(^{18}\) Since we assume inelastic labor supply, we neglect to analyze migration and remittances effects on labor-leisure choice.
and the case where migrants workers are irreplaceable or difficult to replace (that is, a “pessimistic view” or brain-drain view). The “optimistic” case is related to the Lewis surplus-labor hypothesis. As cited by Taylor, et al (1996),

“In the theoretical world developed by Lewis (1954), for example, where migrant-sending areas are characterized by a surplus of workers and a perfectly elastic labor supply, the loss of human resources through migration does not provoke a production decline, nor does it exert upward pressure on wages.”

Related to this case, also Lucas (2005) says:

“…this may occur where migration rate is very low in relation to the overall labor market, where those departing were previously unemployed, or where departing employed workers are easily replaced through migration or training without significant decline in worker quality”.

On other hand, the “pessimistic” case in this setup would reflect the full-employment world without surplus labor or any other labor market imperfections. This case might be a situation in which there is a considerable difficulty to substitute departing workers either by a significant loss of highly skilled workers or by tightness in the local labor market. Hence, in addition to the standard definition of migration rate, there exist for each labor exporting country an “effective” migration rate, which we simplify at the following way. Let migration rate \( \theta \) being taken exogenously from the interval (0,1), then assume that for each local labor market there exists a parameter \( \lambda \in [0,1] \), which characterizes the nature of each labor market in the source countries, such that the “effective” migration rate \( \theta_2 \equiv \theta_2 \lambda \) is bounded by the interval \([0,\theta_1]\). Specifically, \( \lambda \) denotes the rate of departing workers that will not be replaced in the local labor market. In short, “effective” migration rate might represent the relevant migration rate to assess the migration cost in terms of output in the local labor markets19.

---

19 Notice that to get a better understanding about how local labor markets responds to migration is obviously necessary to asses each particular market, where we could consider the type of departing workers (e.g. high skilled or low skilled), how large in the migration rate, labor market participation rate (male and female), how large is the unemployment and underemployment rate, wages reaction to migration, etc. Therefore, by introducing the parameter \( \lambda \) here, we only simplify a complication that needs more attention when we analyze particular source countries. Hence, there exists the probability that migration affects wages and unemployment while at the same time labor market tightness affects migration. Because of the
Even though we would not anticipate the same effect of international migration when migrant workers are easily replaced by local workers or when migrant workers are difficult or impossible to replace, we use for equilibrium definition the standard full-employment view or “pessimistic view”. Nevertheless, for the calibration of the model in the last section of this paper, we use different scenarios to evaluate the impact of migration on local labor markets in the source countries considered in the study.

Therefore, we define equilibrium in the source country as follows. In this small open economy a competitive equilibrium is a sequence \( \{c_{it}, c_{it+1}, e_{it}, h_{it+1}\}_{t=0}^{\infty} \) that given \( \{w_i, w_i^*, r_i\} \), \( \{a_{it}\}_{t=0}^{\infty} \) and the migration rate \( \theta^i \in (0,1) \) satisfies equations (23)-(26), and a sequence \( \{K_{it}, H_{it}\}_{t=0}^{\infty} \) that given \( \{r, w_i\} \) satisfies equations (2) and (3) such that given the aggregate stocks of physical and human capital \( \{H_{i,0}, K_{i,0}\} \) the following equilibrium conditions are satisfied for a source country \( i \):

\[
(1 - \theta^i) s_{it} + \theta^i s_{it}^* = s_{it} \tag{27}
\]

\[
(1 - \theta^i) e_{it} = \tilde{e}_{it} \tag{28}
\]

\[
(1 - \theta^i) H_{it} = \tilde{H}_{it} \tag{29}
\]

\[
\theta^i H_{it} = \tilde{H}_{it}^* \tag{29'}
\]

\[
\tilde{H}_{it} + \tilde{H}_{it}^* = H_{it} = h_{it} \tag{29''}
\]

\[
H_{it+1} = (1 - \theta^i) Y_{it} B_i e_{it} H_{it}^{1-\gamma} \tag{30}
\]

\[
\tilde{Y}_{it} = A_i K_{it}^{-\alpha_i} \left[(1 - \theta^i) H_{it}\right]^{-\alpha_i} \tag{31}
\]

\[
\tilde{Y}_{it}^* = A_i K_{it}^{-\alpha_i} \left(\theta^i H_{it}^*\right)^{-\alpha_i} \tag{31'}
\]

In equilibrium condition (27) we assume that migrants return home in the second period of their life carrying with them without any cost the saving accumulated during the first period. Condition (28) is the human capital investment in the source country \( i \) adjusted
by migration rate, where we assume that migrants’ children do not return to the source
country and then migrant’s human capital investment in their children is a benefit for the
host country. While equilibrium condition (29) represents the “effective” labor supply in
the source country \(i\), condition (29’) denotes “effective” labor supply from the source
country \(i\) in the host country and (29’’) is just the aggregated human capital in the source
country. Likewise, conditions (30) motion law for human capital in the source country,
where we have included equilibrium condition (28) into equation (4). Finally, (31) and
(31’) state that good markets in the source countries and in the host country clear,
respectively\(^{21}\).

Now, let come back to the firms’ maximization problem to find adjusted real wages by
“effective” migration rate. Because of the assumption of the small open economy with
perfect capital mobility\(^{22}\), physical capital in the source countries is completely
determined by equation (2) and assuming that real international interest rate is constant as
well \((r)\), there is no transitional dynamics in this economy, which implies that the
economy holds in its steady state growth path. Thus, from equations (2) and (2’) and
using (32) and (32’), we can obtain the constant relationship between physical capital and
human capital,

\[
\frac{K'}{H'} = \left(1 - \theta^i\right) \left[\frac{\alpha^i A_r}{(1 + r)}\right]^{\frac{1}{1-\alpha}}, \quad (32)
\]

\[
\frac{K'^*}{H'^*} = \theta^i \left[\frac{\alpha^i A_r^*}{(1 + r)}\right]^{\frac{1}{1-\alpha}} \quad (32')
\]

From equation (3) and (32) and using the assumption that host country has a superior
technology relative to source country, we get the following relationship between real
wages in the host country for migrants from the source country \(i\) \(\tilde{w}^*\); and real wages in
the source country \(\tilde{w}_i\), which will be constants over time:

\(^{21}\) Notice that we have not defined equilibrium conditions for migrants in the host country. The reasons to
do this is that we are interested in analyzing only effects of migration with remittances on human capital
formation in the source country and also the migrants from a specific country are assumed to be a very
small fraction of population in the host country.

\(^{22}\) By using small open economy assumption, we simplify computations and allow highlighting human
capital formation as the engines of economic growth in this economy.
\[ \bar{w}_i = \theta_2^i (1 - \alpha^*) A_i^* \left[ \frac{\alpha^* A_i}{1 + r} \right]^{1-\alpha} > (1 - \theta_2^i) (1 - \alpha^i) A_i \left[ \frac{\alpha^i A_i}{1 + r} \right]^{1-\alpha} = \bar{w}_i \quad (33) \]

Likewise, equilibrium conditions (31) and (31'), imply that real wages in the source country \( i \) and in the host country can be written
\[ \bar{w}_i = (1 - \theta_2^i)^{1-\alpha} (1 - \alpha^i) A^i (K_\mu / H_\mu)^{1-\alpha} = (1 - \theta_2^i)^{1-\alpha} w_i \quad (34) \]
\[ \bar{w}_i^* = \theta_2^i (1 - \alpha^*) A_i^* (K_\mu^* / H_\mu^*)^{1-\alpha} = \theta_2^i (1-\alpha^*) w_i^* \quad (34') \]

where \( w_i \) would be the real wages without migration. Thus, expression (34) shows the local labor market cost provoked by migration, which in this case is proportionally adjusted by the labor-income share in the source country. Likewise, equation (34') expresses the migrants’ real wages adjusted by migration rate from the source country \( i \) in the host country.

Now, using (25) into (4) and using equilibrium conditions (28), we get the recursive relationship for human capital in the source country
\[ h_{i+1} = B_i \left[ \frac{(1 - \theta_2^i) \eta \gamma \delta^i}{(1 + \beta + \eta^* \gamma^*) + \delta^i (1 + \beta + \eta \gamma)} \right] (\bar{w}_i + \bar{w}_i^*)^{\gamma'} h_i^{1-\gamma}. \quad (35) \]

Now, using the assumption that \( h_\mu = h_\mu^* \) (that is, homogeneous human capital for each source country) and imposing equilibrium condition \( h_\mu = H_\mu \) and we have that
\[ \frac{H_{i+1}}{H_\mu} = B_i \left[ \frac{(1 - \theta_2^i) \eta \gamma \delta^i}{(1 + \beta + \eta^* \gamma^*) + \delta^i (1 + \beta + \eta \gamma)} \right] (\bar{w}_i + \bar{w}_i^*)^{\gamma'} \quad (36) \]
Let \( 1 + \mu^i = H_{i+1} / H_\mu \) and plugging conditions (34) and (34’) we find that
\[ 1 + \mu^i = B_i \left[ \frac{\eta \gamma \delta^i}{(1 + \beta + \eta^* \gamma^*) + \delta^i (1 + \beta + \eta \gamma)} \right] (1 - \theta_2^i)^{(2-\alpha^i)} w_i + (1 - \theta_2^i)^{\gamma'} w_i^* \quad (37) \]

where \( w_i \) and \( w_i^* \) denote real wages per efficiency unit of effective labor supply in the source country and in the host country, respectively. Thus, the evolution of the economy is governed by the difference equation (35), which implies that output \( Y \), capital \( K \),
human capital $H$ grow to the same rate. Likewise, by aggregating the budget constraints (6) and (7), then it can be verified that consumption grows at the same rate as well.

**Assumption 1.** Assume that condition $\mu \geq 0$ holds for given $\theta_i^i > 0$, then

$$
\left( 1 - \theta_i^{i(2-\alpha)} w_i + (1 - \theta_i^{i(1-\alpha)} w_i^* \right) \geq \frac{1}{B_i^{1/\gamma}} \left( 1 + \beta + \eta^* \gamma^* \right) + \delta^i \left( 1 + \beta + \eta \gamma \right)
$$

must hold.

**Proposition 1.** Under the assumptions 1 we have that

1. \[ \frac{\partial (1 + \mu^i)}{\partial w_i} > 0, \quad \frac{\partial (1 + \mu^i)}{\partial \eta} > 0 \]
2. \[ \frac{\partial (1 + \mu^i)}{\partial w_i^*} > 0, \quad \frac{\partial (1 + \mu^i)}{\partial \eta^*} < 0 \]
3. \[ \frac{\partial (1 + \mu^i)}{\partial \delta^i} > 0 \]
4. \[ \frac{\partial (1 + \mu^i)}{\partial \theta_i^i} = 0 \quad \text{if and only if} \quad \kappa > \frac{(2 - \alpha^i) (1 - \theta_i^i)^{1-\alpha}}{[(1 - \alpha^i) (1 - \theta_i^i) / \theta_i^i \theta_i^{1-\alpha} - \theta_i^{1-\alpha}]}, \]

where \[ \kappa = \frac{w_i^*}{w_i} \]

**Proof.** All results come directly from differentiate equation (37) with respect to indicated parameters.

Proposition 1 reveals some standard results and some novel elements to the analysis of migration and remittances that to the best of our knowledge, they had not highlighted in previous studies. The first part of this proposition shows that human capital formation or economic growth is strictly increasing function with respect to non-migrants’ degree of altruism toward their children $\eta$ and with respect to non-migrants’ real wages $w_i$ in the source country $i$. The second part says that while economic growth rate is a increasing
function of migrants’ real wages $w_i^*$, it is inverse related with the migrants’ degree of altruism toward their children in the host country $\eta^*$. Then, economic growth rate increases when migrants’ degree of altruism toward non migrants increases (that is, higher remittances).

Migration and remittances effect on economic growth not only depend on labor-income share and the own size of migration rate, but also on labor income ratio in the host country relative to labor income in the source country $\kappa$ and, crucially, from migrants’ degree of altruism. In this model, where remittances are altruistically motivated and migration is exogenously determined, there are three critical parameters to asses the effect of migration and remittances on human capital formation and consequently on economic growth in the source countries. The first critical parameter is the migrants’ degree of altruism, which affect welfare in the source countries by means of its effect on the amount of remittances sent by migrants to the source country. Higher migrants’ degree of altruism implies higher amount of remittances per-migrant and therefore higher disposal income for non-migrant agent. Then higher disposal income means higher consumption, higher saving and higher human capital investment in the source country. Thus, higher migrants’ degree of altruism implies higher human capital formation (i.e. higher economic growth) and hence this indicates that non-migrants will be better-off.

The second important aspect of analysis in this framework is the migration rate. The migration rate affect the source country through two different channels. First, higher migration rate implies higher remittances received by the source countries, which increase the human capital investment as pointed out above (see equations 23 and 25)23. On other hand, migration rate negatively affects human capital accumulation in the source country through two different sub-channels. First, the effective investment in human capital decline because part of that investment is realized in the host country by migrants (equilibrium condition 28); however, the main impact comes from the loss in

23 In an empirical paper Faini (2002) finds that remittances are an increasing function of the stock of migrants, which might support our findings.
output for the lower effective labor supply\textsuperscript{24}. In general, we could expect that migration rate has a negative effect on human capital formation in the source country when the income ratio is closer to one or migration rate is relatively high, (that means in this framework $\theta_2 \equiv 0.38$)\textsuperscript{25}. Finally, the third critical factor in our model is the labor income-ratio in the host country relative to the income in the source country. This ratio, to the best of our understanding plays an important role because higher income ratio induce to higher remittances relative to the labor income in the source countries and also higher income ratio implies that a lower opportunity cost in terms of output provoked by departing workers.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Marginal effect of migration rate on economic growth given income-ratio}
\end{figure}

\textsuperscript{24} In addition, we could consider that the magnitude of this impact on economic growth will depend on the way how the local labor market responds to international migration, which is closely related to how large is the migration rate, the skill level of departing workers, whether departing workers were employed or unemployed, etc.

\textsuperscript{25} Faini (2002) estimates a cross-country regression of a modified formula of Borjas (1995)

\[ \frac{\Delta Q}{Q} = -\left(\alpha_L m^2 \varepsilon\right)/2 + \beta m, \]

where $m$ is the migration rate, $\alpha_L$ is the income share of labor, $\varepsilon = (1 - \alpha_L)/\sigma$ where $\sigma$ is the elasticity of substitution from a CES production function and $R = \beta m$ denotes remittances as a function of population living abroad. He finds that if $m$ is not exceedingly large, that is, $m < \beta/\varepsilon \alpha_L \equiv 0.43$, the welfare impact of additional out-migration is positive.
Thus, the differences in income between host country and source countries may magnify the effect of remittances in the source country and diminish the negative effect of migration rate. In short, we might expect that the probability that migration and remittances positively affect a source country is higher when income-ratio is higher between the host country and the source country. In fact, part iv of proposition 1 seems to show those results. In figure 1 we show the marginal effect of migration rate on economic growth for two different levels of income-ratio (i.e. $\kappa = 5$ and $\kappa = 2$).

As a result, from a theoretical point of view, migration and remittances have an ambiguous effect on welfare in the source country. Thus, migration and remittances have two opposite effects on welfare in the source country. First, there is a negative effect which comes from the likely tightness condition in the local labor market, which implies that some workers might be not replaced by local market and consequently it might mean a loss in output. In contrast to the first effect, there is a positive effect which comes from a higher disposal income (i.e. non-migrant labor income plus remittances) that increases consumption, human capital investment and saving in the source country. To see the net effect we develop a simple methodology which is described in the next section.

3. Steady state growth rates: with migration and remittances, and without migration and remittances

In this section we shall investigate the central questions of this paper: under which conditions migration and remittances may contribute to accelerate human capital formation and hence economic growth in the source countries, or alternatively may slow down human capital accumulation in the source countries. We will consequently compare the stationary growth rate at equilibrium in the model with migration and remittances, and the stationary growth rate at equilibrium in a model without remittances,

---

26 This is a familiar result in previous static analysis. For instance, Ribera-Batiz (1986) and Djajic (1986) using a static setup with tradable and non-tradable goods and where non-tradable sector is labor-intensive sector find that migration an remittances have an ambiguous effect on welfare in the source country. Djajic (1986) finds that whether remittances are higher than a critical value, then the source county will be better-off. Moreover, McCormick and Wahba (2000) in a dynamic framework find similar results.

27 In this section and in the rest of the paper we omit subscript $i$ to identify source countries, but the previous identification apply.
which is called the benchmark model. Thus, the maximization problem for the benchmark model without migration and remittances can be written as follows

\[ \text{Max} \ln (w^b h^b_t - e^b_t - s^b_t) + \beta \ln (R s^b_t) + \eta^b \ln B e^b h^b_t^{1-(1-\gamma)} \],

where we have substituted equations (6)-(7) and (4) into (13), we have imposed \( a_i = 0 \) and we have dropped subscript 1 and aggregated superscript \( b \), which identify variables in the benchmark model. Then, the solution for consumption, human capital investment and saving in the source country is given by the set of equations

\[
\begin{align*}
    c^b_{i,t} &= \left( \frac{1}{1 + \beta + \eta^b \gamma^b} \right) (w^b h^b_t) \\
    e^b_{i,t} &= \left( \frac{\eta^b \gamma^b}{1 + \beta + \eta^b \gamma^b} \right) (w^b h^b_t) \\
    s^b_{i,t} &= \left( \frac{\beta^b}{1 + \beta + \eta^b \gamma^b} \right) (w^b h^b_t)
\end{align*}
\]

(38)

(39)

(40)

Now, plugging (40) into (5'') and imposing equilibrium condition \( H^b_t = h^b_t \), we can get the growth rate for human capital accumulation in this small open economy

\[ 1 + \mu^b = B \left( \frac{\eta^b \gamma^b}{1 + \beta + \eta^b \gamma^b} w^b \right) \]

(41)

where \( w^b \) represents the real wage per efficiency unit of labor without migration. Since the ratio between physical capital and human capital is constant (equation (32)), it follows that both factors grow at the same rate. Therefore, output grows at the same rate and from the aggregate budget constraint can be verified that consumption grows at the same rate as well.

As an illustration, we consider three different scenarios to investigate the effect of migration and remittances on human capital formation in the source country, which allow us to do a sensibility analysis of the results with respect to the way how the local labor market responds to international migration. Therefore, in this section we retake the discussion about labor market and migration presented in the previous section. Recall that we defined “effective” migration rate \( \theta^e_2 = \theta_2 \lambda \in [0, \theta_2] \), where \( \lambda \in [0,1] \) is a
parameter that characterizes the labor market reaction to international migration. First, we relate the “optimistic” case to the lower bound case when \( \lambda \to 0 \), which represents a case where all migrant workers are fully replaced by local workers. Then, we name “intermediate” case to the situation when \( \lambda = 1/2 \), where given the migration rate only one half of migrants workers are replaced by local workers. Finally, we link the “pessimistic” case to the upper bound case when \( \lambda \to 1 \), where all migrant workers are no replaced in the local labor market by local workers.

**Proposition 2.** Define \( \kappa \equiv (1 - \alpha^*)Y_r^*/(1 - \alpha)Y_i \geq 1 \) and assume that \( \eta = \eta^b \), \( \gamma = \gamma^b \), and \( \mu \geq 0, \mu^b \geq 0 \), then migration and remittances accelerate human capital formation in the source country if and only if the following conditions are satisfied:

i) In general, \( \delta > \frac{(1 + \beta + \eta^* \gamma^*)}{(1 + \beta + \eta \gamma)} \frac{1}{(1 - \theta \lambda)^{2 - \alpha} + (1 - \theta \lambda) \theta_2^{(1 - \alpha^*)} \kappa - 1} \), provided that \( (1 - \theta \lambda)^{2 - \alpha} + (1 - \theta \lambda) \theta_2^{(1 - \alpha^*)} \kappa > 1 \)

ii) In particular,

a. When \( \lambda \to 0 \), then \( \delta > \frac{(1 + \beta + \eta^* \gamma^*)}{(1 + \beta + \eta \gamma)} \frac{1}{\theta_2^{(1 - \alpha^*)} \kappa} \)

b. When \( \lambda \to (1/2) \), then

\[
\delta > \frac{(1 + \beta + \eta^* \gamma^*)}{(1 + \beta + \eta \gamma)} \frac{1}{(1 - \theta / 2)^{2 - \alpha} + (1 - \theta / 2) \theta_2^{(1 - \alpha^*)} \kappa - 1}
\]

provided that \( (1 - \theta / 2)^{2 - \alpha} + (1 - \theta / 2) \theta_2^{(1 - \alpha^*)} \kappa > 1 \)

c. When \( \lambda \to 1 \), then \( \delta > \frac{(1 + \beta + \eta^* \gamma^*)}{(1 + \beta + \eta \gamma)} \frac{1}{(1 - \theta \lambda)^{2 - \alpha} + (1 - \theta \lambda) \theta_2^{(1 - \alpha^*)} \kappa - 1} \)

provided that \( (1 - \theta \lambda)^{2 - \alpha} + (1 - \theta \lambda) \theta_2^{(1 - \alpha^*)} \kappa > 1 \)

**Proof.** The proof of this proposition comes directly to compare equations (38) and (42). ■■■

Proposition 2 represents one of the main findings in this simple model with migration and remittances, which establishes the optimal condition under which migration and remittances have a positive effect on human capital formation in the host country. Given
the rate of migration, these conditions determine the borderline between positive and negative effect of migration and remittances on economic growth in the source country. In particular, for a given migration rate is valid to say that higher income ratio between migrant workers in the host country and non-migrant workers in the source country requires a lower degree of altruism to obtain a positive effect of migration and remittances on human capital accumulation or equivalently on economic growth (i.e. for low-income countries the altruism threshold is lower than middle-income countries).

Given an arbitrary migration rate (i.e. 0.2) the relationship between $\delta$ and $\kappa$ is shown in figure (2), where the vertical axis represents the altruism degree and the horizontal axis shows arbitrary values for the labor income ratio between migrant workers in the host country and the non-migrant labor income in the source country. It shows that when labor income ratio is closer to one, more significant is the way how local labor market responds to migration. Figure 2 also shows that independently of the response of local labor market to international migration, the degree of altruism required for a positive effect on human capital accumulation in the source country declines when the labor income ratio in the host country to the labor income in the source country increases. Therefore, as long $\kappa \to \infty$, then $\delta \to 0$ and less relevant is the way how the labor market reacts to migration\textsuperscript{28}.

\textsuperscript{28} To get simulations shown in figure 2, without affect the central implications reflected in this figure, we assume that $\eta = \eta^*, \gamma = \gamma^*$, and $\alpha = \alpha^* = 0.4$. 

25
4. Calibration for several Latin American countries

4.1 Confronting theoretical results with data-inferred degree of altruism.

In this section we calibrate the theoretical model constructed in the previous section for some Latin American and Caribbean countries. To do this, we use the optimal conditions given in proposition 2, optimal remittances given in equation (23) and data from 8 Latin American and Caribbean countries. We calibrate conditions (a)-(c) from proposition 2, whose outcomes will give us the minimum level of migrants’ degree of altruism toward non-migrants required to warrantee a positive effect of migration and remittances on economic growth. Now, the arisen question is: how can we infer from the data a value for migrant’s degree of altruism that may be compared with the thresholds fond from calibrating proposition 2? We think that the answer for this question might be found in the equation for optimal amount of remittances given in equation (23). From this equation, we can solve for $\delta$ at equilibrium as a function of migrants’ labor income, non-migrants labor income, migration and remittances. Since remittances are an observed variable on the country-data, we can get a value for $\delta^*$, which reflects the migrants’
degree of altruism inferred from the data for each country considered in the analysis. Thus, by solving from equation (23), we find that
\[ \delta^i = \frac{(1 + \beta + \eta \gamma^*) \bar{w}_i h^*_u + a^*_u}{(1 + \beta + \eta \gamma) \bar{w}_i h^*_u - a^*_u} \] (42)

Using (34) and (34'), the assumption \( h_u = h^*_u \), defining \( a^* \equiv a_u / w_i h_u \), and dividing numerator and denominator of expression (42) by \( w_i h_u \), we can rewrite (42) as
\[ \delta^*_i = \frac{(1 + \beta + \eta \gamma^*) [(1-\theta^*_i \lambda)^{-\alpha'} + a^*_i]}{(1 + \beta + \eta \gamma) (\theta^*_i^{1-\alpha'} \kappa^i - a^*_i)} \] (43)

where \( \kappa \) is the labor income ratio between migrant workers from country \( i \) in the U.S. economy and non-migrant workers from country \( i \) in the source country. At this point, using expression (43) and right hand side of inequalities in proposition 2, we find the empirical-oriented conditions under which migration and remittances have positive effect on economic growth. These conditions are declared in the subsequent proposition.

**Proposition 3.** Under the assumption 1 and assumptions in proposition 2 we have that migration and remittances accelerate human capital formation in the source country \( i \) if and only if the following conditions are satisfied:

a. In general, \[ \frac{[(1-\theta^*_i \lambda)^{-\alpha'} + a^*_i]}{(\theta^*_i^{1-\alpha'} \kappa^i - a^*_i)} \geq \frac{1}{\left[(1-\theta^*_i \lambda)^{-\alpha'} + (1-\theta^*_i \lambda)\theta^*_i^{(1-\alpha')} \kappa^i - 1\right]} \]

b. In particular,

i. When \( \lambda \to 0 \), then \[ \frac{(1 + a^*_i)}{(\theta^*_i^{1-\alpha'} \kappa^i - a^*_i)} \geq \frac{1}{\left[(1-\theta^*_i \lambda / 2)^{-\alpha'} + (1-\theta^*_i \lambda)\theta^*_i^{(1-\alpha')} \kappa^i - 1\right]} \]

ii. When \( \lambda \to (1/2) \), then
\[ \frac{(1 + a^*_i)}{(\theta^*_i^{1-\alpha'} \kappa^i - a^*_i)} \geq \frac{1}{\left[(1-\theta^*_i \lambda / 2)^{-\alpha'} + (1-\theta^*_i \lambda)\theta^*_i^{(1-\alpha')} \kappa^i - 1\right]} \]

iii. When \( \lambda \to 1 \), then
\[ \frac{(1 + a^*_i)}{(\theta^*_i^{1-\alpha'} \kappa^i - a^*_i)} \geq \frac{1}{\left[(1-\theta^*_i \lambda)^{-\alpha'} + (1-\theta^*_i \lambda)\theta^*_i^{(1-\alpha')} \kappa^i - 1\right]} \]
Notice that this proposition confronts the right hand side part of each inequality of proposition 2 with the altruism degree generated from the data by considering the current amount of remittances sent for migrants to the source country. Due to this, this proposition states a weaker condition than proposition 2 (i.e. ≥ instead of >). Proposition 3 states a set of theoretical consistent and testable conditions that link migrants’ degree of altruism, migration rate, labor income in the host country, labor income in the source country and labor share. Thus, those conditions might produce new lights for empirical analysis to assess the relationship between migration, remittances and economic growth. In addition, by evaluating different scenarios on how local labor markets react to international migration, we provide new elements to assess how robust are the effects on migration and remittances on economic growth in the source countries.

4.2 Calibration and empirical results

The countries included in the analysis are Dominican Republic, El Salvador, Guatemala, Haiti, Honduras, Mexico and Nicaragua. There are three reasons for including these countries. First, according to OECD database on foreign-born and expatriates the countries included in the analysis form part of the top 50 countries with highest migration rates. The migration rate for these 8 countries rages between 6.7% and 30.6%. Thus, while Jamaica which has a migration rate of 30.6% is the country number 11 in the top 50 highest-migration rate countries, Honduras which has a migration rate of 6.7% is the country number 46 in the same list. Second, according to the World Bank database these countries in 2000 represented the largest receiver of remittances in Latin American and Caribbean area. As showed in the figure 2, El Salvador is the largest recipient of remittances, which represents almost 20% of the labor income and in all countries the ratio remittances/labor income is higher than the overall Latin American and Caribbean area. Finally, because of a large part of migrants from these 8 countries live in the United States, it simplifies the data collection and allows us to clarify the fundamentally stated relationship between host country (U.S.) and source country (any of these 8 countries) in our theoretical model. Hence, as a host country we will employ data from U.S. economy and as a source country we will utilize data from each country. The data for remittances,

29 The data were obtained in most cases from the 2000 round decennial population censuses
population and GDP was collected from the World Bank’s World Development Indicators (WDI) database. For remittances, we use the WDI data reported by countries from their balance of payments and for GDP per-capita, we use the GDP, PPP (constant international $) per-capita\(^{30}\). Likewise, the data for migration was collected from OECD database on foreign-born and expatriates, which corresponds to the migration rate calculated by dividing the expatriate population (aged 15+) from the source country by the 15+ native-born population of the same country (native-born = expatriates + resident native). Thus table 1 and figure 3 summarize the data used to calibrate both sides of proposition 3.

\[\text{Figure 3. Income ratio, migration and remittances for several Latin American countries}\]

\(^{30}\) There is consensus in the literature that the quality of the data on remittances is poor. In fact, some studies suggest that a significant proportion – estimated from anything between 35 and 70 percent of official remittances – remain unrecorded. Unrecorded remittances are channeled through the informal sector and are not captured in official balance of payment statistics. On other hand, Lucas and Stark (1985) Osili (2006) and Amuedo-Orantes and Pozo (2006) suggest that the amount of recorded remittances include part of self-interest motivated remittances, which does not agree with altruistically motivated remittances used in the theoretical model. At least both data issues affect the quality of the data at opposite direction.
### Table 1

<table>
<thead>
<tr>
<th>Source countries</th>
<th>( a^*<em>{i} = a</em>{i} / (1 - \alpha) Y_{i} )</th>
<th>( \theta^i_z )</th>
<th>( \kappa^i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dominican Republic</td>
<td>0.129</td>
<td>0.110</td>
<td>3.63</td>
</tr>
<tr>
<td>El Salvador</td>
<td>0.196</td>
<td>0.171</td>
<td>4.11</td>
</tr>
<tr>
<td>Guatemala</td>
<td>0.056</td>
<td>0.069</td>
<td>4.55</td>
</tr>
<tr>
<td>Haiti</td>
<td>0.171</td>
<td>0.088</td>
<td>11.56</td>
</tr>
<tr>
<td>Honduras</td>
<td>0.092</td>
<td>0.067</td>
<td>6.93</td>
</tr>
<tr>
<td>Jamaica</td>
<td>0.151</td>
<td>0.306</td>
<td>7.26</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.021</td>
<td>0.113</td>
<td>2.03</td>
</tr>
<tr>
<td>Nicaragua</td>
<td>0.096</td>
<td>0.070</td>
<td>6.30</td>
</tr>
<tr>
<td>Host country</td>
<td>( Y_{i} )</td>
<td>31,824.6</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\( a_{i} \) : Remittances received by source countries \( i \), average 1990-2004  
\( Y_{i} \) : GDP per-capita in the source country \( i \), average 1990-2004  
\( Y_{i} \) : U.S. GDP per-capita, average 1990-2004  
\( \theta^i_z \) : Migration rate  
\( \alpha = \alpha^* = 0.4 \)  

Since the education attainment varies across the 8 countries in the analysis, the migrants’ income in the U.S. also changes across country. In fact, the correlation between median earnings and the percentage of migrants with high school or higher education level for the 8 countries considered in this study is 0.92. Figure (4) shows the earnings of immigrants to U.S. native-born (left side of vertical axis) and the percentage of immigrants with high school or higher education by birth country. While immigrants from Jamaica have the highest education level and the highest median earnings, immigrants from Mexico have the lowest education level and the lowest median earnings. Therefore, in order to calibrate the migrants’ labor income, for labor migrants’ labor income from a specific country we compute the U.S. GDP per-capita adjusted by the ratio of the median earnings of migrants from that country to the median earnings of native-born. For the parameter \( \alpha \) and \( \alpha^* \), we use \( \alpha = \alpha^* = .4 \)

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31 That is, (labor income of migrant workers from the source country \( i \)) = (per capita U.S. GDP)*(median earnings ratio of immigrants from country \( i \) relative to median earnings of native-born in the U.S.)
Using the parameter values reported in table 1, we compute the conditions declared in proposition 3 for each one of the 8 source countries included in this analysis. The main results are reported in figure (5) and table 2. Figure (5) shows the minimum level of
migrants’ degree of altruism or threshold required to warrantee that migration for three different scenarios mentioned above, also show the migration rate and the labor income ration for migrant workers in the U.S. and non-migrant workers in the respective origin country. Figure (5) shows that higher income ratio requires higher migrants’ degree of altruism toward non-migrants (i.e. Mexico). Besides, for countries with lower income ratio, the way how the local labor market responds to international migration does not imply substantial changes in the degree of altruism (i.e. Haiti and Honduras). This is a very interesting result that might explain, partially, why even the high migration during the last years from the countries analyzed here to the U.S., the output on those economies have maintained a almost constant relationship in the last 15 years. On other hand, for countries like Mexico, the way how local labor market responds to migration seems to be significant. In addition, higher migration rate implies lower degree of altruism (i.e. El Salvador and Jamaica) and more sensibility to the mode how labor market reacts to international migration.

Finally, table (2) shows the main findings from calibrating the model for the 8 countries considered in this study. There are mixed results, while migration an remittances have a positive effect on all 8 countries when we assume that all departing workers are easily replaced in the local labor market (i.e “optimistic” case), only Haiti and El Salvador might be experimented positive effects on economic growth for the specific calibration of the parameters used in table 2 (see table1). Likewise, if we assume an intermediate response of local labor markets only Jamaica and Mexico have negative effect. These might obey to two reasons. First Mexico has the lowest income ratio (that is, ≅ 2) and Jamaica has the highest migration rate within the sample of countries. Then, if we see figure (3), we can note that Mexico and Jamaica also have the highest ratio of migration rate to remittances, which might signify that the migrants’ degree of altruism is the lowest for those countries. To understand these results we would do a deeper analysis of Mexican and Jamaican cases, in which we have to include skill level of migrants and non-migrants, unemployment and underemployment in the local labor markets, whether there is a temporal or permanent migration, if migrants leave the country carry with their relatives, etc. In addition, even though in this study we include as migrants’ labor income
the earnings of migrants from different countries, which correspond to the migrants skill level (see figure 4), we did not use an equivalent measure for non-migrant labor income. Instead to do that, we use the per capita income, which for the Mexican case might be higher because the median of non-migrant Mexican workers have a higher skill level than the migrant Mexican workers. In short, the results require more a detailed analysis.

<table>
<thead>
<tr>
<th>Table 2</th>
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<tbody>
<tr>
<td>Country</td>
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<tr>
<td>---------</td>
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<tr>
<td><strong>&quot;Optimistic&quot; case</strong></td>
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<tr>
<td>El Salvador</td>
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<td>Guatemala</td>
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<td>Honduras</td>
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<tr>
<td>Mexico</td>
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<tr>
<td>Nicaragua</td>
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<tr>
<td>Dominican Republic</td>
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<tr>
<td>Haiti</td>
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<tr>
<td>Jamaica</td>
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<tr>
<td><strong>&quot;Intermediate&quot; case</strong></td>
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<tr>
<td>El Salvador</td>
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<td>Guatemala</td>
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<td>Honduras</td>
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<td>Dominican Republic</td>
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<td>Haiti</td>
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<tr>
<td>Jamaica</td>
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<tr>
<td><strong>&quot;Pessimistic&quot; case</strong></td>
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<td>El Salvador</td>
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<td>Guatemala</td>
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<td>Jamaica</td>
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5. Conclusion
In this paper we construct a two-sector growth model with exogenous migration and remittances endogenously determined to analyze the effects of migration and remittances on economic growth. Using this setup we find the conditions under which migration and remittances have a positive effect on economic growth and then we calibrate the model for several Latin American countries. The underlying results show that migration and
remittances effect on economic growth in the source countries not only depend on labor-income share and the own size of migration rate, but also on labor income ratio in the host country relative to labor income in the source country and, crucially, from migrants’ degree of altruism. In this model, where remittances are altruistically motivated and migration is exogenously determined, there are three critical parameters to assess the effect of migration and remittances on human capital formation and consequently on economic growth in the source countries. The first critical parameter is the migrants’ degree of altruism, which affect welfare in the source countries by means of its effect on the amount of remittances sent by migrants to the source country. Higher migrants’ degree of altruism implies higher amount of remittances per-migrant and therefore higher disposal income for non-migrant agent. Then higher disposal income means higher consumption, higher saving and higher human capital investment in the source country.

The second important aspect of analysis in this framework is the migration rate. The migration rate affects the source country through two different channels. First, higher migration rate implies higher remittances received by the source countries, which increase the human capital investment. On other hand, migration rate negatively affects human capital accumulation in the source country through its impact on labor market, which implies loss in output. In general, we could expect that migration rate has a negative effect on human capital formation in the source country when the income ratio is closer to one or migration rate is relatively high, (that means in this framework $\theta_2 \equiv 0.38$). From the empirical evidence presented in this paper, two illustrative examples are Mexico and Jamaica since for those countries the migration and remittances effect on economic growth depend on the assumption about the local labor market. When we assume that departing workers are not easily replaced in the local market, the migration effect is negative in those countries. Finally, the third critical factor in our model is the labor income-ratio in the host country relative to the income in the source country. This ratio, to the best of our understanding plays an important role because higher income ratio induce to higher remittances relative to the labor income in the source countries and also higher income ratio implies that a lower opportunity cost in terms of output provoked by departing workers. Thus, the differences in income
between host country and source countries may magnify the effect of received remittances in the source country and diminish the negative effect of migration rate. In short, we might expect that the probability that migration and remittances positively affect a source country is higher when labor income-ratio is higher between the migrant workers in the host country and the non-migrant workers in the source country.

This paper represents the first step to a more extended research project of migration and remittances impacts in the source countries. We think that a natural extension of this paper may include dropping the perfect capital mobility assumption for analyzing the effect of migrants’ saving in the source country when they return home. Likewise, it would be interesting to elaborate a country-specific study, where the elements of the analysis presented in this paper might be evaluated deeply. In addition, we could consider to take improve the actual calibration and to extend it for a larger number of countries. Others assumptions that could be dropped are inelastic labor supply to assess the moral hazard effect on labor supply decisions and explore the possibility to endogeneize the migration decisions.

6. References