Endogenous Government Spending and Fiscal Stimulus
(Incomplete Version)

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April 2, 2010

Abstract

Government spending multiplier is usually defined as a factor of proportionality that measures how much output changes in response to a change in the exogenous government spending shock. This assumption is inconsistent with the reality that fiscal stimulus are themselves endogenous and state dependent. This paper developed a endogenous regime switching framework in a rational expectation DSGE model to study the counter-cyclical fiscal policy—the unproductive government spending increases only in recession, the regime switch is triggered when certain endogenous variables cross the specified threshold. The model justifies the important role of the counter-cyclical fiscal policy. By comparing the exogenous and endogenous government spending, the model finds out the multiplier effect can be very different in the short and long run.

JEL Classification: E62 H30 H50

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1 Introduction

Conventional wisdom defines fiscal stimulus effects—the output and consumption multipliers derivatives of two endogenous variables with respect to an exogenous fiscal shock, the shock is not a fundamental structural parameter of the economy, invariant to the state of the economy. However, the fiscal stimulus effects are themselves endogenous and state dependent. Two important features are missing if assume government spending is exogenous: first, the exogenous government spending creates symmetric responses, in other words, $1 increase in government spending is treated to have the same effect in boom and recession, this assumption is inconsistent with the facts that most of the countries run counter-cyclical discretionary fiscal policy, it works against the cyclical tendencies in the economy, especially stimulate the economy when it is in a downturn, which delivers an asymmetric fiscal impact only occurs in recession. Second, exogenous government spending usually normalize the exogenous government shock to be $1, however, it is reasonable for people to expect the large size of fiscal expansion under deep recession and small size of fiscal expansion under moderate recession, since the aggressive fiscal expansion is usually given under the exceptional circumstances, but not for more normal fluctuations. Therefore, both the timing and size of the fiscal expansion depend on the current state of the economy, it would be misleading to talk about multiplier effects without taking the state of the economy into account.

Overall, the discretionary fiscal policy, although “discretionary”, has always been implemented implicitly when certain economic conditions occur, for instance a fall in GDP, an increase in unemployment or an increase in inflation. Such rule-based discretion which defined as “formula flexibility” by Musgrave (1959) has regained consensus recently. It is an automatic pilot that is tailored to structural relationships of the economy and its dynamics to reduce the time lapse between recognition and administrative lags and would reduce the discretionary action to narrower limits. This rule capture both endogenous features in discretionary fiscal policy: first, the fiscal stimulus package is triggered only when the recession is believed to begin; second, the size of the fiscal stimulus package depends crucially on the economic condition.

This paper studies the state dependent government spending multiplier effect in a conventional dynamic stochastic general equilibrium (DSGE) model with endogenous government spending regime switch. I consider an economy with nominal pricing frictions, no capital, and a monetary authority that follows a standard Taylor rule. The fiscal policy studied here is with unproductive government spending, the government spending follows an endogenous regime switch rule: low output in the current period, coupled with decreasing expected output, triggers a more aggressive government spending regime, used to reflect the “recession scare” fiscal policy in reality. In addition, the level of aggressive government spending is determined by the deviation of output from some deterministic output target. The endogenous government spending rule is different from anticipated government spending shock because the latter is able to deliver expectation effect by fail to respond systematically to the aggregate economic condition.

First I solved the simplified two period model that admits analytical solution to highlight two distinctive effects—expectation and stimulus effects. In the second part of the paper,
I further investigate the interaction between the two effects by solving the full nonlinear dynamic model, the expectation of higher government spending next period has a distorted contraction effect for the current period, while the stimulus effect generates the government spending path reacts purposefully and systematically to changes in the aggregate economics, which is able to stabilize output well above steady state during recession. There are tradeoffs between the two effects, therefore the definition of multipliers can be important to evaluate exogenous and endogenous government spending. The impact multiplier measures the short run stimulus effect, thus the exogenous government spending has greater effect since the negative expectation effects disappear. The present value multiplier considers the own dynamics of the government spending process within long horizon and measures the long run stimulus effect, the endogenous government spending performs better since the fiscal policy is purposeful to stabilize the output.

In the third step of the analysis I incorporate uncertainty into the endogenous rule. The fiscal policy rule, even if exist, is hardly to be credible for political reasons. In this part, the output lower bound to trigger the fiscal stimulus is not deterministic, the probability to implement the fiscal expansion follows a logistic function, the lower the output, the higher probability to trigger the stimulus. (incomplete)

The paper is organized as follows. Section 2 review the related literature; section 3 lay out the model along with the specification for fiscal policy and monetary policy; section 4 uses a simplified two period model to gain insight into the working of the model; section 5 introduce the numerical method and plot the decision rules; section 6 analyze the dynamic effects of the endogenous government spending and the corresponding asymmetric distribution; section 7 compare the exogenous and endogenous government spending multiplier; section 8 incorporate uncertainty to the threshold that triggers the regime switch; section 9 points out a few directions for future research.

2 Contacts with the Literature

One of the key contribution of this paper is to examine the state dependent government spending effect. The most recent papers begin to concern the multiplier effect when an economy hits the zero nominal interest bound after the severe contraction that hit the U.S and the world economy in 2008. Christiano, Eichenbaum and Rebelo (2009) found the government spending multiplier increases to 3.9 when the zero nominal interest bound is binding because the central bank loses the ability to stimulate the economy to avoid deflation and an increase in government spending lowers desired national savings. This research shed some light on the different fiscal expansion effects when the state of economy is considered (when the monetary policy loses control).

On the other hand, the rule-based discretionary fiscal policy is not a new proposal. There are some discussions about the design of the combination of the automatic stabilizer and discretionary fiscal policy—the “automatic” discretionary fiscal policy. The aim of such proposals is a faster decision making process, shielded from political interference, that ensures a timely fiscal response. Blanchard (2000) suggests the traditional automatic stabilizers is to
have temporary fiscal policy changes triggered by economic developments. Elmendorf and Furman (2008) observe that automatic stabilizers operate like a dial with gradual adjustments to shocks, whereas an automatic discretionary policy operates more like a switch with a substantial fiscal effort being applied above a trigger threshold. Baumsgaard and Symansky (2009) summarize some rule design issues about the economic trigger indicators including “below normal” output and a three-month cumulative decline in payroll employment, also forward-looking triggers. All these policy analysis provide good economic sense but without any theoretical modeling.

This paper combines the rule based fiscal policy with the regime switching model. Some recent work embeds regime switching process for policy in DSGE model, assuming that policy behavior switches exogenously among different exogenous rules for the evolution of policy variables (Andolfatto and Gomme, 2003). Davig and Leeper (2006) use a threshold-style methods for endogenizing regime changes and study the dynamics in monetary policy, monetary policy responses to inflation more aggressively if current inflation reaches the threshold, the endogenous rule delivers dynamics which are absent when regimes switch exogenously. By applying threshold endogenous regime switch to fiscal policy, this paper is able to model the rule based fiscal policy in a DSGE framework.

3 The model

In the following I outline a conventional New Keynesian model with elastic labor supply and inelastic capital. Nominal rigidities are introduced through Rotemberg quadratic adjustment costs (Rotemberg, 1982). Nominal rigidity is introduced to justify the role of government intervention. The model also incorporates habit formation often seen in the class of DSGE models estimated with data.

3.1 Households

We assume a large number of identical households chooses \{C_t, N_t, B_t\}, with separable preferences to maximize:

\[
\max \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - \gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right)
\]

Here \(\sigma\) describes the coefficient of relative risk aversion and the inverse of intertemporal elasticity of substitution, the higher \(\sigma\), the stronger incentive to smooth consumption. The parameter \(\varphi\) is the inverse of Frisch elasticity of labor supply, high value of \(\varphi\) results in large response of labor supply to wage. \(\gamma\) represents the importance of habit persistent, higher \(\gamma\) increases the cost to adjust consumption. Finally, the parameter \(\chi\) controls the overall disamenity of work.

\(C_t\) is composite consumption consisting of differentiated goods \(C_t(i)\)

\[
C_t = \left( \int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} \, di \right)^{\frac{1}{\varepsilon}}
\]
The demand function for each good \( i \) is:

\[
C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t
\]

(3)

The household’s budget constraint is

\[
C_t + \frac{B_t}{P_t} + T_t \leq W_t N_t + \frac{R_{t-1} B_{t-1}}{P_t} + \int_0^1 \Gamma(i)
\]

(4)

where \( B_t \) is one-period nominal bond holdings, \( T_t \) is lump-sum taxes/transfers, \( W_t \) is real wage, \( R_{t-1} \) is the risk-free nominal interest rate between \( t-1 \) and \( t \), \( \Gamma(i) \) is the profit from firm \( i \).

The household maximize (1) subject to (4), the first order conditions are:

\[
\lambda_t = (C_t - \gamma C_{t-1})^{-\sigma} - \beta \gamma (E_t C_{t+1} - \gamma C_t)^{-\sigma}
\]

(5)

\[
\chi N_t = \lambda_t W_t
\]

(6)

\[
\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}
\]

(7)

The household’s optimization problem require (5)-(7) to hold every period. In addition, the transversality condition for bond must hold.

\[
\lim_{T \to \infty} E_t q_{t,T} \frac{B_T}{P_T} = 0
\]

(8)

where \( q_{t,T} = R_{t-1}/(P_T/P_t) \)

### 3.2 Firms

There are two types of firms: competitive final goods producer and monopolistically competitive intermediate goods producers who produce a continuum of differentiated goods.

A final goods producer produces the final composite good using the constant returns to scale technology

\[
Y_t = \left( \int_0^1 y_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{1-\varepsilon}}
\]

(9)

Following Rotemberg(1982), we assume that each intermediate goods-producing firm faces a quadratic cost to changing its nominal price. Since the intermediate firm chooses its price \( p_t(i) \) and the price adjustment cost involves the price in period \( t \) and \( t-1 \), the representative intermediate firm’s maximization problem becomes dynamic:

\[
\max_{n_t(i), p_t(i)} E \sum_{t=0}^\infty \beta^t \lambda_t \left[ \left( \frac{p_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - W_t n_t(i) - \frac{\psi}{2} \left( \frac{p_t(i)}{\pi p_{t-1}(i)} - 1 \right)^2 Y_t \right]
\]

(10)

subject to:

\[
y_t(i) = A_t n_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\varepsilon} Y_t
\]

(11)
The technology $A_t$ follows AR(1) process:

$$\ln A_{t+1} = \ln \bar{A} + \rho \ln A_t + \varepsilon_{t+1}^a$$  \hspace{1cm} (12)

where $\varepsilon_{t}^a \sim N(0, \sigma^2)$

Assuming a symmetric equilibrium, in which all intermediate goods producing firms make the same decision, we can drop the $i$ subscripts so that $y_t(i) = Y_t$, $n_t(i) = N_t$, $p_t(i) = P_t$, after simplifying the first order condition, we can get

$$\psi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = (1 - \varepsilon) + \varepsilon W_t + \beta \psi E_t \left( \lambda_t \left[ \frac{\pi_{t+1}}{\pi} - 1 \right] \left[ \frac{Y_{t+1}}{Y_t \pi} \right] \right)$$  \hspace{1cm} (13)

Finally, the aggregate resource constraint is given by:

$$Y_t = C_t + G_t + \psi \left[ \frac{\pi_t}{\pi} - 1 \right] 2Y_t$$  \hspace{1cm} (14)

### 3.3 Fiscal Policy Specification

Fiscal policy is defined as a set of rules determining government spending. The government’s demand for goods is in the same proportion as households, the government’s demand is:

$$G_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} G_t$$  \hspace{1cm} (15)

where $G_t = \left( \int_0^1 G_t(i)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$

The government budget constraint is:

$$G_t = T_t + B_t + \frac{R_{t-1} B_{t-1}}{P_t}$$  \hspace{1cm} (16)

The government buys the composite final good and throw it to the ocean, paying for it with lump-sum taxes. Government debt makes no difference to the equilibrium here, hence I assume the government clear the budget by lump-sum tax on a period by period basis: $T_t = G_t$.

The indicator of the endogenous fiscal policy rule should be carefully chosen, it seems natural that where a household determines whether an economy is in a recession, the discretionary fiscal measures could be linked to a more aggressive regime. In addition, fiscal regime can not change too frequent, the indicator should also include forward-looking behavior so that only severe recessions trigger the fiscal response. In this section, I choose the switching indicator as output gap, defined as the deviation of real output from the potential output, the potential output is normalized to steady state output in the beginning and is subject to change in different experiments.

I assume a fiscal policy process that permits the government to vary its response to contemporaneous output as well as expected output, the expected output is obtained conditional
on keeping the government spending unchanged:

\[
\ln G_{t+1} = \ln \bar{G} + \rho \ln G_t + \phi(s_t)(Y_t - \bar{Y})
\]

\[
\phi(s_t) = \begin{cases} 
\phi & \text{if } Y_t - \bar{Y} < 0 \text{ and } EY_{t+1} < Y_t \\
0 & \text{otherwise}
\end{cases} \quad (\phi < 0)
\]

The government spending is determined only by its own lagged value in the boom and determined by both the lagged government spending and the current output gap in recession. This rule reveals two facts: first, the regime switching rule could be understood as nonlinear policy rule deliver asymmetric response, the fiscal expansion is only triggered in recession; second, the degree of fiscal expansion is also state contingent determined by the current output gap, the more severe of the recession, the stronger response of the government spending.

In this model, the household with rational expectation has complete information regarding the policy making process. At period \(t\) they observe all past and current variables; they form expectation of future output incorporate the effects that supply shocks have on the probability distribution over the policy rules. Although at period \(t\), the household know the policy regime at period \((t+1)\) with certainty, that does not mean they know all the possible future regimes since they depends on the sequence of realizations of technology shock \(A_t\).

### 3.4 Monetary Policy Specification

Monetary authority follows a simple Taylor rule, implements its policy by adjusting the nominal interest rate \(R_t\), in response to the inflation rate:

\[
R_t = R\pi_t^\alpha
\]

with \(\alpha > 1\), interest rate responses to inflation more than one for one.

### 4 A Two-Period Version

Before launching into the numerical solution of the infinite-horizon model, I consider a simplified two period version with flexible price to gain insight into the working of the two distinctive effects, expectation and stimulus effect in the model.

The representative household maximizes the expected discounted sum of utilities:

\[
U(C_1, N_1) + \beta U(C_2, N_2)
\]

The household chooses consumption in the first period, and the consumption in the second period incorporates the consumption habit:

\[
U(C_1, N_1) = \frac{C_1^{1-\sigma}}{1-\sigma} - \frac{N_1^{1+\varphi}}{1+\varphi}
\]
\[ U(C_2, N_2) = \frac{(C_2 - \gamma C_1)^{1-\sigma}}{1 - \sigma} - \chi \frac{N_1^{1+\varphi}}{1 + \varphi} \]  

(22)

Subject to the budget constraints:
\[ C_1 + B = W_1 N_1 - T_1 + \Gamma_1 \]  

(23)
\[ C_2 = W_2 N_2 + (1 + r) B - T_2 + \Gamma_2 \]  

(24)

When prices are flexible, each monopolistic producer i sets its relative price as a markup over the marginal cost
\[ \frac{P_t(i)}{P_t} = \mu_t \frac{W_t}{MPN_t} \]  

(25)

where \( MPN_t \) is the marginal product of labor and \( \mu_t \) is the markup. In a symmetric equilibrium the above expression reduces to:
\[ W_t = \frac{MPN_t}{\mu_t} \]  

(26)

Flexible price allocates resource efficiently, the markup is constant at steady state \( \varepsilon/(\varepsilon-1) \).

Since the production function has constant returns to scale \( Y_t = A_t N_t \), \( MPN_t = A_t \). In the equilibrium, hours are pinned down by the labor market condition:
\[ \lambda_1 = C_1^{-\sigma} - \beta \gamma (C_2 - \gamma C_1)^{-\sigma} \]  

(27)
\[ \lambda_2 = (C_2 - \gamma C_1)^{-\sigma} \]  

(28)
\[ \chi N_t^{\sigma} \lambda_t = A_t \frac{\varepsilon - 1}{\varepsilon} \quad t = 1, 2 \]  

(29)

The aggregate resource constraint is \( Y_t = C_t + G_t, t = 1, 2 \), the same endogenous government spending rule is applied in the simplified model. Assume the technology is exogenous and stochastic following a two state Markov process with \( A \in \{A_L, A_H\} \) and transition matrix:
\[
\begin{bmatrix}
P_{LL} & P_{LH} \\
P_{HL} & P_{HH}
\end{bmatrix}
\]  

(30)

In period 1, household observes the low technology level and output decreases below the potential, if household also expects the low technology is persistent, then the higher government spending will be triggered in period 2. Without loss of generality, if assume \( A_1 = A_2 = A_L \), then \( G_1 = G_L \) and \( G_2 = G_H \), agents expect higher government spending in period 2 with certainty. The two period model allows us to concentrate specifically on the role of expectation effect in period 1.
Log-linearize the first order conditions (27), (28), (29) and the aggregate resource constraint yield

\[\hat{\lambda}_1 = -\frac{\sigma(1 + \beta\gamma^2\sigma(1 - \gamma)^{-\sigma})}{1 - \beta\gamma(1 - \gamma)^{-\sigma}}\hat{c}_1 + \frac{\beta\gamma\sigma(1 - \gamma)^{-\sigma}}{1 - \beta\gamma(1 - \gamma)^{-\sigma}}\hat{c}_2\]

(31)

\[\hat{\lambda}_2 = \frac{\sigma\gamma}{1 - \gamma}\hat{c}_1 - \frac{\sigma}{1 - \gamma}\hat{c}_2\]

(32)

\[\hat{n}_t = \frac{1}{\varphi}(\hat{\lambda}_t + \hat{a}_t) \quad t = 1, 2\]

(33)

\[\hat{c}_t = \frac{1}{c}(\hat{a}_t + \hat{n}_t - g\hat{g}_t) \quad t = 1, 2\]

(34)

Using the method of minimal state variable, the equilibrium can be solved as a function of \(a_t\) and \(g_t\). Because the real wage in this simplified model is constant, the key effect at work here is the expectation effect of higher government spending. With flexible prices, the expectation effect arises from habit persistence and wealth effect. In this model Ricardian equivalence holds, hence expected higher government spending in period 2 crowd out private consumption (equation(34)), the marginal utility of consumption in period 1 is affected by habit persistence (equation(32)), therefore the wealth effect takes place and affects the equilibrium labor in period 1.

\[\hat{n}_1 = \psi_{na}\hat{a} + \psi_{ng1}\hat{g}_1 + \psi_{ng2}\hat{g}_2\]

(35)

The expectation effect of higher government spending on current equilibrium labor is determined by the parameter \(\psi_{ng2}\). To provide additional intuition for the determinants of the parameter \(\psi_{ng2}\), Figure 1 displays \(dn_1/dg_2\) for various parameter configurations. In each case I perturb one parameter at a time relative to the benchmark parameter values.
Since $g_2$ is positive, it is unambiguous that expectation of higher government spending reduces current equilibrium labor under all parameter choices. The (1,1) element of Figure 1 shows the habit persistence is the driving force of the expectation effect, if $\gamma = 0$, government spending in period 2 has no effect on equilibrium labor in period 1. When $\gamma$ starts to increase, the decreased future consumption dominates the marginal utility of consumption, current labor begins to decrease, however, there is a kink when $\gamma$ is around 0.5, when $\gamma$ is higher, the incentive of consumption smooth is very strong, consumption in period 1 also decrease, offsets the negative effect on marginal utility of consumption as well as labor. The (1,2) element shows $dn_1/dg_2$ is an increasing function of $\sigma$, $\sigma$ is the coefficient of relative risk aversion, bigger $\sigma$ indicates the stronger incentive to smooth consumption, therefore the smaller change in marginal utility of consumption brings the smaller change in labor. The (2,1) element shows $dn_1/dg_2$ decreases more as $\varphi$ increases, represents the lower the Frisch elasticity of labor supply, the more decrease in labor. The (2,2) element shows that as discount factor $\beta$ increases the labor decreases. The smaller is $\beta$, the more impatient of consumers, hence they tend to consume more and work more today.

The expectation effect generates contraction in period 1, however, higher government spending in period 2 causes labor and output to rise. The relationship can be observed by the equilibrium labor in period 2:

$$\hat{n}_2 = \phi_{na}\hat{a} + \phi_{ng1}\hat{g}_1 + \phi_{ng2}\hat{g}_2$$ (36)

The stimulus effect of higher government spending is determined by the parameter $\phi_{ng2}$. Figure 2 displays $dn_2/dg_2$ for various parameter configurations. Same as before, in each case I perturb one parameter at a time relative to the benchmark parameter values.
The sign of $dn_2/dg_2$ is always positive and not sensitive to the parameter choice, although the size might be affected by parameter combinations. The $(1,1)$ and $(1,2)$ elements represent the desire to smooth consumption, higher $\gamma$ and $\varphi$ increase marginal utility of consumption in period 2, increase labor more through wealth effect. Notice here the habit persistence parameter is an increasing function to labor but the dynamic does not crucially depends on $\gamma$, the stimulus effect exists even when exclude habit. The effects from element $(2,1)$ and $(2,2)$ are similar with Figure 1, only the effect of discount rate $\beta$ reverses now, if agent is impatient ($\beta$ is small), he chooses to work more in period 1 thus work less in period 2.

5 Dynamic Model: Numerical Solution

The expectation effect is highlighted in the static model through the habit persistence and wealth effect. The mechanism remains at work in the dynamic model, in the presence of monopolistic competition and sticky prices, the interaction among hours, consumption, inflation and real wage become richer, but the same expectation effect plays important role. In this section, the infinite-horizon model is solved numerically, the nature of expectation effect is described by the comparison between endogenous regime switching and fixed regime.

5.1 Calibration

<table>
<thead>
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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<td>$\beta$</td>
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<td>$\gamma$</td>
<td>parameter of habit persistence</td>
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<tr>
<td>$\rho_g$</td>
<td>autoregressive parameter of g process</td>
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</tr>
</tbody>
</table>

TABLE 1. Calibration in Numerical Simulation

The model is calibrated at a quarter frequency, the parameter describing preferences, technology and price adjustment are set to be consistent with Rotemberg and Woodford (1997) and Woodford (2003), these parameter values are typical of macro business cycle
literature. $\chi$ is set so the state state share of time spent in employment is 0.33. Steady state output is set to 1 and steady state inflation is set to 0.5 percent. If parameter adjustment parameter $\psi = 70$ means 66 percent of firms cannot reset their price each quarter, here I set $\psi = 20$ to reflect lower level of price stickiness. I limit a discussion to $\phi$, the switching parameter in the endogenous government spending rule, $\phi$ is negative to describe high government spending once the regime switching is triggered. Larger absolute value of $\phi$ has two distinctive effects, stronger expectation effects as well as stronger response of government spending level to output gap.

5.2 Decision Rule

The full nonlinear model is solved using numerical algorithm monotone map method (Davig and Leeper, 2006). This method discretizes the state space requires a set of initial guess and finds a fixed point in decision-rules for each point in the state space. Let $\Theta_t$ denote the state space at date $t$, the solutions converge to functions that map the minimum set of state variables $\Theta_t = (\ln A_t, \ln G_t, C_{t-1})$ into values for the endogenous variables: hours $N_t$, inflation $\pi_t$ and marginal utility of consumption $\lambda_t$.

The endogenous threshold switching model delivers two distinctive features do not observe in fixed regime:

**Asymmetry:** With fixed regime, agents know that fiscal policy will respond symmetrically next period to technology shock regardless of the sign of the shock. Threshold regime switching induces agents to expect stronger fiscal responses next period whenever the negative technology shock is severe enough to push the current output below the threshold and expect an even lower output, this asymmetric expectation has a distortion effect on agent’s behavior in the current period even before the aggressive fiscal policy takes place, which can not observe in the fixed regime. In contrast, a positive technology shock increase the output, the fiscal expansion is not triggered, the solution under switching rule and fixed regime should converge, the expectation formation effects disappear.

**Nonlinearity:** The discretionary government spending rule is designed to respond systematically to the deviation of current output from the target, for example, there can be dramatic increase in government spending during severe recession, therefore the magnitude of the expectation effect can be sensitive to the current economic state, generates the non-linear equilibrium which depends upon the size of negative technology shock.
Contemporaneous response of hours, inflation and marginal utility of consumption to a technology shock: when fix $G_t = 0.2$ and $C_{t-1} = 0.8$

Figure 3 describes the contemporaneous response for hours, inflation and marginal utility of consumption as a function of technology shock, the equilibrium is a four-dimensional decision rule, here I fix two state variables $G_t$ and $C_t$ to isolate the technology shock effect. The steady state technology on the x-axis is about 3.03, to the left of 3 indicates a negative technology shock and to the right of 3.03 indicates a positive technology shock. A large negative technology shock, can cause agent to expect more aggressive government spending in the subsequent period. In this model with sticky prices and flexible wages, the expected high government spending crowd out expected consumption, notice the habit formation plays an important role for the dynamics, drives the contemporaneous consumption down, this negative demand shock push down the real wage and price level, the inflation rate with switching is lower than the fixed regime. In addition, the lower consumption today...
increases the marginal utility of consumption in the current period and decreases it in the next period, we can observe higher marginal utility of consumption than fixed regime after a negative technology shock. Since wage is flexible, the negative supply shock drives the wage rate down, when the decrease in wage exceed the increase in marginal utility of consumption, hours decrease. As the technology begin to recover, the expectation formation effects become weaker and weaker and converge to fixed regime in the end.

FIGURE 4.
Contemporaneous response of hours, inflation and marginal utility of consumption to a negative technology shock at different G level: when fix $A_t = 2.97$ and $C_{t-1} = 0.8$.

The decision rule also depends on the current state of government spending. Figure 4 illustrate the response of hours, inflation and marginal utility of consumption to a negative technology shock at different current government spending level. The higher the government
spending at period $t$, the less likely that the fiscal expansion will be triggered by the same negative technology shock, because the higher level of government spending is able to maintain output above steady state output.

6 Dynamic Impacts of Government Spending

One special feature of the rule-based discretionary fiscal policy is to respond automatically to different processes of supply shocks, a well-designed policy rule should be robust regardless of the own dynamic of exogenous shocks. In this section, I will simulate the impulse response function under endogenous government spending, exogenous government spending and no policy regime, testing different effects of rule-based counter-cyclical discretionary fiscal policy.

![Graphs showing dynamic impacts of government spending](Image)

**FIGURE 5.**
Responses to a negative technology shock. In deviations from steady state.
Figure 5 describes three policies to a 4 standard deviation negative shock to technology, which translates to a 3 percent drop in the level of steady state technology, which can be treated as a severe recession. The endogenous switching government spending (blue dashed line) responses automatically to the current output, while the exogenous government spending (red solid line) repeat the first period response of endogenous switching, the no policy case (green dashed line) keeps government spending at steady state level. The first striking difference exists between the endogenous government spending and no policy regime. With counter-cyclical discretionary fiscal policy, output almost recovers to the steady state level after 6 quarter, while output is still below steady state after 35 quarters if no fiscal policy is conducted. At the same time, due to the higher level of government spending, consumption is crowded out and inflation rises. However, the large increase in output (+1.5%) is in the cost of small changes in consumption (−0.8%) and inflation (−0.3%), this facts illustrate the strength and provide justification for the counter-cyclical fiscal policy. The second difference lies on endogenous government spending and exogenous government spending. The exogenous government spending is usually a one time increase, while the endogenous government spending generates a smooth path. The difference between the two can be observed by the comparison of the pictures in the lower left corner and upper left corner that one time exogenous government spending shock corresponding to the supply shock is not sufficient to close the output gap, as the endogenous government spending continue to rise until the 8th period, it is able to recover the output steadily. Even I increase the 1 period response of exogenous government spending, the output will still increase in the first period then decrease, increase the volatility of output, and this is because the exogenous government spending fails to respond systematically to the economic condition.

As the decision rule imply, the endogenous government spending regime switch creates the asymmetry and nonlinearity. The distribution under fixed regime is usually symmetric if assume normal shocks, the distribution under exogenous regime switching is a combination of two symmetric normal distribution, while the distribution under endogenous regime switching is skewed since the high government spending is only triggered under extreme economic conditions, the skewness reflects the fact that what are more likely to occur using the endogenous government spending rule.
Distributions of Output, Labor, Consumption and Government Spending with less active response to output ($\phi = -1$, left) and more active response to output ($\phi = -5$, right).

Figure 6 illustrates the different distributions of economic variable with less active response to output ($\phi = -1$, left column) and more active response to output ($\phi = -5$, right column) from 3000 random draws of technology shock, for comparison purpose, the red solid line is the corresponding normal distribution curve. Under both circumstances, government spending responses asymmetrically to the output, government spending increases less when it responds less active, and vice versa. Therefore, the more active response (right column) produces a severely skewed distribution, output and hours is more right skewed, with tails extend far above the steady state, while consumption is more left skewed, with high government spending, it is more likely that consumption is below the state state. As the absolute value of $\phi$ decreases, the asymmetric government spending response becomes weaker and the skewness also becomes weaker, produce more volatilities in the output. This experiment is an example that symmetric distributed technology shock can produce asymmetric response of economic variables if certain endogenous fiscal policy is applied. The skewness arises mainly from the expansion effect of government spending, even the contraction expectation
effect exist under, the endogenous government spending dampens the volatility of output and stabilize it above the steady state.

7 Government Spending Multiplier

The conventional way to estimate government spending multiplier the ratio of change in output and change in government spending, this multiplier is usually called impact multiplier. In Christiano, Eichenbaum and Rebelo (2009), the multiplier is given by:

\[
\text{Multiplier} = \frac{dY_t}{dG_t}
\]  

(37)

This definition is a little problematic since the process of government spending is usually serially correlated, only consider the contemporaneous effect is biased.

Blanchard and Perotti (2002) defined the government spending multiplier as the increase in the level of output k periods ahead in response to an increase in government spending at time t:

\[
\text{Multiplier}(k) = \frac{dY_{t+k}}{dG_t}
\]

(38)

This definition does not consider the discount rate implies a unit increase in output 50 years in the future is treated as equivalent to a unit increase in output in the current year. Without discounting, the multiplier can be misleading. Here I follow Mountfor and Uhlig (2009), Davig and Leeper (2009), use a present value government spending multiplier:

\[
\text{Present Value Multiplier} = \frac{E_t \sum_{j=0}^{k} \prod_{i=0}^{j} R_{t+i} \Delta Y_{t+k}}{E_t \sum_{j=0}^{k} \prod_{i=0}^{j} R_{t+i} \Delta G_{t+k}}
\]

(39)

This multiplier report the change in the present value of additional output over different horizons generated by a unit change in the present value of government spending. In the rest of the paper, I report both the impact multiplier and present value multiplier, the effect of exogenous and endogenous government spending can be reversed if different multiplier is applied.

<table>
<thead>
<tr>
<th></th>
<th>(\frac{\Delta Y}{\Delta G})</th>
<th>1st quarter</th>
<th>5 quarters</th>
<th>10 quarters</th>
<th>25 quarters</th>
<th>(\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous Multiplier</td>
<td>-.2270</td>
<td>-.0402</td>
<td>-.0709</td>
<td>-.1026</td>
<td>-.1063</td>
<td></td>
</tr>
<tr>
<td>Endogenous Multiplier</td>
<td>-.3465</td>
<td>-.0273</td>
<td>-.0293</td>
<td>-.0321</td>
<td>-.0322</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2.
Exogenous and Endogenous Output Multipliers
Table 2 reports both impact multiplier and present value output multiplier when government spending is exogenous and endogenous, the technology process and dynamic responses is the same as Figure 5 described. The first column is the impact multiplier, it is easy to notice that the exogenous multiplier is greater than the endogenous multiplier, this is due to the absence of contractionary expectation effect in the exogenous multiplier. Column 2 to 4 are present value multipliers under different time horizon, for the exogenous multiplier, it always has the greatest impact the first period, then decay gradually regardless of the economic condition, in contrast, by systematically responding to the output, the endogenous government spending is able to close the output gap within 6 quarters thus produces larger present value multiplier than exogenous government spending.

<table>
<thead>
<tr>
<th></th>
<th>(\frac{\Delta C}{\Delta G}) after 1st quarter</th>
<th>5 quarters</th>
<th>10 quarters</th>
<th>25 quarters</th>
<th>(\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous Multiplier</td>
<td>-1.2746</td>
<td>-.0977</td>
<td>-.1689</td>
<td>-.2421</td>
<td>-.2506</td>
</tr>
<tr>
<td>Endogenous Multiplier</td>
<td>-1.3782</td>
<td>-.1063</td>
<td>-.1917</td>
<td>-.2858</td>
<td>-.2975</td>
</tr>
</tbody>
</table>

**TABLE 3.**
Exogenous and Endogenous Consumption Multipliers

Table 3 reports the the consumption multiplier. In the current setup of the model, government spending crowd out private consumption, thus both the expectation effect and stimulus effect push consumption down toward the same direction. The exogenous consumption multiplier is strictly greater than the endogenous consumption multiplier. If expected government spending is higher in the future, it crowd out private consumption in the current period as well as all future periods. The higher output multiplier is in the cost of lower consumption, however, the endogenous government spending is still rewarded in terms of the increase in output overweight the decrease in consumption, this is because the expectation effect pushes the price and inflation down, therefore reduces firm’s adjustment cost.

8 Uncertain Threshold

Although it is nontrivial for the agents to form rational expectations of regimes two or more periods in the future, the next period regime is known exactly. However, profound uncertainty surrounds monetary and fiscal policy behavior, it can be qualitatively important. In order to incorporate the uncertainty in fiscal behavior and the corresponding expectation effect, I posit that the probability of switching to higher government spending is a decreasing function of that current output, the lower the output, it is more likely that government spending will go up next period. A logistic function governs the evolution of the probability.

\[
p = \frac{1}{1 + A \exp(B(X - C))}
\]

where parameters A, B and C determine the shape of the logistic function. To isolate the expectation effect, I specify two logistic functions to represent the low uncertainty and high uncertainty.
The red line represents low uncertainty, once the output is below steady state, it is more likely that the high government spending regime will be triggered next period, when output is 0.95, the regime switching next period is almost certain. For comparison purpose, the blue line represents relative high uncertainty about the future regime.

Incomplete.

9 Future Work

The endogenous government spending is able to close the output more efficiently by systematically respond to the output gap. However, the policy specification is not really attractive for the contractionary expectation effect, and the transmission mechanism is the crowding out of private consumption. Empirically, several authors (Blanchard and Perotti (2002), Perotti (2007)) have found that government spending shocks cause real wage to rise, hence crowd in private consumption, while other authors (Ramey (2008)) found consumption multiplier is slightly negative. However, all the empirical work is limited in its ability to measure multipliers for the period from 1948 onward by the lack of variation in government spending and it can vary during different time horizons.

The endogenous government spending would be more appealing if high government spending crowd in private consumption from the model. Therefore the expectation effect can move towards the same direction to stimulus effect. There could be several ways to obtain more
interesting results. First, introduce productive government spending, the revision can create
demand side effect as well as supply side, deliver more expansion effect. Second, the fiscal
policy can be more expansionary with the interaction of monetary policy. If the monetary
policy switches to “passive” together with the increasing in government spending when the
threshold is reached, the expectation effect can be reversed. By sacrificing the price stabil-
ity, the real interest rate decrease to crowd in consumption, therefore creates expansionary
expectation effect.

Overall, the asymmetric policy behavior is an interesting area to explore. The business
cycle research is very mature but the asymmetry economic behavior between boom and re-
cession is rarely modeled. Endogenous regime switching is the good starting point to think
about the asymmetry phenomenon, it is also interesting to look for the evidence from the
data.
A Appendix

A.1 Solving the Two-Period Model

Log linearize the first order condition and aggregate resource constraint yield

\[
\hat{\lambda}_1 = \frac{\sigma (1 + \beta \gamma \sigma (1 - \gamma)^{-\sigma - 1})}{1 - \beta \gamma (1 - \gamma)^{-\sigma}} \hat{c}_1 + \frac{\beta \gamma \sigma (1 - \gamma)^{-\sigma - 1}}{1 - \beta \gamma (1 - \gamma)^{-\sigma}} \hat{c}_2
\]

(41)

\[
\hat{\lambda}_2 = \frac{\sigma \gamma}{1 - \gamma} \hat{c}_1 - \frac{\sigma}{1 - \gamma} \hat{c}_2
\]

(42)

\[
\hat{n}_t = \frac{1}{\varphi} (\hat{\lambda}_t + \hat{a}_t) \quad t = 1, 2
\]

(43)

\[
\hat{c}_t = \frac{1}{c} (\hat{a}_t + \hat{n}_t - g \hat{g}_t) \quad t = 1, 2
\]

(44)

Substitute equation (41)(42)(44) to (43), the equilibrium labor in period 1 can be solved

\[
\psi_n \hat{n}_1 = \psi_a \hat{a} + \psi_{g_1} \hat{g}_1 + \psi_{g_2} \hat{g}_2
\]

(45)

where

\[
\psi_n = \varphi + \frac{b_1}{c} + \frac{b_2 b_3}{\varphi c^2 + b_4 c}
\]

\[
\psi_a = 1 + \frac{b_2 - b_1}{c} + \frac{b_2 (c + b_3 - b_4)}{\varphi c^2 + b_4 c}
\]

\[
\psi_{g_1} = \frac{g b_1}{c} - \frac{g b_2 b_3}{\varphi c^2 + b_4 c}
\]

\[
\psi_{g_2} = \frac{g b_2 b_4}{\varphi c^2 + b_4 c} - \frac{g b_2}{c}
\]

The expectation effect of government spending in period 2 on equilibrium labor in period 1 can be described by the parameter \(\psi_{g_2}\), the magnitude of \(\psi_{g_2}\) depends upon the parameter choice in the model (Figure 1).

The equilibrium labor in period 2 can be solved

\[
\phi_n \hat{n}_2 = \phi_a \hat{a} + \phi_{g_1} \hat{g}_1 + \phi_{g_2} \hat{g}_2
\]

(46)

where

\[
\phi_n = 1
\]

\[
\phi_a = \frac{c + b_3 - b_4}{\varphi c + b_4} - \frac{b_3}{\varphi c + b_4} \times \frac{\psi_a}{\psi_n}
\]

\[
\phi_{g_1} = -\frac{\psi_{g_1}}{\psi_n} \times \frac{b_3}{\varphi c + b_4} - \frac{g b_3}{\varphi c + b_4}
\]

\[
\phi_{g_2} = -\frac{\psi_{g_2}}{\psi_n} \times \frac{b_3}{\varphi c + b_4} + \frac{g b_4}{\varphi c + b_4}
\]

The stimulus effect of government spending on equilibrium labor in period 2 can be described by the parameter \(\phi_{g_2}\), the magnitude of \(\phi_{g_2}\) depends upon the parameter choice in the model (Figure 2).
A.2 Equations in Numerical Simulation

The model is solved using the monotone map method from Coleman (1991). The algorithm is to discretize the state space around the steady state $\Theta_t = \{lnA_t, lnG_t, C_{t-1}\}$, then conjecture an initial guess of decision rules for labor, inflation and marginal utility of consumption, the initial guess is obtained by solving the simplified model with habit persistence and price stickiness. The other endogenous variables can be solved using the initial decision rules. At each point in the state space, substitute the decision rules to household and firm’s first order condition $(47)(49)(50)$, the state variables in period $t+1$ is determined endogenously by equation $(51)(52)(53)(54)$. The expectation is calculated by Gaussian quadrature. At each point of the state space, numerical integration is used to yield a nonlinear system of equations until the decision rules converge.

$$\lambda_t = (C_t - \gamma C_{t-1})^{-\sigma} - \beta\gamma(E_t C_{t+1} - \gamma C_t)^{-\sigma}$$ (47)

$$\chi N_t = \lambda_t W_t$$ (48)

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}$$ (49)

$$\psi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = (1 - \varepsilon) + \frac{\varepsilon W_t}{A_t} + \beta \psi E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{\pi_{t+1}}{\pi} - 1 \right] \frac{Y_{t+1} \pi_{t+1}}{Y \pi} \right)$$ (50)

$$Y_t = C_t + G_t + \psi \left[ \frac{\pi_t}{\pi} - 1 \right]^2 Y_t$$ (51)

$$\ln A_{t+1} = \ln \bar{A} + \rho^a \ln A_t + \varepsilon^a_{t+1}$$ (52)

$$\ln G_{t+1} = \ln \bar{G} + \rho^g \ln G_t + \phi(s_t)(Y_t - \bar{Y})$$ (53)

$$\phi(s_t) = \begin{cases} 
\phi & \text{if } Y_t - \bar{Y} < 0 \text{ and } EY_{t+1} < Y_t \quad (\phi < 0) \\
0 & \text{otherwise}
\end{cases}$$ (54)
B References


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