

# Competition of Online Rebate Sites

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## Abstract

In this paper we provide a model to measure the expected payoff for online rebate sites in a duopoly market. We find that if the demand of buyers is uniformly distributed, then site A could make more profits than site B given the cost per buyer is small. In the general case, the profitability of rebate sites may depend on the distribution of consumer's demand. We conclude that if the distribution of demand is non-increasing, site A is able to make more profits in the equilibrium. However, it is ambiguous in the case the distribution of demand is not nonincreasing. If the retailer increases the commission, the profit difference between two sites become larger under the assumption of the non-increasing distribution of demand.

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# 1 Introduction

More and more people prefer online purchase. Compared with usual purchase in local stores, internet purchases have many advantages, including saving transportation cost, easily comparing prices across different online stores and receiving more product information. There is a special group of online sites nowadays. They don't sell any products, but they make profits by attracting customers to buy products on some particular online retailers. They act like commercial companies and make profits via customers' purchases. If buyers purchase products through the links they provided they can receive commission from the online retailers. Even though the commission is a small part of purchase, they still can make a large amount of money if the demand volume is huge. Those sites attract more and more attention from major newspapers. "These savings sites aren't retailers themselves. Rather, they link you to online merchants and collect commissions when you buy things." (Wall Street Journal November 26, 2005)

This group of saving sites belongs to affiliates of online retailers. Affiliates marketing is a method of promoting web businesses in which an affiliate is rewarded for every visitor, subscriber, customer, or sale provided through its efforts. A recent survey by J.D. Power found that 48% of the local retailers surveyed perceived internet referral services to be a threat to the existing system (Chen, Iyer and Padmanabhan 2002).

Basically there are three types of sites in this special group. The first type, those sites collect all commission or referral fee paid by online retailers, although buyers made a purchase by clicking through the links on the site, buyers obtain nothing except for the product. Best examples are Pricegrabber and techbargains.com, the former use the comparison of prices to attract buyers and the latter win buyers by posting deals information. Pricegrabber receives referral fee even though buyers just

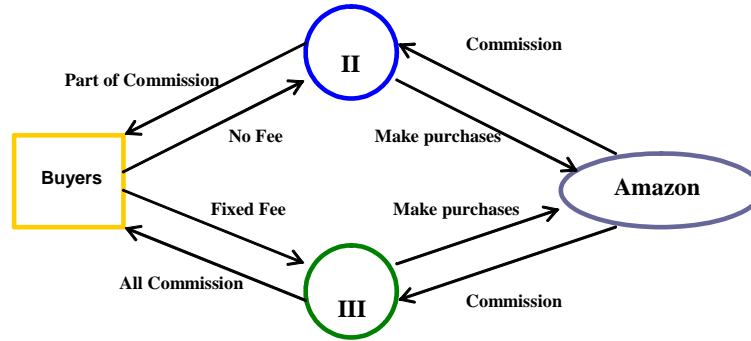
click the link and read the information. Techbargains.com obtains commission from the retailer if buyer purchased some products on the site of that retailer via the link provided.

The second type, sites send a part of commission back to buyers as a way to attract more customers. For example, like fatwallet.com and ebates.com, it provides rebates to buyer if they make a purchase through the links on the site. Of course, the rebates is less than then the commission from the online retailers. Since the commission vary across online stores, the rebates also vary over stores. If we go to the fatwallet.com and click "Store" on the front page, you could find rewards information for different online stores. For instance, buyers can receive 6% rebate if they buy products from cooking.com via fatwallet.com. To illustrate the entire procedure, it is worth to give an example here. If you wanted to buy a \$300 KitchenAid mixer at Cooking.com, you could go to a popular rebate Web site such as FatWallet.com. Click on FatWallet's link for Cooking.com. That transports you to the retail site, where you shop as you would normally. But now, you'll get 6 percent of your purchase deposited in cash in your FatWallet account, according to recent rebate rates. Two extra clicks to shop through the portal just earned you \$18.

The last type, some sites return all the commission back to the buyers who make a real purchase through the link provided. However, they charge a annual membership fee. The best example for this type is Bountyzoo.com. On the front page of this site, they state "we are the most generous rebate site in the us, we give customer 100% back". However, they do charge a membership fee of \$25. If we click rebate information in the site, we find that if we buy from cooking.com, we are able to get 9% of total purchase back. we assume that is all commission paid by cookings.com. The rebate increases by \$9 in the previous example.

We call the second and the third type rebate sites. We summarize these two types

of rebate sites in figure 1 . In figure 1, we assume the online retailer is *Amazon*.



The procedure of rebates

Many recent literatures discuss topics on manufacturer rebates or coupon. Gerstner and Hess (1991) stand on the manufacturer side and compare the case of providing trade discounts with that of giving rebates. They argue that manufacturer rebates can be profitable even if price discrimination does not occur. But the model is built on a monopoly manufacturer and retailers without competition. Nevo and Wolfram (2002) carry out an empirical analysis on coupons in cereal industry. They report the shelf prices are lower during periods when coupon are available. They find lagged coupons have a positive effect on current sales. Rochet and Tirole (2004) define this online market as a two-sided market. In their paper, two-sided markets are markets in which one or several platforms enable interactions between end-users, and try to get two sides on board by appropriately charging each side. For instance, American Express connects two sides, merchant and buyers. American Express charges a merchant discount to the merchant, and charges nothing on buyers. In the sense, the rebate sites like platforms. They receive online retailers commission and charge a negative amount of money (rebates) on buyers. Most of their study focuses on a market of credit cards. Information gatekeepers in Baye and Morgan (2001) is too a platform in two-sided markets as mentioned in Rochet and Tirole (2004). Pricegrabber charges buyers nothing, but obtains referral fee from online retailers. The paper

does not discuss the referral mechanism. In Baye and Morgan (2001), the gatekeeper is a monopoly company providing price comparison and no competition exists for the gatekeeper. They assume the firms who want list information of products pay the gatekeeper a fixed fee. They define two types of buyers, loyals and shoppers. Shoppers always buy products from the gatekeeper if they can find the lowest price on it. In our model, we examine competition of two rebate sites and discuss the behavior of a proportion of shoppers defined in Baye and Morgan (2001) and some buyers who knows where to buy. We call those buyers informed buyers. They not only search the price on gatekeeper but also know buy products through rebate sites. Those informed buyers typically do what is described by Rand (2005, Forbes). He says that "one good way to find deals is to find the cheapest price on a comparison-shopping site and then check at Ebates.com to see if there's a rebate offered for the merchant you found." Ebates is a rebate site we mentioned in the second type. Those rebate sites play a similar role as the gatekeeper, providing links of online retailer to buyers. Although they have no price comparison service, they connect online retailers and informed buyers who know the price of product and where to buy. In this paper, we focus on a duopoly market in which two types of reabte sites compete each other.

In the first section, we lay out the setup of the model. Then, we analyze the optimal strategies in some special cases. In the second section, we compare the profitability of two rebate sites in equilibrium. We further provide comparative-static analysis in the third section. Conclusion and future work are given in the last section.

## 2 Model

### 2.1 Model Setup

We assume that there are two online sites provide rebates to buyers on the market. They face  $N$  informed buyers in the market. Without loss of generality, the number of buyers is normalized to 1. Only one online retailer exists in the market. It provides a fixed commission that is a proportion of the purchase,  $\alpha$ , to both sites if customers click the link they provided and make purchases. There are two types of rebate sites. The rebate site that only charge membership fee,  $\phi$ , say site A, will return all the commission to buyers. But the rewards site which doesn't charge membership fee, say site B, return  $\beta$  of purchase back to buyers. Both  $\beta$  and  $\alpha$  are proportions to total purchase, and they satisfy that  $0 \leq \beta \leq \alpha$ . Rebate site A and B incur same fixed cost  $C$  per customer<sup>1</sup>. All buyers know their future demand for the entire coming year. We assume buyer's demand,  $q$ , is drawn from a common distribution  $F(q)$ ,  $q \in [0, w]$ . The distribution of buyer's demand is common knowledge to rebate sites. We assume  $F(q)$  has a density function  $f(q)$ , which is positive anywhere on  $[0, w]$ .

Since site B provides rebates back to buyers without charging any additional fee, all buyers would not buy products from the online retailer directly. The time frame of decisions by buyers, rebate sites and the online retailer are as follows. First, the online retailer announce its commission ( $\alpha$ ). It offers this commission to both rebate sites. Second, given the fixed commission and knowledge of distribution of buyer's demand, site A and site B determine their optimal fee policy,  $\phi$  and  $\beta$  respectively; Due to rebates provided by two sites, buyers choose one site to complete the online purchases according to the volume of their future demand.

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<sup>1</sup>Once buyers request money back, rebate sites has to send the money by check or via Paypal. The process is automatic in most cases. That means that the cost for buyers with more frequent and larger amount purchases is as same as that for buyers with less frequent purchases.

Our analysis starts from the decision of two rebate sites. Both rebate sites want to maximize their expected profit by choosing optimal fee policies. Given assumptions made above, the expected profit of site A and site B could be written as,

$$\Pi_A = P(q(\alpha - \beta) \geq \phi)(\phi - C)$$

$$\Pi_B = P(q(\alpha - \beta) \leq \phi)\{(\alpha - \beta)E[q|q \leq \frac{\phi}{\alpha - \beta}] - C\}$$

In the expected profit function of site A,  $P(q(\alpha - \beta) \geq \phi)$  gives a proportion of buyers who are using site A. The resource of profits for this site only comes from the membership fee. From the second equation, a  $P(q(\alpha - \beta) \leq \phi)$  proportion of buyers choose site B. Site B makes profits from that the difference between the commission and the rebate times the conditional demand.

Let  $\theta$  denote  $\frac{\phi}{\alpha - \beta}$ . we notice that  $\frac{\partial \theta}{\partial \phi} = \frac{1}{\alpha - \beta}$  and  $\frac{\partial \theta}{\partial \beta} = \frac{\phi}{(\alpha - \beta)^2}$ .  $\theta$  measures the cutoff of the annual demand for consumer's decision. When buyers know their future demand is great than  $\theta$ , they choose site A. Otherwise, they go to site B. If the demand is equal to  $\theta$ , buyers would randomly choose one site. We assume that  $C$  is very small such that site A makes positive profits as long as  $\phi > C$ . In other words, site A have a capability to change  $\phi$  close to  $C$  to attract a proportion of consumers using its service. Therefore,  $\theta$  always falls in the support of buyers' demand.

When  $\theta$  is in  $[0, w]$ , we can rewrite the expected profit function for site A and site B as

$$\Pi_A = (\phi - C)[1 - F(\theta)] \tag{1}$$

$$\Pi_B = (\alpha - \beta) \int_0^\theta q dF(q) - F(\theta)C \tag{2}$$

With competing each other, both sites want to realize the maximal profit in the online market. They, however, need balance the fee policy and the number of buyers

they could attract. Since two sites use different fee mechanisms to obtain the revenue, they may have different strategies.

## 2.2 Equilibrium Conditions and Profit

To explore behaviors of two sites, we first look at the first order conditions for both rebate site. The first order condition of expected profit of site A can be written as

$$\frac{\partial \Pi_A}{\partial \phi} = 1 - F(\theta) - (\phi - C)f(\theta)\frac{1}{\alpha - \beta} = 0 \quad (3)$$

Similarly, the first order condition for site B is

$$\frac{\partial \Pi_B}{\partial \beta} = f(\theta)\theta\left(\theta - \frac{1}{\alpha - \beta}C\right) - \int_0^\theta qdF(q) = 0 \quad (4)$$

We assume that the second order conditions are less than 0. The optimal fee choices must satisfy two equations above. Next, we start with initial analysis with 0 cost per buyer. We then provide analysis on general assumptions.

If both sites have no cost for each buyer, it is easy to see the following proposition.

**Proposition 1** *if the fixed cost  $C = 0$ ,  $\phi = 0$  and  $\beta = \alpha$  is a Nash equilibrium.*

**Proof.** The proof is straightforward. Suppose site A charge 0 membership fee. If site B choose a positive  $\beta$ , no customers would use this site to make purchases. If site B send  $\alpha$  rebate to buyers, half of customers may come to this site but site B makes 0 profit. So, site B is indifferent in any values in  $\beta$ 's support. Similarly, if site B set  $\beta = \alpha$ , site A has no incentive to charge a positive membership fee. In this equilibrium, both sites have half market share but make no profits. ■

Besides this "bad" Nash equilibrium, both sites may have many equilibria. Given the setup of 0 cost per buyer, it is worth to revisit the first order conditions,

$$\frac{\partial \Pi_A}{\partial \phi} = (1 - F(\theta) - f(\theta)\theta) = 0 \quad (5)$$

$$\frac{\partial \Pi_B}{\partial \beta} = f(\theta)\theta^2 - \int_0^\theta q dF(q) = 0 \quad (6)$$

We notice that two first order conditions become two equations of ratio of two optimal choices only, i.e.  $\theta$ . We may have infinite pure strategies as long as the ratio of optimal choice satisfies two equations. However, it should depend on the shape of distribution of customer's demand. Suppose we can find  $\tilde{\phi}$  and  $\tilde{\beta}$  as the optimal choices for both sites. Then  $\tilde{\theta} = \frac{\tilde{\phi}}{\alpha - \tilde{\beta}}$ .  $\tilde{\phi}$ ,  $\tilde{\beta}$  and  $\tilde{\theta}$  are functions of the exogenous variables. The optimal choices must be satisfied equation 5 and 6. If we substitute the first order conditions in expected expected profit function, we have

$$\Pi_A = \Pi_B = \tilde{\phi} f(\tilde{\theta}) \tilde{\theta}$$

With 0 cost per customer, any equilibrium generated by both equation 5 and 6 guarantee that both sites makes identical profits. In the next section, we focus on profitability of two rebate site under equilibrium conditions.

### 3 Results on Profitability

We first present an analysis on monopoly market with a simplest setting. We assume that site A and site B are monopoly in the market. The buyer's demand,  $p$ , is distributed uniformly on  $[0, 1]$ . We then would compare their profits if there is only one rebate site in the market.

For convenience, we list the expected profit functions as the following if  $p$  is uniformly distribution,

$$\Pi_A = (\phi - C)[1 - \theta] \quad (7)$$

$$\Pi_B = ((\alpha - \beta)\frac{\theta^2}{2} - \theta C) \quad (8)$$

we then look at the first order condition under the uniform distribution,

$$\frac{\partial \Pi_A}{\partial \phi} = 1 - \frac{\phi}{\alpha} - (\phi - C)\frac{1}{\alpha} = 0 \quad (9)$$

where  $\theta' = \frac{\phi}{\alpha}$ .

The optimal choice is  $\phi = \frac{\alpha + C}{2}$ . We plug it in the profit function, we should have  $\Pi_A = \frac{(\alpha - C)^2}{4\alpha}$ . If site B is monopoly in the rebate site market, site B could set  $\beta$  a little bit higher than 0 to avoid that buyers make purchase directly from the retailer's site. Therefore site B expected profit is  $\Pi_B = \frac{\alpha - 2C}{2}$ . Since  $C$  must satisfy that  $C < \frac{\alpha}{2}$ , we would have following cases,

$$\Pi_A^m - \Pi_B^m = \begin{cases} > 0 & \text{if } (\sqrt{2} - 1)\alpha < C < \frac{\alpha}{2} \\ = 0 & \text{if } C = (\sqrt{2} - 1)\alpha \\ < 0 & \text{if } 0 < C < (\sqrt{2} - 1)\alpha \end{cases} \quad (10)$$

If  $C$  is smaller enough i.e.  $C < (\sqrt{2} - 1)\alpha$ , then site B would make more profits than site A. The result reverses if  $C$  is close to  $\frac{\alpha}{2}$ . When  $C$  is equal to  $(\sqrt{2} - 1)\alpha$ , both site made identical profits,  $\frac{3-2\sqrt{2}}{2}a$ , in the monopoly case. Since we assume a uniform distribution for  $q$ , the profit comparison is only related to  $C$  and  $\alpha$ . We then go back to the general setup for the distribution of demand in the duopoly market to examine the profitability. We find that if  $q$  follows some particular distributions, the profits made by site A is strictly greater than that made by site B.

Let optimal choices be denoted by  $\tilde{\phi}$ ,  $\tilde{\beta}$  and  $\tilde{\theta}$ . Combining equation 3 and equation 4 with the expected profit functions, we have the maximal profits for both rebate sites

$$V_A = (\tilde{\phi} - C)^2 f(\tilde{\theta}) \frac{1}{\alpha - \tilde{\beta}} \quad (11)$$

$$V_B = (\alpha - \tilde{\beta})f(\tilde{\theta})\tilde{\theta}(\tilde{\theta} - \frac{1}{\alpha - \tilde{\beta}}C) - F(\tilde{\theta})C \quad (12)$$

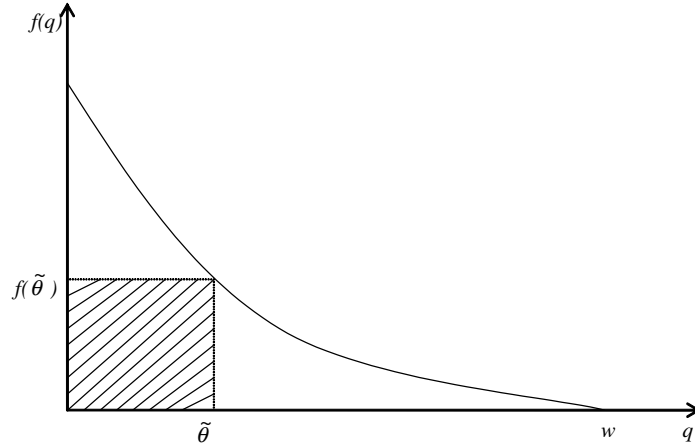
The profit difference is defined by  $D = V_A - V_B$ . It then can be expressed as

$$D = C(F(\tilde{\theta}) - (1 - \frac{C}{\phi})f(\tilde{\theta})\tilde{\theta}) \quad (13)$$

We notice that  $1 - \frac{C}{\phi} < 1$ . The profitability difference in the equilibrium depends on not only the optimal fee policies, also the shape of distribution of the demand. From the equation 13, it is easy to see the following proposition.

**Proposition 2** *If the density function of  $q$  is non-increasing on  $[0, w]$ , site A makes more profitable in the equilibrium.*

**Proof.** The proof is simple. If the density function of  $q$  is non-increasing, the density function should like the figure shown below,



The area of shadowed square is the value of  $f(\tilde{\theta})\tilde{\theta}$ . It is directly showed from the picture that that  $F(\tilde{\theta}) \geq f(\tilde{\theta})\tilde{\theta}$ . Due to  $1 - \frac{C}{\phi} < 1$ , the profit difference  $D > 0$ . ■

Intuitively, this type of distribution passes us information that the number of buyers is non-increasing (or diminishing) as the demand goes large. For example, if  $F(q)$  is uniform or exponential, the result mentioned above holds. For site A, the

profit comes from the number of buyers whose demand is higher than the cutoff,  $\theta$ , as long as membership fee is higher than cost. Site B attract buyers with relative less demand. However it profits come from the demand and the commission difference,  $(\alpha - \beta)$ . Due to the fixed cost per buyer, the profits for site B could be small if everyone consumes limited products online. The reason is that site B has to use the part of commission to offset the cost per buyer occurred. To illustrate this profitability difference in the equilibrium, we present an example in the next part by using uniform distribution again.

**Example 3** *Let  $q$  be distributed uniformly on  $[0, 1]$ , site A makes profit of  $\frac{C}{3}$ ; but site B make 0 profit.*

If the demand is uniform distributed, the first order conditions for site A and site B are

$$\frac{\partial \Pi_A}{\partial \phi} = 1 - \theta - (\phi - C) \frac{1}{\alpha - \beta} = 0 \quad (14)$$

$$\frac{\partial \Pi_B}{\partial \beta} = \theta \left( \theta - \frac{1}{\alpha - \beta} C \right) - \frac{\theta^2}{2} = 0 \quad (15)$$

From the first order condition of site B, we find the equation rules out  $\beta$  so that we are able to identify  $\phi$  directly.

If  $\frac{\alpha}{3} < C < \frac{\alpha}{2}$ , no equilibrium exists. If  $0 < C \leq \frac{\alpha}{3}$ , we have  $\phi = 2C$ , and  $\beta = \alpha - 3C$ . We notice that the optimal choice of site A only depends on the fixed fee, while that of site B is determined by the commission and the fixed fee jointly. site A makes profits of  $\Pi_A = \frac{C}{3}$ , which is less than  $\Pi_A^m = \frac{(\alpha - c)^2}{4\alpha}$  in the monopoly case. Site B has 0 profit.

When  $0 < C \leq \frac{\alpha}{3}$ , site B is indifferent in possible values for  $\beta$  as long as site A set its membership fee equal to  $2C$ . Although site B possesses  $\frac{2}{3}$  of entire buyers using its services, all revenue from commission has to pay the cost occurred in the process of

returning rebates to buyers. Site A has advantages in this case. It can make positive profits once the membership fee is greater than the fixed cost.

What if the distribution is not non-increasing ? The results are not explicit. The profitability for both sites depends the location of the ratio of optimal choice,  $\theta$ .

## 4 Comparative-Static Analysis

In this section, we carry out comparative-static analysis on the optimal fee policies. The optimal fee choices may determine by two exogenous variables, the commission paid by retailer, the cost occurred per buyer and the distribution of buys' demand. A comparative-static analysis could provides managerial implications on how to set different fee policies as the market setting changes.

**Proposition 4** *In equilibrium, the optimal  $\beta$  for site B goes up by the same amount of  $\alpha$ . On the other hand, the optimal membership fee,  $\phi$ , stays same as commission paid by the retailer increases.* <sup>2</sup>

Given the distribution of buyer's consumption and cost, the strategy of site A only depends on the difference between the commission and the rebate site B return to buyers. As the commission increases, site B keep the same commission margin in the equilibrium. As a result, site A won't change its optimal fee policy. Therefore, if the retailer set a higher commission, the profits for both sites stays the same. The buyers would benefit from an increase in the commission instead.

**Proposition 5** *If the density function of  $q$  is non-increasing on  $[0, w]$ ,  $D$  increases as  $C$  becomes larger on  $(0, \frac{\alpha}{2})$ .*

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<sup>2</sup>see appendix for proof.

**Proof.**  $D$  is defined as the profitability difference between site A and site B. As shown previously,  $D = C(F(\tilde{\theta}) - (1 - \frac{C}{\phi})f(\tilde{\theta})\tilde{\theta})$  in the equilibrium. Therefore we have

$$\frac{\partial D}{\partial C} = (F(\tilde{\theta}) - (1 - \frac{2C}{\phi})f(\tilde{\theta})\tilde{\theta}) > 0 \quad (16)$$

The difference becomes larger when  $C$  increases. ■

This result can be verified by the case of uniform distribution. Since the profit of site A only depends on the cost, i.e.  $\Pi_A = \frac{C}{3}$ , and site B has no profit,  $D$  increases as  $C$  goes up. Intuitively, the change of an increase in  $C$  drive site A to rise the membership fee but have site B reduce the rebates. However, the proportion of buyers choosing both sites does not change. Thus, site A directly benefits from the increase in cost and buyers lost surplus from this change.

## 5 Conclusion and Future Work

In this paper we provide a simple model to measure the expected payoff for online rebate sites. In monopoly market, we find that if the demand of buyers is uniformly distributed, then site B could make more profits than site A given the cost per buyer is small. If the cost is close to a half of the commission, site A is more profitable in this case. In the general case, the profitability of rebate sites may depend on the distribution of consumer's demand. We conclude that if the distribution of demand is non-increasing, site A is more profitable in the equilibrium. However, it is ambiguous in the case the distribution of demand is not nonincreasing. We further discuss the change of optimal fee polices made by two sites when the commission increases in the case of non-increasing distribution of  $p$ , . An increase in commission paid by retailer does not lead to an increase in profits for both sites. Both rebates sites keep the same optimal strategies. It turns out that buyers are the only beneficiary from this change. In the meanwhile, cost per buyer may affect profits for both sites. Intuitively, the

membership fee should go up and rebates returned to buyer should decline when the cost increases. The profit difference between two rebate sites becomes larger when the cost per buyer increases. Buyers lost surplus from this change in the cost.

The future work for this paper will focus on adding the expected profit function of the online retailer in the model and a possible utility function of buyers. Therefore, we are able to find how change in commission affects behaviors of buyers. It may possible to find the optimal commission choice for the online retailer. Also we need more work on analyzing the profits of two sites if the distribution of demand is not non-increasing. It may be possible to find implications.

## 6 Appendix

Proposition 4.

**Proof.** We list two first order conditions as follows

$$\frac{\partial \Pi_A}{\partial \phi} = 1 - F(\theta) - (\phi - C)f(\theta)\frac{1}{\alpha - \beta} = 0 \quad (17)$$

$$\frac{\partial \Pi_B}{\partial \beta} = f(\theta)\theta\left(\theta - \frac{1}{\alpha - \beta}C\right) - \int_0^\theta qdF(q) = 0 \quad (18)$$

we can rewrite the total derivative of two equations in a matrix form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d\phi \\ d\beta \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix} d\alpha$$

where

$$\begin{aligned} a &= f(\theta)\frac{1}{\alpha - \beta} - f'(\theta)\frac{1}{\alpha - \beta}(1 + (\phi - C)\frac{1}{\alpha - \beta}), \\ b &= -\frac{1}{(\alpha - \beta)^2}(f'(\theta) + (\phi - C)(f'(\theta)\theta + f(\theta))), \\ c &= f'(\theta)\theta\left(\frac{\theta}{\alpha - \beta} - \frac{C}{(\alpha - \beta)^2}\right) + f(\theta)\frac{1}{\alpha - \beta}\left(\theta - \frac{C}{\alpha - \beta}\right) \\ d &= f'(\theta)\theta^2\frac{1}{(\alpha - \beta)^2}(\phi - C) + f(\theta)\frac{1}{(\alpha - \beta)^2}\left(\theta - \frac{2C}{\alpha - \beta}\right) \end{aligned}$$

$$\text{Denote } \det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \Delta$$

therefore, by Cramer's rule, we have

$$\frac{d\phi}{d\alpha} = \frac{\begin{vmatrix} b & b \\ d & d \end{vmatrix}}{\Delta} = 0$$

$$\frac{d\beta}{d\alpha} = \frac{\Delta}{\Delta} = 1$$

As  $\alpha$  increases,  $\phi$  remains the same but  $\beta$  increases by a same amount. ■

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