

Information Gatekeeper with Asymmetric Firms ^{*†}

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Abstract

Asymmetry exists among the firms selling homogeneous product in online market. This paper provides a theoretical study on the equilibrium behavior of the firms and the information gatekeeper in a duopoly market with asymmetric sizes of loyal consumers. In equilibrium, the firm with larger number of loyal consumers tends to advertise less frequently but charges a lower average price whenever advertising. And the optimal advertising fee charged by the information gatekeeper depends on the degree of asymmetry of the market. As the market becomes more asymmetric, the gatekeeper charges a higher advertising fee.

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1 Introduction

The boom of e-commerce in the past decade has dramatically changed the ways in which consumers obtain information and purchase products. The firms, in the mean time, have been adjusting their advertising and retailing strategies. This provides opportunities for the price comparison service to burgeon and develop into a business of significant scale. According to E-consultancy, in the UK, the price comparison services made revenues between 120 to 140 Million Euros in 2005. Nowadays, for a popular product, there are usually dozens of firms that list their prices on the price comparison websites (will be also referred to as “information gatekeepers ”). The continuous increasing of the price comparison business makes the economic analysis of information gatekeeper models more and more important.

Baye and Morgan (2001)’s seminal paper is the first to investigate the market equilibrium when consumers, firms and the information gatekeeper make their own decisions to maximize their own utility/profit. In their model, the firms can list their price information at the information gatekeeper’s site so that they can have an opportunity to attract consumers outside their local markets. In equilibrium, to maximize profit, the information gatekeeper charges a positive listing fee to induce the price dispersion in the market. This result explains the persistent price dispersion even in the information era, when the cost of obtaining price information is small.

The firms in Baye and Morgan model are symmetric in the sense that they have the same number of local customers. In the real world, however, firms are asymmetric. Even if the firms are competing with a homogeneous product, they differ significantly in their service quality, website design and reputation. As a result, the firms build their own loyal consumer groups of different sizes. This asymmetry in the size of loyal consumer groups has an significant impact on the pricing and advertising decisions of the firms.

Some casual observations of the price comparison websites indicate that asymmetry widely exists in the pricing and advertising strategies of the firms. First, some firms list their prices significantly less frequently than others. Second, some firms list lower prices more frequently than others on the price comparison websites. For example, we examine the price listing of a single product (SanDisk 2 GB Secure Digital Card) at Shopper.com, the pricing lists were recorded 19 times in the period from 10/21/2006 to 1/7/2007. Figure 1 shows the listed price for Dell and Buy.com. Over the period, Dell lists its price 16 times while buy.com only lists

8 times. And whenever buy.com lists its price, it charges a lower price than Dell. This is just one example of the asymmetries in pricing and advertising that is ubiquitous on the pricing comparison sites.

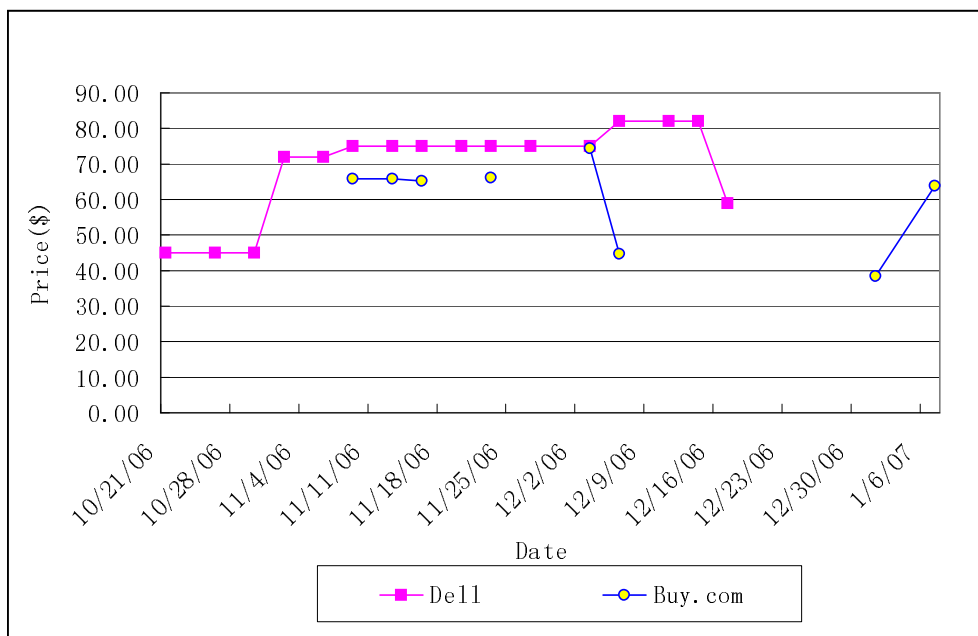


Figure 1: Listed price of SanDisk 2 GB Secure Digital Card of Buy.com and Dell on Shopper.com

Therefore, to characterize the asymmetric behavior of firms, studies on asymmetric models are necessary. However, the complexity introduced by asymmetry makes the models difficult to solve even for a duopoly market. In the literature, there are limited research on asymmetric models and most of them are been focused only on pricing strategy of the firms. Narasimhan(1988) analyzes a duopoly pricing model with asymmetry in the sizes of loyal consumers. The equilibrium behavior of the duopoly turns out to be determined by the characteristics of the shoppers. If the shoppers are extremely price sensitive and only buy products from the firm offering the lowest price, the equilibrium price distribution of the firm with less loyal consumers is first-degree stochastic dominated by that of the firm with more loyal consumers. Baye, Kovenock and de vries (1992) generalize Narasimhan's model to N firms case and find that if firms have different sizes of loyal consumers, then two of the firms with the least loyal consumers will continuously randomize over some price interval and

all other firms set prices equal to reservation price with probability one. However, in above models, there is no role for the information gatekeeper, or in other words, the cost of advertising is zero. And “little is known about the general clearinghouse model with asymmetric consumers. ”(Baye, Morgan & Scholten 2006)

Our paper contributes to the literature by studying a duopoly clearinghouse model with consumer asymmetries and positive listing fees. The model gives insights on the asymmetries in firms’ advertising frequencies and pricing strategies observed on the price comparison sites. The model is based on the clearinghouse model in Baye and Morgan(2001). There are two types of consumers in the model, loyal consumers and comparison shoppers. The two firms are asymmetric in their numbers of endowed loyal consumers.¹ Shoppers first consult price comparison website and buy from the firm offering the lowest price. If there is no price listed on the website, they will randomly choose from one of the firms. Loyal consumers, however, buy directly from their own preferred firm. The information gatekeeper charges a fixed fee from the firm that lists price on its website. And firms make pricing and advertising decisions given the fixed fee.

We find that there are two sets of equilibria of firms’ behavior, each corresponds to a certain range of listing fee charged by the information gatekeeper. The firm with more loyal consumers will advertise less frequently, but whenever it decides to advertise, it charges lower prices more frequently than the firm that has less loyal consumers. This pricing behavior of the firms significantly differs from that when advertising fee is zero, which is predicted in Narasimhan(1988). The intuition for this result is as follows. For the firm with more loyal consumers, the opportunity cost of advertising and charge a price lower than r is higher. Therefore, in equilibrium, it will advertise less frequently. Also because of the larger number of loyals, when the firm decides to advertise, it needs smaller profit margin to balance the cost of advertising. Therefore, it gives higher discount when it advertises. In short, in an economy where firms can choose both advertising and pricing strategy, the firm with larger market share tends to use a less aggressive advertising strategy to protect the profits from its own loyal customers.

We also find that as the market becomes more asymmetric (the difference of the number of loyal customers increases), the gatekeeper charges a higher listing fee since the firm with

¹There are other types of asymmetries, such in costs, service qualities, but we will focus on the asymmetry in the size of the loyal consumers.

less loyal is willing to pay more to advertise as the market gets more asymmetric.

The rest of the paper is organized as follows. Section 2 lays out the setup of the model. Section 3 solves the firm's equilibrium strategy given fixed advertising fee. Then the optimal fee for the gatekeeper is analyzed in section 4. Section 5 concludes.

2 Model Setup

There are two price-setting firms ($i = 1, 2$) competing in a homogeneous market. Firms have unlimited capacity to supply this product at a constant marginal cost, m , which is assumed to be zero without loss of generality. This market is served by a price information gatekeeper, who provides a information portal for the firms to advertise their price for a fixed fee ϕ .²

There is a continuum of consumers with size normalized to one, each has a unit demand up to a reservation price r . We assume there are two types of consumers, loyal and shoppers. The loyal will only purchase from their favorite firm given the price charged by the firm does not exceed r . The shoppers, however, will consult the information gatekeeper to compare the price charged by different firms, and purchase from the firm charging the lowest price, given it does not exceed r . If there is no firm listing at the information gatekeeper, or the price listed are the same for both firms, the shopper will randomly choose a firm and purchase from there, given the price does not exceed r .³ Let the size of the loyal for each firm be L_1 and L_2 , respectively. To introduce asymmetric structure into the model, we assume $L_1 \geq L_2$ without loss of generality. And let S denote the size of shoppers, which equals to $1 - L_1 - L_2$. The firms are not allowed to price discriminate between different consumers.

There are a few things regarding the assumption of the model that we would like to discuss. First, the unit demand assumption is made for computational simplicity. A more general assumption would be a downward sloping demand schedule $q(p)$. The firms strategy under these two assumptions, however, are quite similar. Under both setups, the firm's

²There are different business models for online pricing comparison website, retailers can either pay a fixed fee, a click-through fee or a fee when a purchase is completed. We are focusing on the fixed fee case in our model.

³This is a reasonable assumption, since in equilibrium, if a firm does not list price on the price comparison site, it will always charge r . As a result, there is no incentive for consumers to do sequential search among the firms.

profit in their loyal market is maximized at certain price (r for the unit demand case and the monopoly price for the downward sloping demand case). To gain the business from the shoppers, they must sacrifice the profit in the loyal market price by charging a lower price. Second, our model is a generalized model in the sense that it will converge to existing models with parameters take extreme values. When $L_1 = L_2$, our model converges to Baye and Morgan (2001) with $n = 2$ and unit demand assumption. When $\phi = 0$, our model converges to Narasimhan(1988) with advertising options.

The market runs in three stages. First, the information gatekeeper decides its advertising fee, ϕ , for the firms. Second, the firms make their pricing decisions and advertising decisions simultaneously. Finally, the consumers make their purchasing decisions according to the price and their type. Since the consumer's behavior is exogenously assumed, we will first focus on the decisions of the firms.

3 Equilibrium Analysis of the Firms

Given the advertising fee, ϕ , there are two decisions that the firms need to make: whether to list the price information at the gatekeeper, and what price to charge. Neither of the decisions is trivial. For advertising, the firm needs to weigh the potential profit gain of advertising against the cost of advertising fee. For pricing, there is a trade off between extracting more profit per sale and attracting more sales from the shoppers. We will show the existence of a mixed strategy equilibrium in this subgame by construction.

To accommodate for the mixed strategy equilibrium, let α_i stand for the probability that firm i will advertise. And let $F_i(p)$ be the price distribution that firm i will draw from if it decides to advertise. Clearly, pure strategy for advertising will be characterized by $\alpha_i = 0$ or 1 , and pure strategy for pricing will be characterized by a degenerate price distribution function.

To analyze the potential gain of advertising, we first consider the strategy and expected profit of a firm when it does not advertise. Given the assumptions, if the firm does not advertise, as long as it charges price that is below r , it can get sales from its own loyals, and half of the shoppers if the other firm does not advertise either. In other words, the firm can not change its demand by changing price if it does not advertise. Obviously, the firm's optimal pricing strategy when it does not advertise is to charge r . And the expected profit

that it can get is:

$$E\pi_i^{NA} = r[L_i + \frac{1}{2}(1 - \alpha_j)S]. \quad (1)$$

On the other hand, the expected profit when the firm advertises and charges price p is:

$$E\pi_i^A(p) = p\{L_i + [(1 - \alpha_j) + \alpha_j(1 - F_j(p))]S\} - \phi, \quad (2)$$

where F_j is the price distribution of the other firm. Basically, the equation says that if the firm advertises and charges price p , it will get sales from its own loyal L_i , and all the shoppers if the other firm does not advertise (with probability $1 - \alpha_j$), or the other firm advertises but charges a higher price than p (with probability $\alpha_j(1 - F_j(p))$). And it has to pay the advertising fee ϕ .

The level of the advertising fee plays a critical role in the decision of the firms. When $\phi > \frac{rS}{2}$, neither firm advertises is a Nash Equilibrium, since the cost of advertising, ϕ is greater than the maximum possible gain in revenue the firm can get by deviating from not advertising, which is $\frac{rS}{2}$. When $\phi = 0$, then firm i is indifferent between not advertising and advertising with charging r if the other firm does not have mass point at r . This fact makes the existence of multiple equilibria possible. We will discuss this in detail in later part of the paper. For the following analysis, we will focus on the range of $0 < \phi \leq \frac{rS}{2}$.

At this point, it is useful to introduce \underline{p}_i , which is defined as the minimum price that firm i will ever consider charging when advertising. In other words, pricing at \underline{p}_i , firm i 's maximum possible profit equals to the profit when it does not advertise. Therefore, we have:

$$r[L_i + \frac{1}{2}(1 - \alpha_j)S] = \underline{p}_i(L_i + S) - \phi, \quad i = 1, 2 \quad (3)$$

It can be shown that when $\alpha_i = 1$, we have $\underline{p}_1 \geq \underline{p}_2$.

Under the specified range of ϕ , it turns out that there does not exist an equilibrium with pure pricing strategies when the firms advertise. And there does not exist an equilibrium with both firm having pure strategies in advertising. The results are given in Lemma 1 and Lemma 2.

Lemma 1 *In equilibrium, if $0 < \alpha_i \leq 1$, i.e., both firm advertise with positive probability, then neither firm uses pure strategy in pricing when it decides to advertise.*

Proof. We first prove that it is impossible for both firm to use pure strategy in price whenever advertising. Assume otherwise, then firm i will charge $p_i \in [\underline{p}_i, r]$ and firm j will charge $p_j \in [\underline{p}_j, r]$.

If $p_i = p_j$, then we argue that firm j can increase its profit by deviating and charge $p_j - \delta$, where δ is a very small positive number. We have $E\pi_j^A(p_j) = p_j[L_j + (1 - \alpha_i)S + \frac{\alpha_i S}{2}] - \phi$ and $E\pi_j^A(p_j - \delta) = (p_j - \delta)(L_j + S) - \phi$. Therefore,

$$E\pi_j^A(p_j - \delta) - E\pi_j^A(p_j) = \frac{p_j \alpha_i S}{2} - \delta(L_j + S). \quad (4)$$

Since $\alpha_i > 0$ by assumption, and δ can be arbitrarily small, we have $E\pi_j^A(p_j - \delta) - E\pi_j^A(p_j) > 0$, which means when $p_i = p_j$, the pure strategy can not be an equilibrium.

If $p_i \neq p_j$, assume $p_i < p_j$ without loss of generality, then firm i will be better off by charging $p_j - \delta > p_i$ instead of charging p_i . To see this, notice when $p_i < p_j$ the expected profit for firm i , $E\pi_i^A(p_i) = p_j(L_j + S) - \phi$, is a strictly increasing function in p_i . Therefore, firm i can always increase its expected profit by increase its price when p_i is strictly lower than p_j .

It is also impossible for one firm to have pure strategy and the other firm to have mixed strategy in equilibrium. If firm i charge p_i whenever it advertises, then $E\pi_j(p)$ is a strictly increasing function on $[\underline{p}_j, p_i)$ and on $(p_i, r]$, respectively. Therefore, there does not exist two prices that are both best responses of firm j to p_i . Consequently, firm j does not have mixed strategy in equilibrium.

Summarizing from the above, it is impossible for any firm to have pure pricing strategy when both firm advertise with positive probability. ■

Lemma 2 *When $0 < \phi \leq \frac{rS}{2}$, there does not exist equilibrium such that (1) either firm does not advertise, (2) both firms advertise with probability one.*

Proof. First, if firm i does not advertise, it is always optimal for firm j to advertise with probability one and charge r given $\phi \leq \frac{Sr}{2}$. Then, firm i can increase its profit by advertising with probability one and undercut r by a small amount δ . Since $\phi < Sr$, $\phi < S(r - \delta)$ for an arbitrarily small δ . Therefore, any of the firms does not advertises is not an equilibrium strategy.

Second, if both firm advertise, by Lemma 1, there is no pure strategy in pricing. However, if both firm apply mixed strategies in pricing, then according to Narasimhan (1988), the price

distribution of at least one of the firms will have positive density at r , and the other firm will not have positive mass at r . So for the firm that have positive density at r , advertising and charging r will not generate any more sale than not advertising and charging r , but will incur a positive fixed advertising cost. In other words, advertising and charging r is dominated strategy and can not be part of the mixed strategy equilibrium. Therefore, both firm advertise with probability one can not be an equilibrium. ■

Lemma 1 and Lemma 2 together rule out the existence of pure strategy equilibria in price when firms advertise. And they also narrow down the possible combinations of advertising strategy to two cases: (1) both firm mix in advertising, (2) one firm mix in advertising and the other firm advertise with probability one.

The following analysis investigate the equilibrium strategies of the firms.

If either firm is mixing in price, it must be true that $E\pi_i^A(p)$ is a constant for any price in the domain of its price distribution. Also, if the firm is mixing also in advertising, it must be true that $E\pi_i^{NA} = E\pi_i^A(p)$, i.e., the expected profit of advertising equal to the expected profit of not advertising.

The relative size of \underline{p}_i turns out to be very critical to the equilibrium behavior of the firms. Notice that in equilibrium, if $\underline{p}_i > \underline{p}_j$, firm j can not mix in advertising. By definition, we have $E\pi_j^{NA} = \underline{p}_j(L_j + S) - \phi = E\pi_j^A(\underline{p}_j)$. However, since firm i will not charge any price below \underline{p}_i , firm j can strictly increase its price to \underline{p}_i and still sell to all the shoppers, which gives a higher profit than $\pi_j^A(\underline{p}_j)$. In other words, not advertising is a strictly dominated strategy for firm j if $\underline{p}_i > \underline{p}_j$.

Given this insight, there are three possible cases for mixed strategy equilibrium to exist:

1. $\underline{p}_1 > \underline{p}_2, 0 < \alpha_1 < 1, \alpha_2 = 1$.
2. $\underline{p}_1 < \underline{p}_2, \alpha_1 = 1, 0 < \alpha_2 < 1$.
3. $\underline{p}_1 = \underline{p}_2, 0 < \alpha_1 < 1, 0 < \alpha_2 < 1$.

It is straight forward to show that case 2 is not possible. And case 1 and case 3 are both equilibria for the firms' strategies, each corresponds to a different range of the advertising fee ϕ . The details of equilibrium strategies of the firms are described in Proposition 1.

Proposition 1 *If the gatekeeper sets the listing fee ϕ , and firms make optimal pricing and advertising decisions, then there exist an asymmetric Nash equilibrium which is dependent*

on the level of ϕ :

1. (Range 1) If $0 \leq \phi < rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$,

- Firm 2 will advertise its price with probability one and firm 1 will advertise its price with probability

$$\alpha_1 = \frac{L_2 + S}{L_1 + S} \left(1 - \frac{\phi}{rS}\right). \quad (5)$$

- When a firm chooses to advertise, its will charge price from the distribution that is characterized by the following c.d.f.:

$$F_1(p) = \frac{p(L_1 + S) - (rL_1 + \phi)}{pS(1 - \frac{\phi}{rS})}, \text{ on } [\underline{p}, r], \quad (6)$$

$$F_2(p) = 1 - \frac{L_1(r - p) + \phi}{pS}, \text{ on } [\underline{p}, r], \quad (7)$$

where $\underline{p} = \frac{rL_1 + \phi}{L_1 + S}$. And if a firm chooses not to advertise, it will charge r .

- And the profits for the two firms are:

$$E\pi_1 = rL_1, \quad (8)$$

$$E\pi_2 = \frac{rL_1(L_2 + S) + \phi(L_2 - L_1)}{(L_1 + S)}. \quad (9)$$

2. (Range 2) If $rs \frac{L_1 - L_2}{1 + L_1 - 2L_2} \leq \phi \leq \frac{rS}{2}$, then

- Both firms will have mixed strategies in advertising, and the probabilities are:

$$\alpha_1 = \frac{rS - 2\phi}{rS}, \quad (10)$$

$$\alpha_2 = \frac{(rS - 2\phi)(2L_1 + S - L_2)}{rS(L_2 + S)}. \quad (11)$$

- When a firm chooses to advertise, its will charge price from the distribution that is characterized by the following c.d.f.:

$$F_1(p) = \frac{1}{\alpha_1} \left(1 - \frac{rL_2 - pL_2 + \frac{1}{2}rS(1 - \alpha_1) + \phi}{pS}\right), \text{ on } [\underline{p}, r], \quad (12)$$

$$F_2(p) = \frac{1}{\alpha_2} \left(1 - \frac{rL_1 - pL_1 + \frac{1}{2}rS(1 - \alpha_2) + \phi}{pS}\right), \text{ on } [\underline{p}, r], \quad (13)$$

where, $\underline{p} = \frac{rL_2 + 2\phi}{L_2 + S}$. And when a firm chooses not to advertise, it will charge r .

- And the profits for the two firms are:

$$E\pi_1 = \frac{rL_2(L_1 + S) + \phi(2L_1 + S - L_2)}{L_2 + S}, \quad (14)$$

$$E\pi_2 = rL_2 + \phi. \quad (15)$$

Proof. See Appendix. ■

Figure 2 plots the price distributions $F_i(p)$ for the two firms for certain values of the parameters. In both ranges of ϕ , firm 2's price distributions have positive mass at the reservation price r . According to the analysis given in Narasimhan(88), firm 1's price distribution will have zero probability density at r . The reason is when firm 2 has mass point at r , for firm 1, charging r is strictly dominated by charging $r - \delta$, where δ is an arbitrarily small number.

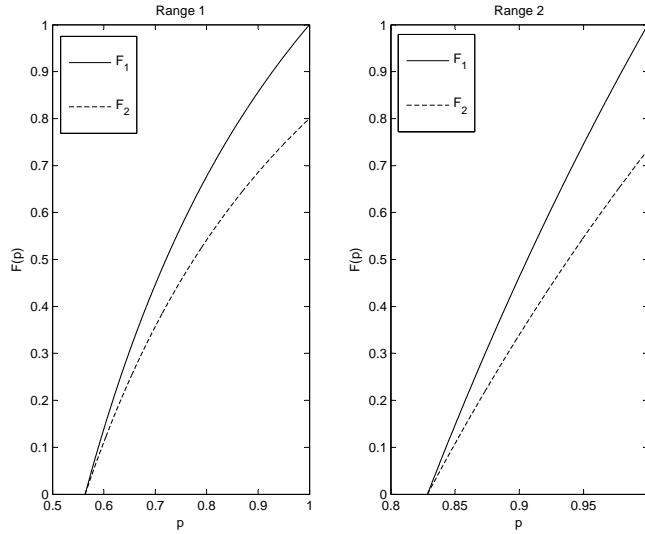


Figure 2: Price Distribution of two firms ($r = 1$, $S = 0.5$, $\theta = 1/3$, $\phi = 0.1$ (range 1) or 0.2 (range 2))

The primary purpose of this paper is to see the effect of asymmetry on the equilibrium strategies of the firms, or to be more specific, the differences in strategies for firms with different numbers of loyal. In both ranges of ϕ , we have $\alpha_1 \leq \alpha_2$ and $F_1(p) \geq F_2(p)$, i.e., the firm with more loyal will advertise less frequently. But whenever it decides to advertise, it is more likely to give a bigger discount off the reservation price. The intuition for this result is as follows. Since firm 1 has more loyal consumers, its opportunity cost of advertising and charge price lower than r is higher. Therefore, in equilibrium, firm 1 will advertise less frequently. Also because of the larger number of loyal, when firm 1 decides to advertise, it needs smaller profit margin to balance the cost of advertising. Therefore, it gives higher discount when it advertises.

Since our model is a generalized version of Narasimhan(1988), and Baye and Morgan

(2001) with $n = 2$. It is interesting to see whether our equilibrium results will converge to the special cases when the parameters take special values. First, when $L_1 = L_2 = L$, the equilibrium will always in range 2 since the threshold value that separates the two ranges becomes zero. And we have

$$\alpha_1 = \alpha_2 = \alpha = \frac{rS - 2\phi}{rS}, \quad (16)$$

$$F_1(p) = F_2(p) = \frac{1}{\alpha} \left(1 - \frac{(r-p)\frac{1-S}{2} + \frac{1}{2}rS(1-\alpha) + \phi}{pS} \right), \text{ on } [\underline{p}, r], \quad (17)$$

$$\underline{p} = \frac{r(1-S) + 4\phi}{1+S}, \text{ and} \quad (18)$$

$$E\pi = \phi + rL. \quad (19)$$

These results are consistent with Baye and Morgan (2001), with $n = 2$ and adjustment for the unit demand assumption that is made in our paper.

Second, when $\phi = 0$, we will be in range 1 of the equilibrium. And we have:

$$\alpha_1 = \frac{L_2 + S}{L_1 + S}, \quad (20)$$

$$F_1(p) = \frac{pL_1 + pS - rL_1}{pS} \text{ on } [\underline{p}, r], \quad (21)$$

$$F_2(p) = 1 - \frac{L_1(r-p)}{pS} \text{ on } [\underline{p}, r], \quad (22)$$

where $\underline{p} = \frac{rL_1}{L_1+S}$. On the face, the equilibrium that is characterized here is different from Narasimhan(1988), which does not allow for the possibility of not advertising. So in the equilibrium described in Narasimhan(1988), we have $\alpha_1 = 1$ and $F_1(p) = 1 + \frac{L_2}{S} - \frac{L_1r(L_2+S)}{Sp(L_1+S)}$. However, these two seemingly different equilibria generate the same profits for the two firms. When the advertising fee is zero, we have $F_2(r) = 1$, which means that firm 2 no longer has mass point at the reservation price. Then for firm 1, advertising and charging r is the same as not advertising and charging r , since in either case, firm 1 is selling only to its loyal to earn a profit of rL_1 . From another perspective, firm 1's total pricing distribution (taking into account both the advertising case and non-advertising case) is:

$$\alpha_1 F_1(p) = 1 - \frac{L_2}{S} - \frac{L_1r(L_2+S)}{Sp(L_1+S)}, \quad (23)$$

which matches the pricing distribution given by Narasimhan(1988). Actually, any convex combination of the equilibrium that is the special case of Proposition 1 and the equilibrium

that is characterized in Narasimhan(1988) is an equilibrium in our model. And all of the equilibria generate the same profits for both firms.

To study the effect of the parameters (ϕ, r, θ, S) on the equilibrium behavior of the firms, we compute the comparative statics of α_i and $F_i(p)$ w.r.t. the parameters. Here $\theta = L_1 - L_2$ is a measure of the degree of asymmetry of the market. Basically, the loyal and shopper sizes of the market can be fully characterized by the pair (S, θ) . The comparative statics results are summarized in Table 1. We also include the comparative statics for the symmetric case as a benchmark.

	Range 1				Range 2				Symmetric	
	α_1	α_2	F_1	F_2	α_1	α_2	F_1	F_2	α_i	F_i
ϕ	-	o	-	-	-	-	-	?	-	-
r	+	o	-	-	+	+	-	?	+	-
θ	-	o	-	-	o	+	+	?	N/A	N/A
S	+	o	+	+	+	+	+	?	+	+

+:positive effect, -: negative effect, o: no effect

N/A: not applicable

Table 1: Comparative Statics

We are able to calculate analytically the comparative statics for all the variables with respect to the four parameters except for $F_2(p)$ in range 2. Most of the comparative statics results are consistent with economic intuitions. As the advertising fee ϕ increases, the advertising frequency reduces(except α_2 in range 1, which is constantly one) due to the higher cost of advertising, and when the firm advertises, the average price charged by the firms increases to cover the higher cost. As the reservation price r increases, firms will advertise more often since the potential gain by advertising is higher. Also, the raise of the reservation price allows the firms to charge a higher average price in the equilibrium. The increase of the number of shoppers, S , will increase the incentive of the firms to competing for the shoppers. So in equilibrium, the probabilities to advertise increase, and the amounts of discounts are larger for both firms.

The above comparative statics are all consistent with the results for the symmetric case. A nice feature of the model is that the asymmetric structure of our model enables us to study

the effect that the degree of asymmetry, θ on the equilibrium strategies of the firms. As θ increases, we see a decrease of advertising probability for firm 1 in range 1 and an increase of advertising probability for firm 2 in range 2. The reason is that the increased degree of asymmetry reduces the incentive of the larger firm to advertise while raise the incentive of the smaller firm to advertise. A less intuitive result is for the price distributions. As θ increases, in range 1, firm 1 will shift its price distribution to the right; in range 2, however, firm 1 will shift its price distribution to the left. The reason for this result is as follows. The direct effect of increase of the degree of asymmetry will make firm 1 charge a higher price and firm 2 charge a lower price. However, if firm 1 increases its price, it will have an indirect effect on firm 2 to cause firm 2 to increase the price in equilibrium. The final results of the comparative statics will be dependent on which effect has a larger magnitude. In range 1, the direct effect on firm 2 is smaller than the indirect effect so firm 2 will raise its average price. In range 2, the direct effect on firm 2 is larger than the indirect effect so firm 2 will lower its price.

4 Information Gatekeeper's Optimal Fee Decision

Foreseeing the firm's behavior given the fixed advertising fee, ϕ , the gatekeeper will set the value of ϕ to maximize his own profit in the equilibrium. The gatekeeper has a fixed setup cost, K , which is assumed to be zero without loss of generality. Then the expected profit the gatekeeper get is:

$$E\pi_G = (\alpha_1 + \alpha_2)\phi. \quad (24)$$

While choosing the optimal advertising fee, the gatekeeper faces the trade off between profit per advertising and the frequency of advertising. Higher fee will increase the profit per advertising but decrease the frequency of advertising. Proposition 2 describes the gatekeeper's best strategy.

Proposition 2 *Assume the gatekeeper can set the advertising fee $\phi \in [0, \frac{rS}{2}]$ to maximize its profit. Then at equilibrium, the optimal fee of the gatekeeper is:*

1. When $3L_1 \leq 1 + 2L_2$,

$$\phi^* = \frac{rS}{4}. \quad (25)$$

The market will be within range 2 as described in proposition 1. And the expected profit of the gatekeeper is:

$$E\pi_G = \frac{1}{4}rS\frac{L_1 + S}{L_2 + S}. \quad (26)$$

2. When $3L_1 > 1 + 2L_2$,

$$\phi^* = \frac{rS(L_1 - L_2)}{1 + L_1 - 2L_2}. \quad (27)$$

The market will be at the boundary of range 1 and range 2 described in proposition 1. And the expected profit of the gatekeeper is:

$$E\pi_G = rS\frac{2(1 - L_2)(L_1 - L_2)}{(1 + L_1 - 2L_2)^2} \quad (28)$$

Proof. See Appendix. ■

Figure 3 demonstrates the optimal strategy of the gatekeepers in the loyal size space $L_1 - L_2$. The shaded area represents all the possible combinations that L_1 and L_2 can take, given the restriction that $L_1 \geq L_2$ and $L_1 + L_2 < 1$. The closer the pair (L_1, L_2) is to the 45-degree line, the less asymmetric is the market. When the pair is in the vertical shaded area, the optimal fee for the gatekeeper is $\frac{rS}{4}$. And when the pair is in the horizontal shaded area, the optimal fee for the gatekeeper is $\frac{rS(L_1 - L_2)}{1 + L_1 - 2L_2}$. When $3L_1 > 1 + 2L_2$, we have $\frac{rS(L_1 - L_2)}{1 + L_1 - 2L_2} > \frac{rS}{4}$, which implies that as the market gets more asymmetric, the gatekeeper will charge higher advertising fee. The primary reason for this is that as the market gets more asymmetric, firm 2's willingness to advertise increases and the gatekeeper increases the advertising fee to extract more profit from firm 2.

5 Conclusions

The paper studies the equilibrium behavior of the firms and the information gatekeeper in a duopoly market with asymmetric sizes of loyal consumers. Our findings show that the asymmetry in the size of loyal consumer base does affect firms' advertising and pricing strategy. In equilibrium, the firm with more loyal consumers tends to advertise less frequently but charge a lower average price whenever advertising. The reason it advertises less is due to the higher opportunity cost by advertising and charge price that is lower than the reservation price. When it decides to advertise, however, it can afford to charge a lower price since it have a larger consumer base.

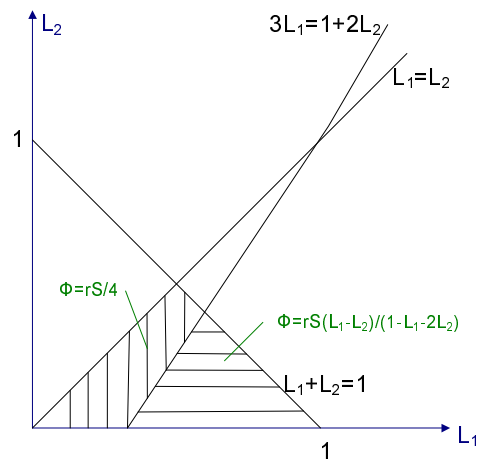


Figure 3: Illustration of Gatekeeper's Optimal Fee Decision

The gatekeeper's behavior will also be affected by the asymmetry structure of the market. The optimal advertising fee charged by the information gatekeeper will depend on the degree of asymmetry of the market. As the market becomes more asymmetric, the gatekeeper will charge a higher advertising fee. The reason is that as the market becomes more asymmetric, the firm with less loyal customers is willing to pay a higher amount to compete for the shoppers.

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6 Appendix

6.1 Proof of Proposition 1

Case 1

For case 1, we have $\alpha_2 = 1$ and $0 < \alpha_1 < 1$. Therefore, firm 1 must be indifferent between advertising and not advertising, i.e. $E\pi_1^{NA} = E\pi_1^A(p)$. If we plug in the expression for the profits, we have

$$p(L_1 + (1 - F_2(p))S) - \phi = L_1r. \quad (29)$$

Solve this gives the expression for $F_2(p)$ as in Proposition 1. Plug in the limits value for the domain of $F_2(p)$, we have $F_2(\underline{p}_1) = 0$ and $F_2(r) = 1 - \frac{\phi}{rS} < 1$. This implies that firm 2's price distribution has mass point at the reservation price r . Therefore it must hold for firm 2 that: (1) $E\pi_2^A(\underline{p}_1) = E\pi_2^A(r)$, (2) $E\pi_2^A(p) = E\pi_2^A(\underline{p}_1)$. Plugging in the profit expression gives us:

$$\underline{p}_1(L_2 + s) - \phi = r(L_2 + (1 - \alpha_1)s) - \phi, \quad (30)$$

$$p(L_2 + (1 - \alpha_1 F_1(p))s) - \phi = \underline{p}_1(L_2 + s) - \phi. \quad (31)$$

Solving these two equations yields the solution for firm 1's strategy in Proposition 1. It is easy to verify that $F_1(\underline{p}_1) = 0$ and $F_1(r) = 1$. Notice that from the argument given in Narasimhan (1988), since $F_2(p)$ has mass point at r , $F_1(p)$ will not have positive density at r , for the reason that firm 1 can always do strictly better than charging r by undercutting r by a small amount.

Notice that this equilibrium will hold only when $\underline{p}_2 < \underline{p}_1$ holds, i.e.,

$$\frac{rL_2 + \frac{1}{2}(1 - \alpha_1)rS + \phi}{l_2 + S} < \frac{rL_1 + \phi}{L_1 + S}, \quad (32)$$

which simplifies to $\phi < rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$. Since $\frac{L_1 - L_2}{1 + L_1 - 2L_2} < \frac{1}{2}$, the equilibrium that we characterized will only exist under a limited range of listing cost.

The equilibrium profit can be calculated as: $E\pi_1 = E\pi_1^{NA}$, $E\pi_2 = E\pi_2^A(\underline{p})$.

The above proof for Case 1 is also valid when $\phi = 0$.

Case 3

For case 3, we have $0 < \alpha_i < 1$. This can happen only when $\underline{p}_1 = \underline{p}_2$, which is equivalent to:

$$\frac{rL_1 + \frac{1}{2}rS(1 - \alpha_2) + \phi}{L_1 + S} = \frac{rL_2 + \frac{1}{2}rS(1 - \alpha_1) + \phi}{L_2 + S}. \quad (33)$$

Since both firm have mixed strategies in pricing, both firms will have positive probability density at the lowest price boundary, we have $E\pi_1^A(p) = E\pi_1^A(\underline{p}_1)$ and $E\pi_2^A(p) = E\pi_2^A(\underline{p}_2)$, which give us:

$$pL_1 + pS(1 - \alpha_2 F_2(p)) - \phi = \underline{p}_1(L_1 + S) - \phi \quad (34)$$

$$pL_2 + pS(1 - \alpha_1 F_1(p)) - \phi = \underline{p}_2(L_2 + S) - \phi \quad (35)$$

Until now in our model, we are not sure which firm will have a positive density at r . There are three possibilities: (1) firm 1 has a mass point (2) firm 2 has a mass point (3) neither firm has a mass point so at equilibrium both firm will have a positive density at r .

If firm i has positive probability density at r , then the condition $E\pi_i^A(r) = E\pi_i^A(\underline{p}_i)$ has to hold. So we have:

$$rL_i + rS(1 - \alpha_j) - \phi = \underline{p}_i(L_i + S) - \phi, \quad (36)$$

which yields

$$\alpha_j = 1 - \frac{2\phi}{rS} \quad (37)$$

First we argue that the third possibility can not happen. If both firm have positive density at r , then (37) will hold for both firms. Therefore we have $\alpha_1 = \alpha_2$. If we plug this into the expression for $\underline{p}_1, \underline{p}_2$, we would find $\underline{p}_1 \neq \underline{p}_2$ as long as $L_1 \neq L_2$, which contradicts with (33).

Now we consider possibility 1, if firm 1 has a mass point at r , then (37) will be valid for firm 1. Basically, the system of equilibrium strategies is fully described by equations (33),(34),(35), and (37) for firm 1.

We can solve for $F_1(p)$:

$$F_1(p) = \frac{1}{\alpha_1} \left(1 - \frac{rL_2 - pL_2 + \frac{1}{2}rS(1 - \alpha_1) + \phi}{pS} \right) \quad (38)$$

$$= \frac{r(pSL_1 + pS^2 + pL_1L_2 + pL_2S - rL_1L_2 - 2\phi S - rSL_1 - 2\phi l_2)}{p(rS - 2\phi)(2L_2 + S - L_1)} \quad (39)$$

$F_1(\underline{p}_1) = 0$. However, $F_1(r) = \frac{S+L_2}{S+2L_2-L_1} > 1$. This contradicts with our assumption that firm 1 will have mass point at r . So possibility 1 is not a valid solution for equilibrium.

Now lets consider possibility 2, this time the system of equilibrium strategies is fully described by equations (33),(34),(35), and (37) for firm 2. Solving the systems of equations gives the following equilibrium solution as described in Proposition 1.

Notice that the solution satisfies required properties. It is obvious that $0 \leq \alpha_1 \leq 1$. When $\phi > rs \frac{L_1 - L_2}{1 + L_1 - L_2}$, we have $0 \leq \alpha_2 < 1$. And it can be verified that $F_1(p_1) = 0$, $F_1(r) = 1$, $F_2(p_2) = 0$ and $F_2(r) = \frac{L_1 + S}{2L_1 + S - L_2} < 1$. Therefore, at equilibrium, firm 2 will have mass point at r and firm 1 will not have positive density at r .

And the expected profits can be calculated as: $E\pi_1 = E\pi_1^{NA}$, $E\pi_2 = E\pi_2^{NA}$ since both firms are mixing in advertising in equilibrium.

6.2 Proof of Proposition 2

Since there are two ranges in our equilibrium. The approach of this proof is to find the optimal fee within each range, then to choose the larger one of the two local maximums.

The gatekeeper's expected profit:

$$E\pi_G = (\alpha_1 + \alpha_2)\phi. \quad (40)$$

In range 1 of the equilibrium, we have:

$$E\pi_G = \left[\frac{L_2 + S}{L_1 + S} \left(1 - \frac{\phi}{rS}\right) + 1 \right] \phi. \quad (41)$$

This function is concave in ϕ . Solving the first order condition w.r.t. ϕ gives the optimal fee, $\phi^* = \frac{rS}{2} \left(1 + \frac{L_1 + S}{L_2 + S}\right) > \frac{rS}{2}$. This optimal fee is out of the domain where range 1 equilibrium is sustained. So we have a corner solution for range 1:

$$\phi^1 = \frac{rS(L_1 - L_2)}{(1 + L_1 - 2L_2)}, \quad (42)$$

with the corresponding profit: $E\pi_G^1 = rS \frac{2(1-L_2)(L_1-L_2)}{(1+L_1-2L_2)^2}$

In range 2 of the equilibrium, we have

$$E\pi_G = \frac{rS - 2\phi}{rS} \left(1 + \frac{2L_1 + S - L_2}{L_2 + S}\right) \phi. \quad (43)$$

The first order condition w.r.t. ϕ yields the local maximum:

$$\phi^* = \frac{rS}{4} \quad (44)$$

Therefore, if $\frac{rS}{4} \geq rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$, i.e., $3L_1 \leq 1 + 2L_2$, the optimal fee in range 2 is: $\phi^2 = \frac{rS}{4}$. And the corresponding profit $E\pi_G^2 = \frac{1}{4} rS \frac{L_1 + S}{L_2 + S} > E\pi_G^1$. Therefore $\frac{rS}{4}$ is the advertising fee that global maximum is achieved. On the other hand, if $\frac{rS}{4} < rS \frac{L_1 - L_2}{1 + L_1 - 2L_2}$, i.e., $3L_1 > 1 + 2L_2$, we have $\phi^2 = \frac{rS(L_1 - L_2)}{(1 + L_1 - 2L_2)}$, which is the same solution as in range 1. As a result, it will also be the global maximum.

Summarizing the above arguments leads to the results in Proposition 2.