Characterizing the Effects of Entitlement Reform in a Perpetual Youth Model*

(Preliminary and Incomplete)

Alexander W. Richter†

April 2, 2010

ABSTRACT

The U.S. federal government is facing the prospect of exponentially rising entitlement obligations that are threatening to push the debt-to-GDP ratio to historically unparalleled levels. In an effort to understand the economic consequences of these projected debt run-ups, this paper takes a stand on how future policy must adjust and examines its implications for the aggregate economy. Assuming the monetary authority never relents on its policy to aggressively fight inflation, delaying reform forces taxes continuously higher and increases the probability of the economy reaching the fiscal limit, the point at which increases in taxes are no longer feasible. Recognizing the potential economic consequences of reaching the fiscal limit, this model places the fiscal authority in a horse race to pass reform prior to the economy hitting the fiscal limit. If reform is passed prior to reaching the fiscal limit, tax policy is able to remain passive, as transfers switch to a stable process that holds indefinitely. If, on the other hand, the fiscal limit is reached, taxes become active and, assuming government default is not possible, the monetary authority is forced to passively adjust the nominal interest to rising inflation. Although this policy adjustment helps to stabilize debt, it fails to address the underlying problem. Without reform, transfers continue to follow an unstable path, and, without a clear direction from Congress or the aid of monetary policy, inflation begins to pose a significant challenge for the aggregate economy.

Keywords: Finite Lifetime, Monetary Policy, Fiscal Uncertainty, Fiscal Limit
JEL Classification: E62; E63; H63

*Prepared for the annual Jordan River Conference at Indiana University, April 16, 2010. I thank my advisors Eric Leeper and Todd Walker for many helpful conversations.
†Correspondence: Department of Economics, Indiana University, Bloomington, IN 47405, USA. Phone: (812) 855-8580. E-mail: richtera@indiana.edu.
I INTRODUCTION

According to its most recent budget outlook, the Congressional Budget Office (CBO) is projecting an explosive path for the evolution of federal debt held by the public relative to GDP (see Figure 1). Specifically, under the assumption that current law remains in effect, the CBO projects that by 2035 federal debt held by the public will reach nearly 80 percent of GDP and by 2050 it will swell to over 125 percent of GDP. Nearly all of these projected increases in government debt are the result of increases in the growth of spending on the largest three entitlement programs—Medicare, Medicaid, and Social Security. By 2050, CBO projections indicate that total spending on entitlement programs is expected to rise from 10.1 to 17.9 percent of GDP, of which roughly three-quarters can be attributed to increases in Medicare spending (see Figure 2). For historical context, debt has exceeded 100 percent of GDP only for a short period of time during World War II, peaking at 113 percent of GDP in 1945. However, unlike the period following the war, when debt relative to GDP fell sharply over the following two decades, budget shortfalls are projected to continue widening for the foreseeable future (Congressional Budget Office, 2009).

Although these figures are illustrative of the fiscal stress that may lie ahead, they are rather uninformative about the potential consequences of debt run-ups and the implications for future economic outcomes. The basic reason is because projections of this nature fail to account for the fact that agents are forward-looking. Well before the debt-to-GDP ratio reaches these projected levels, the perceived riskiness of government debt will rise and cause substantial increases in nominal interest rates on Treasury securities, particularly at the long end of the yield curve. Given investors’ awareness of these projections, the lack of a response of interest rates indicates that agents are placing little, if any, probability on the government defaulting. Thus, it seems reasonable to conclude that agents strongly believe there will be an adjustment in future fiscal policy that will place the debt-to-GDP ratio on a sustainable path. However, since policymakers fail to provide agents with any indication of if, and when, policy might adjust, agents are faced with a tremendous amount of uncertainty surrounding fiscal policy. In an effort to understand the potential consequences of debt run-ups, this paper builds on the work of Davig, Leeper, and Walker (2010) by taking a stand on how future policy will adjust and examining its implications for the aggregate economy.

In order to characterize adjustments in future fiscal policy, it is important to first consider what policy options are possible. Assuming sovereign default is not possible, if the central bank remains committed to fighting inflation by adjusting the nominal interest rate more than one-for-one with inflation (Taylor, 1993), the only policy options are for the fiscal authority to either adjust taxes or pass reform that reduces the growth rate in entitlement spending. It is clear, however, that

1 This assumes that the tax cuts of 2001 and 2003 will not be renewed and income guidelines for AMT will not be changed. Of course, an even more grim picture is portrayed if expiring tax provisions are renewed or AMT income limits become indexed to inflation.
2 High projected growth rates in Medicare spending are the result of both an aging population and substantial increases in the cost of health care. Even after passing through the baby-boomers generation, Medicare costs are still projected to rise exponentially.
3 The CBO also points out that projected increases in the debt-to-GDP ratio are larger than any developed countries have experienced in the post-World War II era.
4 For a detailed analysis of the implications of default within the context of a DSGE model with an imbedded fiscal limit see Bi (2010) and Bi and Leeper (2010)
a long-term solution to rising growth rates of debt cannot be obtained by consistently adjusting future tax policy. Eventually, as taxes continue to rise, the economy will either reach the peak of its Laffer curve, where higher taxes no longer yield increased revenues, or the political resistance will become so great that increasing tax rates is no longer a viable policy option.

Of course, it may be the case that the fiscal authority is seeking reform while using tax policy to only temporarily finance the rise in government debt. Recognizing the potential economic consequences of reaching the fiscal limit, this approach places the fiscal authority in a horse race to pass reform prior to the economy hitting the fiscal limit. If reform is passed prior to reaching the fiscal limit, government transfers are placed back on a stable trajectory and the economic consequences of a rapidly rising debt are lessened. However, as time passes without reform, the economic consequences of inaction rise and the likelihood of reaching the fiscal limit becomes increasingly probable.

Upon reaching the fiscal limit, regardless of whether it is due to political or economic reasons, the monetary authority is forced to passively adjust the nominal interest rate to rising inflation. By switching to a passive monetary policy, inflation is allowed to drift away from its target level and, with rapidly rising debt, begins to creep upward. The higher price level then reduces the relative value of past debt while also increasing the amount of revenues from taxes. Thus, the fiscal limit approaches.

---

5 Trabandt and Uhlig (2009) find that some countries are already at or near the peaks of their Laffer curve.

6 This terminology follows Leeper (1991). Passive monetary policy implies that the monetary authority only weakly adjusts the nominal interest rate to changes in inflation, whereas an active monetary authority targets inflation by sufficiently adjusting nominal interest rates to pin down inflation. Active fiscal policy implies that the fiscal authority sets the tax rate independently of the size of government debt, while passive fiscal policy implies that the fiscal authority adjusts taxes in response to changes in the level of debt in an effort to stabilize the debt-to-GDP ratio.
authority is able to lean on the monetary authority to stabilize the increases in debt that are caused by the explosive transfers process while continuing to forego entitlement reform.

This approach, however, is not without economic consequences. First, it has been well-documented that even moderately higher inflation can reduce growth and employment levels (Fischer, 1993; Bruno and Easterly, 1998). Thus, even if this policy is able to stave off hyper-inflationary situations, the economic effects of a prolonged period of heightened inflation may have significant implications for the aggregate economy. Second, it fails to address the underlying problem. Without reform, transfers continue to follow an unstable path, and, without a clear direction from Congress or the aid of monetary policy, inflation continues to pose a significant challenge.

Even after reaching the fiscal limit, the possibility of reform remains. By passing reform, transfers are indefinitely placed on a stable path, drastically helping to reduce the growth rate of government debt and inflation. Although reform has the potential to be passed before or after the fiscal limit, the economic implications of delaying reform are clear. By reaching the fiscal limit, taxes are at their maximum level and no longer able to freely respond to any further increases in government debt. Given that the fiscal authority is pinned to an active policy, the existence of a unique bounded equilibrium requires the monetary authority to remain passive. Thus, by reaching the fiscal limit, the economy is relegated to a world where the fiscal authority is determining the price level for the foreseeable future.

1.1 Economic Framework

In order to analyze the impact of entitlement reform, I incorporate an infinite period overlapping generations (OLG) setup into a dynamic stochastic general equilibrium (DSGE) model with cap-
Characterizing the Effects of Entitlement Reform in a Perpetual Youth Model

Capital accumulation, monopolistic competition, costly price adjustment, distorting taxes on capital and labor, monetary policy, and fiscal uncertainty. The primary advantage of using this setup over more traditional OLG frameworks is its flexibility. Since the average lifetime of each agent can be parameterized, this model allows for the possibility of either finitely- or infinitely-lived agents while still maintaining clean analytical results at the aggregate level. The presence of finitely-lived agents allows this model to capture important distributional effects across labor types and generations that are otherwise not attainable in a representative agent framework. Another advantage of this model is that it breaks down Ricardian equivalence, which is a critical component for analyzing the consequences of debt run-ups. More specifically, the death parameter, which also measures the deviation of this model from the traditional Ricardian equivalent framework, can be adjusted to manipulate the relative consequences of higher debt levels.

In the benchmark model, the monetary authority is assumed to remain active by following a simple Taylor rule, switching to a passive policy only in the event that the fiscal limit is reached. This model contains several mechanisms that deliver fiscal uncertainty. The fiscal authority has control over tax and transfers policy, whose states are governed by first-order two-state Markov processes. Prior to passing reform, government transfers, which serve as a proxy for entitlement benefits, are assumed to initially follow an unstable path that is both persistent and stochastic. Before a potential fiscal limit is reached, taxes are assumed to remain passive, by following a simple rule that sets the distortionary tax rate in response to deviations of lagged real government debt (as a percentage of GDP) from its steady-state level. The probability of passing reform is an increasing function of the debt-to-GDP ratio, a feature that is consistent with the increasing political pressure for reform that is associated with a rising debt. As is consistent with the notion of a Laffer curve, the probability of reaching the fiscal limit is a positive function of the tax rate. In both cases, the evolution of the probabilities is governed by a logistic function, whose parameters can be adjusted to analyze the effects of both probabilities sensitivity to its response variable.

Agents are unable to predict if, and when, reform will occur and whether the fiscal limit will be reached. If reform is passed prior to reaching the fiscal limit, tax policy is able to remain passive, as transfers switch to a stable process that holds indefinitely. If, on the other hand, the fiscal limit is reached, taxes become active and, with no other policy option available, the monetary authority is forced to fall into an absorbing state under passive policy. Given that taxes are positively related to debt, the outcome of the race to pass reform prior to reaching the fiscal limit will be critically dependent on the sensitivity of tax rates to increases in debt. The greater the response of taxes, the greater the likelihood of reaching fiscal limit and, therefore, the urgency to pass reform. In order to further understand the implications of delaying reform, the model also allows for the possibility of reform after the fiscal limit is reached. In this event, the monetary authority continues to passively adjust the nominal interest rate, since the fiscal authority is no longer able to adjust its tax rate in response to increases in debt.

The following section lays out the complete nonlinear model. Section 3 considers a simpler model, that helps the reader gain some intuition for how the price level is determined within the context of a perpetual youth model. Specifically, I derive analytical results and conduct simulations that show how the conventional policy regions are affected by a positive probability of death and the presence of a fiscal limit. The remaining section lays out a plan for future work and concludes.
2 Complete Model

The following model is a discrete time variant of the Yaari (1965)-Blanchard (1985) perpetual youth model\(^\text{7}\). The model has the following features. It includes both an endogenous labor supply decision and a choice of money holdings. Consumption goods and labor services are both supplied under monopolistic competition. Firms are subject to costly price adjustments. Agents face uncertainty regarding not only the duration of their lifetime and the trajectory of their economic variables but also regarding fiscal policy. Specifically, each period agents are subject to the possibility of the economy reaching its fiscal limit and/or the fiscal authority passing entitlement reform. Within the context of a modern DSGE model, this framework allows for the evaluation of how agents’ decision paths and the central bank’s ability to control inflation are altered in the face of growing fiscal uncertainty.

2.1 Individuals

Assume all agents are subject to identical probabilities of death, \(\vartheta\). Population dynamics are eliminated from the model, so that birth and death rates are constant and equalized. The size at birth of generation \(s\) is normalized to \(\vartheta\), implying that the time \(t\) size of generation \(s\) is \(\vartheta(1 - \vartheta)^{t-s}\). Thus, the total population size at time \(t\) over all generations is \(\sum_{s=-\infty}^{t} \vartheta(1 - \vartheta)^{t-s} = 1\). The average lifetime of a member of generation \(s\) is given by \(\sum_{t<s} \vartheta(1 - \vartheta)^{t-s-1} = 1/\vartheta\). Thus, when \(\vartheta \to 0\), this model reduces to the more traditional infinite horizon setup.

Each member, \(j\), of generation \(s \leq 0\) maximizes expected lifetime utility of the form\(^\text{8,9}\)

\[
E_0 \sum_{t=0}^{\infty} [\beta(1 - \vartheta)]^t \log \left[ c_{s,t}(j)^{1-\kappa} \left( \frac{m_{s,t}(j)}{P_t} \right)^{\kappa} - V(n_{s,t}(j)) \right], \quad 0 < \kappa < 1
\]

where \(\beta \in (0, 1)\) is the subjective discount factor, \(P_t\) is the aggregate price index, and \(c_{s,t}(j), m_{s,t}(j),\) and \(n_{s,t}(j)\) are, respectively, consumption of the final good, nominal money balances, and the quantity of labor supplied at time \(t\) by agent \(j \in [0, 1]\) born at time \(s\).\(^\text{10}\) Agent \(j\)’s disutility from work is determined by the strictly increasing and convex function \(V\).

Following Dixit and Stiglitz (1977), assume the consumption bundle at \(t\) for an agent born at time \(s \leq t\), denoted \(c_{s,t}(j)\), is composed of a continuum of differentiated goods, \(c_{s,t}(i,j)\), \(i \in [0, 1]\). Using a CES production function, aggregate consumption is then given by \(c_{s,t}(j) \equiv \int_0^1 c_{s,t}(i,j)(\epsilon-1)/\epsilon d\hat{i}\epsilon/\epsilon-1\), where \(\epsilon > 1\) measures the elasticity of intertemporal substitution across different varieties of consumption goods. The corresponding aggregate price index, \(P_t\),

\(^{7}\)A nonstochastic discrete time variant of the Blanchard-Yaari model was first considered by Frenkel and Razin (1986).

\(^{8}\)This unconventional utility function follows Ascari and Rankin (2007), which they note is needed in order to rule out the possibility of a negative labor supply. Alternatively, a non-negativity constraint could used, but this approach would compromise the clean analytical form of aggregate equations.

\(^{9}\)A continuous time monetary version of the Blanchard-Yaari model was first introduced by van der Ploeg and Marini (1988). For a discrete time variant see Cushing (1999).

\(^{10}\)In general, I denote individual values by lower case letters and aggregate values by capital letters.
is given by \( P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} di \right]^{1/(1-\varepsilon)} \), where \( P_t(i) \) denotes the price of good \( i \). The familiar demand for good \( i \) by agent \( j \), \( c_{s,t}(i,j) = [P_t(i)/P_t]^{-\varepsilon} c_{s,t}(j) \), corresponds to the agent’s maximum attainable consumption bundle given a specific level of expenditures.

As is conventional in the Blanchard-Yaari setup, assume agents have no bequest motive and, instead, sell contingent claims on their assets to insurance companies. Each period, assets are collected from \( \vartheta \) agents who passed away and subsequently transferred to the remaining survivors. Assuming a perfectly competitive life insurance industry, each surviving agent receives a premium payment of \( \vartheta/(1-\vartheta) \). Incorporating the gross return on the insurance contract, \( 1 + \vartheta/(1-\vartheta) = 1/(1-\vartheta) \), into the per-period budget constraint for a surviving agent yields

\[
\int_0^1 P_t(i)c_{s,t}(i,j)di + P_t k_{s,t}(j) + m_{s,t}(j) + b_{s,t}(j) \leq P_t \omega_{s,t}(j) + (1-\vartheta)^{-1}P_t a_{s,t}(j),
\]

where \( b_{s,t}(j) \) and \( k_{s,t}(j) \) denote the level of nominal riskless government bonds and the stock of capital carried into period \( t+1 \) and \( R_t \) is the gross nominal interest rate on bonds purchased at time \( t \). \( \omega_{s,t}(j) \) represents individual \( j \)'s net labor income and is given by

\[
\omega_{s,t}(j) \equiv [1 - \tau_{s,t}(j)] \frac{w_{s,t}(j)}{P_t} n_{s,t}(j) + z_{s,t}(j) - \frac{d_{s,t}(j)}{P_t},
\]

where \( w_{s,t}(j) \) is the nominal wage received by agent \( j \), \( \tau_{s,t}(j) \) is a distorting tax rate, \( z_{s,t}(j) \) are transfers that the government is legally obligated to pay to agent \( j \), and \( d_{s,t}(j) \) denotes the share of nominal firm profits received by agent \( j \). \( a_{s,t}(j) \) represents beginning of the period financial wealth and is denoted by

\[
a_{s,t}(j) \equiv [1 - \tau_{s,t}(j)]R_t^k k_{s,t-1} + (1-\delta)k_{s,t-1} + \frac{m_{s,t-1}}{P_t} + \frac{R_{t-1}b_{s,t-1}}{P_t},
\]

where \( \delta \) is the time-invariant depreciation rate and \( R_t^k \) denotes the nominal rental rate on capital holdings, \( k_{s,t}(j) \).

Following Annicchiarico, Giammarioli, and Piergallini (2006), each agent’s labor is supplied in a monopolistically competitive setting, whereby each agent \( j \) is assumed to face the following demand function for her labor services\(^{11}\)

\[
n_{s,t}(j) = \left( \frac{w_{s,t}(j)}{W_t} \right)^{-\eta_t} N_t, \tag{5}
\]

where \( \eta_t > 1 \) is the elasticity of substitution between differential labor units, \( N_t \) is the aggregate level of employment, and the corresponding aggregate wage rate is given by

\[
W_t = \left[ \sum_{s=-\infty}^t \int_0^{\vartheta(1-\vartheta)^{t-s}} w_{s,t}(j)^{1-\eta_t} dj \right]^{\frac{1}{1-\eta_t}}. \tag{6}
\]

\(^{11}\)This relationship is derived from firms maximizing the amount of labor they hire for a given level of expenditures and then aggregating across all firms. Further details are given in Section 2.3.
Given a set of prices, \( \{P_t, R_t, R^k_t\} \), individual \( j \) from generation \( s \) chooses a sequence of quantities \( \{c_{s,t}(j), m_{s,t}(j), n_{s,t}(j), k_{s,t}(j), b_{s,t}(j)\} \) to maximize (1) subject to (2)-(5). Optimality then yields the following first order conditions

\[
1 = \beta R_t E_t \left\{ \frac{\bar{c}_{s,t}(j)}{c_{s,t+1}(j)} \frac{P_t}{P_{t+1}} \right\},
\]

\[
1 = \beta E_t \left\{ \frac{\bar{c}_{s,t}(j)}{c_{s,t+1}(j)} \left[ (1 - \tau_{s,t+1}(j))\frac{R^k_{t+1}}{P_{t+1}} + (1 - \delta) \right] \right\},
\]

\[
m_{s,t}(j) = \frac{\kappa}{1 + \kappa} \frac{R_t}{1 - \kappa} c_{s,t}(j),
\]

\[
\frac{w_{s,t}(j)}{P_t} = (1 - \kappa)^{-1} \left( 1 + \mu^w \right) \frac{P_t c_{s,t}(j)}{m_{s,t}(j)} V'(n_{s,t}(j)),
\]

which are satisfied only if agent \( j \) is still alive at time \( t \). Adjusted consumption, \( \bar{c}_{s,t}(j) \equiv c_{s,t}(j) - \left[ P_t c_{s,t}(j)/m_{s,t}(j)\right]^n V(n_{s,t}(j)) \), represents agent \( j \)’s level of consumption above a given subsistence level\(^{12}\) and \( \mu^w \equiv 1/(q_t - 1) \) denotes the exogenous optimal wage markup. Note that these conditions reveal that certain variables are invariant across both individuals and generations. Given that individuals from every generation face identical interest rates, equation (9) shows that individuals from all generations will choose the same ratio of real money balances to consumption, a condition that is required for money to serve as an adequate medium of exchange. This result, along with the fact that perfectly flexible wages in a symmetric equilibrium imply identical wage rates across all individuals and generations (i.e. \( w_{s,t}(j) = W_t \)), implies that the equilibrium labor supply is also constant across generations. These conditions will become critically important when aggregating across individuals and generations.

Define the real stochastic discount factor (SDF) for agent \( j \) who was born at time \( s \) as

\[
q_{t,t+1}(s,j) \equiv \beta \frac{\bar{c}_{s,t}(j)}{c_{s,t+1}(j)}.
\]

Then the no Ponzi game condition can be written as

\[
\lim_{T \to \infty} E_t \left\{ (1 - \varrho)^{T-t} q_{t,T}(s,j) a_{s,T}(j) \right\} = 0,
\]

where \( q_{t,T}(s,j) \equiv \prod_{k=1}^{T} q_{k-1,k}(s,j) \) and \( q_{t,t}(s,j) \equiv 1 \). Combining (7) and (11) yields

\[
E_t \left\{ q_{t,t+1}(s,j) \frac{P_t}{P_{t+1}} \right\} = \frac{1}{R_t},
\]

for each agent \( j \) from generation \( s \in (-\infty, t] \). Thus, the average SDF is also invariant across individuals and generations. To derive individual \( j \)’s consumption function, first use (8), (11), and (13) to rewrite (2) in terms of the period-\( t \) price of her portfolio, which has a random value \( a_{s,t+1}(j) \)

\(^{12}\)This interpretation was introduced in Ascari and Rankin (2007). They point out that this specification of the utility function can be rewritten into a form that resembles a Stone-Geary utility function.
in the next period. Then solve the resulting budget constraint forward, impose the no Ponzi game condition, and use (11) to obtain
\[ \tilde{c}_{s,t}(j) = \chi \left[ \frac{a_{s,t}(j)}{1 - \vartheta} + h_{s,t}(j) - \frac{1}{1 - \kappa} E_t \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} q_{t,T}(s,j) \psi_{s,T}(j)^{-\kappa} V(n_{s,T}(j)) \right], \quad (14) \]

where \( \chi \equiv (1 - \kappa)[1 - \beta(1 - \vartheta)] \) is a time- and generation-invariant parameter, \( \psi_{s,t}(j) \equiv [\kappa/(1 - \kappa)][R_t / (R_t - 1)] \) denotes real money balances in terms of consumption goods, and \( h_{s,t}(j) \equiv E_t \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} q_{t,T}(s,j) \omega_{s,T}(j) \) represents human (non-asset based) wealth.

2.2 AGGREGATION

The aggregate values, denoted by upper case letters, are obtained by summing across individuals within a particular generation and then summing across all generations. That is, the aggregate value \( X_t \) for a generic economic variable \( x_{s,t} \) is given by 
\( X_t \equiv \sum_{s=-\infty}^{t} \int_{0}^{1} (1 - \vartheta)^{s-x} x_{s,t}(j) dj \). Given that agents are born with zero assets, assuming government policies are equally distributed across all individuals, the aggregate budget constraint can be written as
\[ C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \Omega_t + A_t, \quad (15) \]

where \( \Omega_t \equiv (1 - \tau_t)W_tN_t/P_t + Z_t + D_t/P_t \) is aggregate net income and \( A_t = [(1 - \tau_t)R_t^k + 1 - \delta_t]K_{t-1} + M_{t-1}/P_t + R_{t-1}B_{t-1}/P_t \) is aggregate financial wealth. Since the ratio of real money balances to consumption (and therefore employment) only varies with respect to time, aggregating equations (9), (10), and (12)-(14) yields the following relationships
\[ \frac{M_t}{P_t} = \frac{\kappa}{1 - \kappa} \frac{R_t}{R_t - 1} C_t, \quad (16) \]
\[ \frac{W_t}{P_t} = \frac{1 + \mu_t^w}{1 - \kappa} \Phi_t^{-\kappa} V'(N_t), \quad (17) \]
\[ \lim_{T \to \infty} E_T \{ Q_{t,T}A_T \} = 0, \quad (18) \]
\[ E_t \left\{ Q_{t,t+1} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{R_t}, \quad (19) \]
\[ \tilde{C}_t = \chi \left[ A_t + H_t - \frac{1}{1 - \kappa} E_t \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} Q_{t,T} \Phi_T^{-\kappa} V(N_T) \right], \quad (20) \]

where \( Q_{t,T} \) is the aggregate SDF and \( H_t = \sum_{T=t}^{\infty} (1 - \vartheta)^{T-t} E_t Q_{t,T} \Omega_T \) is aggregate human wealth. In order to derive the aggregate law of motion for consumption, follow the techniques applied at the individual level to rewrite (15) and substitute the resulting budget constraint into (20). Then move the original version of (20) one period ahead, apply expectations, and combine with the other result to obtain
\[ \tilde{C}_t = \frac{1}{\beta} E_t \{ Q_{t,t+1} \tilde{C}_{t+1} \} + \frac{\vartheta \chi}{\beta(1 - \vartheta)} E_t \{ Q_{t,t+1} A_{t+1} \}. \quad (21) \]
Unlike the conventional infinite lifetime model, consumption now responds to the aggregate level of financial wealth net of taxes and, therefore, debt run-ups resulting from explosive government transfers will have a positive effect on aggregate consumption. This result also suggests that Ricardian equivalence will not hold when $\vartheta \neq 0$, since each agent’s rate of return on bonds is different from how they discount taxes.

2.3 **FIRMS**

The production sector consists of monopolistically competitive intermediate goods producing firms who produce a continuum of differentiated inputs and a representative final goods producing firm.

2.3.1 **INTERMEDIATE GOODS PRODUCING FIRMS**

Each firm $i \in [0, 1]$ in the intermediate goods sector, produces a differentiated good, $Y_t(i)$, with identical technologies given by

$$Y_t(i) = K_{t-1}(i)\alpha N_t(i)^{1-\alpha},$$

where $K(i)$ and $N(i)$ denote the capital stock and level of employment used by firm $i$ and $\alpha$ is the cost share of capital. The total amount of labor services employed by each firm is composed of labor supplied by individuals across all generations according to the following CES production function

$$N_t(i) = \left[ \sum_{s=-\infty}^{t} \int_{0}^{\vartheta(1-s)} n_{s,t}(i,j) \frac{\eta_{s,t}}{\eta_t} \right]^{\frac{\eta_t}{\eta_t-1}}.$$

Each firm maximizes the amount of labor they hire for a given level of expenditure. After solving for each firm’s optimal labor input, aggregating across all firms yields (5).

When choosing inputs, each firm seeks to minimize its total costs given a particular level of output. That is, by choosing consumption and labor each firm minimizes

$$TC_t(i) = W_t N_t(i) - R_t^k K_{t-1}(i)$$

subject to the production function specified in (22). Optimality then requires

$$\frac{K_{t-1}(i)}{N_t(i)} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}. \tag{24}$$

This result shows that the capital-labor ratio, and hence marginal costs, are identical across intermediate goods producing firms.

2.3.2 **PRICE SETTING**

The representative final goods producing firm purchases inputs from the intermediate goods producing firms in order to produce a composite good according to CES technology given by, $Y_t \equiv \int_{0}^{1} Y_t(i)^{\varepsilon/(\varepsilon-1)}/\varepsilon di/\varepsilon/(\varepsilon-1)$, where $\varepsilon > 1$ is the elasticity of substitution between goods and $Y_t$ denotes
aggregate output. Maximizing profits for a given level of output yields firm $i$'s demand function for intermediate inputs given by $Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t$.

Following Rotemberg (1982), assume each firm faces a quadratic cost to adjusting its nominal price level, which emphasizes the potentially negative effect that price changes can have on customer-firm relationships. Then, using the above results and adopting the functional form used in Ireland (1997), real profits of firm $i$ are given by

$$D_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} Y_t - \Phi_t \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t - \frac{\varphi}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 Y_t,$$

where $\varphi \geq 0$ determines the magnitude of the adjustment cost and $\pi$ is the steady-state gross inflation rate. Each intermediate goods producing firm then chooses their price level, $P_t(i)$, to maximize the expected discounted present value of real profits given by

$$E_0 \sum_{t=0}^{\infty} \beta^t Q_{t,t+1} \frac{D_t(i)}{P_t},$$

where prices of future profits are determined by the aggregate household’s SDF, since profits are distributed among individuals across all generations. The first order condition is given by

$$0 = (1-\varepsilon) Q_{t,t+1} \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t + \varepsilon Q_{t,t+1} \Phi_t \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon+1} Y_t - \varphi Q_{t,t+1} \left( \frac{P_t(i)}{P_t \pi P_{t-1}(i)} - 1 \right) \frac{P_t(i)}{\pi P_t(i)}.$$

In a symmetric equilibrium, all intermediate goods producing firms will make identical decisions. Thus, in the absence of costly price adjustment (i.e. $\varphi = 0$), real marginal costs reduce to $(\varepsilon-1)/\varepsilon$, which is equivalent to the inverse of the firm’s markup factor.

2.4 Monetary and Fiscal Policy

The fiscal authority finances discretionary spending, $G_t$, and government transfers, $Z_t$, through lump-sum taxation, seigniorage revenues, and by issuing one-period nominal debt. The government’s flow budget constraint is given by

$$\frac{M_t}{P_t} + \frac{B_t}{P_t} + \tau_t \left( \frac{W_t}{P_t} N_t + R_t K_{t-1} \right) = G_t + Z_t + \frac{R_{t+1} B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t}.$$  

I now describe how the model incorporates the various layers of uncertainty. Figure 3 illustrates the wide range of potential outcomes that are possible in the model. The price level is initially determined within the conventional policy region where the monetary authority actively fights inflation by following a simple Taylor rule (AM) while the fiscal authority passively adjusts the tax rate to increases in the level of real debt (PF). Each period, before and after reaching the fiscal limit, there is a probability, $p_R > 0$ of passing reform. The fiscal limit occurs at some unknown date, $T$, with probability, $p_L > 0$, which is only reached if reform is not passed for $T$ successive periods. If
Figure 3: Possible evolution of monetary and fiscal policy assuming the monetary authority is forced to abandon its inflation target if, and when, the fiscal limit is reached.

the fiscal limit is reached, in order to obtain a unique bounded equilibrium, the monetary authority is forced to abandon its commitment to an inflation target in favor of a passive monetary policy rule (PM).

The precise mechanisms governing monetary and fiscal policies before and after the fiscal limit are as follows. Government transfers, which initially follow a non-stationary path, are governed by

\[
Z_t = \begin{cases} 
\phi Z_{t-1} + \varepsilon_t, & \text{for } S_{Z,t} = 1 \text{ (Before Reform)}, \\
(1 - \rho) \bar{Z} + \rho Z_{t-1} + \varepsilon_t, & \text{for } S_{Z,t} = 2 \text{ (After Reform)}, 
\end{cases}
\]

where \(\phi > 1, |\rho| < 1\), and \(\varepsilon_t \sim N(0, \sigma_Z^2)\). The agent knows transfers evolve according to a first-order two-state Markov chain given by

\[
P_{R,t} = \begin{bmatrix} \text{Pr}[S_{Z,t} = 1 | s_{Z,t-1} = 1] & \text{Pr}[S_{Z,t} = 2 | s_{Z,t-1} = 1] \\
\text{Pr}[S_{Z,t} = 1 | s_{Z,t-1} = 2] & \text{Pr}[S_{Z,t} = 2 | s_{Z,t-1} = 2] \end{bmatrix} = \begin{bmatrix} 1 - p_{R,t} & p_{R,t} \\
0 & 1 \end{bmatrix},
\]

where \(p_{R,t}\) is the probability of passing reform that would place transfers on a stable trajectory indefinitely. Even after the fiscal limit is reached, the model continues to allow for the possibility of reform. Upon passing reform, transfers switch from an unstable process, denoted active transfers (AT), to a stable process, denoted passive transfers (PT). The assumption that the stable transfers state is absorbing implies that any reform undertaken by Congress is sufficient to ensure long-term solvency. Given the potential severity of the fiscal situation facing the United States and the political challenges associated with passing entitlement reform, this seems like a reasonable assumption. However, adapting the model to allow for incremental reform that addresses only the short-term problem could be easily accommodated by changing the stationary transfers state to a
semi-reflecting state.

In order to capture the increasing political pressure for reform that is associated with a rising debt, assume the probability of reform is positively related to the debt-to-GDP ratio. More specifically, assume the probability of reform is governed by a logistic function given by

\[ p_{R,t} = \frac{\exp(\zeta_0 + \zeta_1 (b_{t-1} - b^*))}{1 + \exp(\zeta_0 + \zeta_1 (b_{t-1} - b^*))}, \]

where \( b_t \equiv B_t/P_t \) is real debt at time \( t \), \( b^* \equiv B/P \) is the target level of real debt, and the parameter \( \zeta_1 < 0 \). This function captures the fact that as debt rises relative to GDP, media outlets increase their coverage of the potential severity of the problem, which leads to a more hostile electorate demanding reform.

The monetary authority sets interest rate policy according to

\[ R_t = \begin{cases} \bar{R} + \alpha (\pi_t - \pi^*), & \text{for } S_{R,t} = 1 \text{ (Before Fiscal Limit)}; \\
\bar{R}, & \text{for } S_{R,t} = 2 \text{ (After Fiscal Limit)}, \end{cases} \quad (29) \]

where \( \pi^* \) is the target inflation rate, \( \bar{R} \) is the steady-state nominal interest rate, and \( \alpha \) is a sensitivity parameter controlling the response of the nominal interest rate to changes in inflation.

The distortionary tax rate, which is initially set in response to deviations of lagged real government debt (relative to GDP) from its steady-state level is governed by

\[ \tau_t = \begin{cases} \bar{\tau} + \gamma \left( \frac{b_{t-1}}{\pi_{t-1}} - b^* \right), & \text{for } s_{\tau,t} = 1 \text{ (Before Fiscal Limit)}; \\
\tau^\text{max}, & \text{for } s_{\tau,t} = 2 \text{ (After Fiscal Limit)}, \end{cases} \quad (30) \]

where \( \bar{\tau} \) is the steady-state tax rate, \( \gamma \) determines the sensitivity of taxes to real debt, and \( \tau^\text{max} \) is the tax rate associated with the fiscal limit. Each period in which Congress fails to pass reform, the explosive transfers process places tremendous upward pressure on government debt, forcing tax rates continually higher. As tax rates rise, policymakers face increasing political resistance and, eventually, either the resistance will become so great that higher tax rates are no longer a viable option or the economy will reach the peak of its Laffer curve, whereby higher taxes no longer yield increased revenues. At this point, the fiscal limit is hit and the monetary authority is forced to adopt a pure interest rate peg while Congress continues with its attempt to pass reform. Following Davig, Leeper, and Walker (2010), the state governing whether the fiscal limit has been hit also follows a two-state Markov chain given by

\[ P_{L,t} = \begin{bmatrix} \Pr[S_{\tau,t} = 1 | S_{\tau,t-1} = 1] & \Pr[S_{\tau,t} = 2 | S_{\tau,t-1} = 1] \\
\Pr[S_{\tau,t} = 1 | S_{\tau,t-1} = 2] & \Pr[S_{\tau,t} = 2 | S_{\tau,t-1} = 2] \end{bmatrix} = \begin{bmatrix} 1 - p_{L,t} & p_{L,t} \\
0 & 1 \end{bmatrix}, \]

where probability of hitting the fiscal limit, \( P_{L,t} \), follows a logistic function given by

\[ p_{L,t} = \frac{\exp(\xi_0 + \xi_1 (\bar{\tau} - \bar{\tau}))}{1 + \exp(\xi_0 + \xi_1 (\bar{\tau} - \bar{\tau}))}; \]
and the parameter $\xi_1 < 0$. This function makes explicit the fact that there exists an upper bound to taxes, whether it is the result of political or economic forces at hand. Each agent’s preferences determine the maximum tax rate, but they face uncertainty about if, and when, this rate will be hit. Each period in which Congress fails to pass reform the consequences of inaction rise as higher debt levels place upward pressure on inflation. Overall, by building in a horse race between reform and the fiscal limit, this setup clearly establishes the prevailing tradeoff currently facing Congress—either pass reform in the near future and face little or no economic consequences of debt run-ups or fail to pass legislation until the after fiscal limit is reached and face potentially high economic consequences from a prolonged state of heightened debt and rising inflation.

2.5 Resource Constraint

Since there exists a continuum of intermediate goods producing firms, indexed by $i \in [0, 1]$, the total amounts of labor and capital supplied by the agent are defined as $N_t = \int_0^1 N_t(i)di$ and $K_t = \int_0^1 K_t(i)di$. The aggregate resource constraint is given by

$$C_t + I_t + G_t + \frac{\phi}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 Y_t = Y_t,$$

where the law of motion for capital is governed by

$$K_t = I_t + (1 - \delta)K_{t-1}.$$

3 Analytical Examples

Prior to solving the complete nonlinear model, it is useful to gain an understanding of some of the underlying dynamics in the model using a simpler setup that can be solved analytically. In order to derive a Fisher equation that is readily comparable to the infinite horizon setup, the first simplified model removes all uncertainty. I then return to a stochastic model but still eliminate any degree of uncertainty surrounding fiscal policy. Within this simplified framework, I consider how price level is jointly determined by monetary and fiscal policy under the two policy mixes that deliver a unique bounded equilibrium. Although these results are very well-known in the conventional infinite horizon setup, they have not been thoroughly examined in the finite horizon setup. Finally, I consider two scenarios that examine the effects of the economy reaching the fiscal limit. In the first scenario, the transfers process never adjusts and, upon reaching the fiscal limit, the monetary authority is forced to abandon its commitment to targeting inflation. In the second scenario, reform passes, but not until the fiscal limit is reached. In this case, transfers become stable at the expense of the monetary authority having to abandon its inflation target. In each of these scenarios, I show how the level of real debt and the unique price level is determined.

3.1 Fisher Equation Comparison

One drawback of working with a stochastic version of the perpetual youth model is that it does not allow for an analytical derivation of the functional form of the aggregate SDF and, as a consequence, does not allow for the derivation of a Fisher equation. In order to provide a comparison
between the Fisher equation in the perpetual youth setup and the conventional infinite horizon setup, where the probability of death is zero, consider a cashless, non-stochastic version of the perpetual youth model, where all agents have identical preferences, receive the same endowment, face the same fiscal policies (i.e. transfers and lump-sum taxes), and are subject to identical probabilities of death, \( \vartheta \). To further simplify the derivations assume labor is inelastically supplied and the nominal interest rate is pegged by the monetary authority in all time periods. The government chooses sequences of taxes, transfers, and nominal bonds to satisfy its flow budget constraint given by

\[
b_t + \tau_t = Z_t + \frac{R_{t-1}}{\pi_t} b_{t-1}.
\]  

(32)

In this simplified model, a representative agent born at time \( s \) chooses \( \{c_{s,t}, b_{s,t}\} \) to maximize

\[
\sum_{t=0}^{\infty} [\beta(1-\vartheta)]^t \ln c_{s,t},
\]  

(33)

subject to the following flow budget constraint

\[
c_{s,t} + \frac{b_{s,t}}{P_t} + \tau_t - Z_t \leq Y_t + R_{t-1} \frac{b_{s,t-1}}{P_t} (1-\vartheta)^{-1}.
\]  

(34)

Solving the individual’s maximization problem yields the following Euler equation

\[
c_{s,t+1} = \beta R_t \pi_t^{-1} c_{s,t}.
\]  

(35)

Iterating (34) forward using (35) and imposing the no Ponzi game condition, yields a closed-form solution for the optimal consumption choice of an agent born at time \( s \) given by

\[
c_{s,t} = \chi \left\{ \frac{R_t \vartheta b_{s,t-1}}{(1-\vartheta) P_t} + \sum_{k=0}^{\infty} \left( \frac{1-\vartheta}{R} \right)^k \frac{P_{t+k}}{P_t} [Y_{t+k} + Z_{t+k} - \tau_{t+k}] \right\},
\]  

(36)

where \( \chi = 1 - \beta(1-\vartheta) \). Assuming a representative agent for each generation, aggregate values are given by \( X_t \equiv \sum_{s=\infty}^{t} \vartheta(1-\vartheta)^{t-s} x_{s,t} \). Given that agents initially have zero non-human wealth, the aggregate budget constraint and consumption function are given by

\[
C_t + \frac{B_t}{P_t} + \tau_t - Z_t = Y_t + R_{t-1} \frac{B_{t-1}}{P_t},
\]  

(37)

\[
C_t = \chi \left\{ \frac{R_{t-1} B_{t-1}}{P_t} + \sum_{k=0}^{\infty} \left( \frac{1-\vartheta}{R} \right)^k \frac{P_{t+k}}{P_t} [Y_{t+k} + Z_{t+k} - \tau_{t+k}] \right\}.
\]  

(38)

Combining (37) into (38) and performing tedious algebra yields the following Fisher equation

\[
C_{t+1} = R \beta \pi_{t+1}^{-1} C_t - \vartheta \chi \frac{RB_t}{1-\vartheta} P_{t+1}^{-1}.
\]  

(39)

It is easy to see that real government debt now influences the path of aggregate consumption. When the probability of death \( \vartheta \neq 0 \), for a given interest rate an increase in the stock of government debt increases present consumption. When \( \vartheta = 0 \), (39) reduces to the traditional Fisher equation that relates the nominal interest rate to inflation and is independent of the level of nominal debt.
3.2 Price Level Determination

In an effort to characterize how the price level is jointly determined by monetary and fiscal policy, I now return to a stochastic model with a time varying interest rate, while retaining the other features of the simplified setup from Section 3.1. Given the inherent non-linearities of the perpetual youth setup, in order to characterize the price level under the various policy mixes, it is necessary to first log-linearize the equations that characterize the equilibrium. Assuming government spending is zero each period, after imposing the goods market clearing condition \( C_t = Y_t \), the consumption function, (21), reduces to

\[
Y_t = \frac{1}{\beta} E_t \{ Q_{t,t+1} Y_{t+1} \} + \frac{\vartheta}{\beta (1 - \vartheta)} E_t \{ Q_{t,t+1} R_{t+1} \pi_{t+1}^{-1} b_t \}. \tag{40}
\]

Assuming a constant aggregate endowment \( Y_t = Y \), log-linearizing (40) around the deterministic steady-state yields\(^1\)

\[
\hat{R}_t = E_t \hat{\pi}_{t+1} + \lambda \hat{b}_t, \tag{41}
\]

where a circumflex denotes log-deviations from the deterministic steady-state\(^2\) and \( \lambda \equiv \frac{\vartheta \pi}{1 - \vartheta} \frac{R \beta^2}{y^2} \).

Log-linearizing the government budget constraint, (32), yields

\[
\hat{\pi}_t = \hat{b}_{t-1} + \hat{R}_{t-1} - \frac{\beta}{1 + \lambda} \left[ \hat{b}_t + \hat{\tau}_t - \hat{Z}_t \right], \tag{42}
\]

where \( \hat{\tau} \) and \( \hat{Z} \) respectively denote the steady-state ratios of lump-sum taxes- and transfers-to-debt.

3.2.1 Active Monetary and Passive Fiscal Policy

In order to derive the equilibrium price level, assume the monetary authority aggressively targets inflation by following a simple Taylor rule of the form

\[
\hat{R}_t = \alpha \hat{\pi}_t, \tag{43}
\]

where \( \alpha > 1 \). Lump-sum taxes, which are set in response to deviations of real government debt from its steady-state level, are given by

\[
\hat{\pi}_t = \gamma \hat{b}_{t-1}, \tag{44}
\]

where \( \gamma \) is set to ensure that any increases in government debt will be met with the expectation that future taxes will rise by a sufficient amount to service the higher debt and retire it back to its steady-state level. Combining (41) with the simple Taylor rule given in (43) implies

\[
\alpha \hat{\pi}_t = E_t \hat{\pi}_{t+1} + \lambda \hat{b}_t.
\]

\(^1\)For the purposes of this analytical example, steady-state values are denoted by an asterisk. Targets of the central bank and fiscal authority are set to their corresponding steady-state values.

\(^2\)That is, for some generic variable \( X \), \( \hat{X}_t = \ln X_t - \ln X \approx (X_t - X)/X. \)
Thus, under active monetary policy the unique bounded solution for inflation is given by

\[ \hat{\pi}_t = \lambda \sum_{k=0}^{\infty} \left( \frac{1}{\alpha} \right)^k E_t \hat{b}_{t+k}, \]  

so that deviations of equilibrium inflation from target are proportional to the deviations of real debt from its target. This shows that even if the monetary authority is aggressively targeting inflation, the fiscal authority still has influence over equilibrium inflation, as large debt run-ups relative to steady-state will cause the monetary authority to lose control of inflation. In the conventional setup where agents are infinitely-lived, \( \hat{\pi}_t = 0 \) so that equilibrium inflation, regardless of the level of debt, is always on target when the monetary authority follows a Taylor-type rule.

Given that the monetary authority is unable to reach its inflation target when real government debt is not pinned down, it must be the case that the fiscal authority responds to disturbances in transfer payments in such a way that eventually pins down the level of real government debt. Combining (42) and (44), applying expectations conditional on time \( t-1 \), and imposing (41) yields the expected evolution of real government debt

\[ E_{t-1} \hat{b}_t = (\tilde{\beta}^{-1} - \gamma \tau) \hat{b}_{t-1} + E_{t-1} \tilde{Z} \hat{Z}_t, \]

where \( \tilde{\beta} \equiv \beta / (1 + \lambda)^2 < 1 \) represents the adjusted discount factor, which accounts for the strictly positive probability of death. The tax rule implies that any increases in the level of debt will be met by higher taxes. However, if the response is not sufficiently strong, disturbances to transfers will lead to continually higher levels of debt and compromise the effectiveness of the monetary authority’s inflation targeting policy. If, on the other hand, \( \tilde{\beta}^{-1} - \gamma \tau < 1 \), the effect of any disturbance to transfers will slowly decay, causing real government debt to eventually return to its steady-state level. The steady-state level of real government bonds is, \( b^* = (\tau^* - z^*) / [\beta^{-1}(1 + \lambda) - 1] \), equal to the primary surplus discounted by the net real interest rate, \( r^* \equiv R^*/\pi^* = \beta^{-1}(1 + \lambda) \).

Further insight regarding the financing of government transfers can be obtained from the intertemporal equilibrium condition that relates real government debt to the discounted present value of primary surpluses. In order to derive this condition, advance (42) one period ahead, apply expectations conditional on information at time \( t \), substitute in (41), solve forward, and impose the aggregate no Ponzi game condition to obtain

\[ \hat{b}_t = \sum_{k=1}^{\infty} \tilde{\beta}^k E_t \left[ \hat{\tau}_{t+k} - \tilde{Z} \hat{Z}_{t+k} \right]. \]  

(46)

In the conventional infinite horizon setup, where the probability of death is zero, the intertemporal equilibrium condition, (46), reduces to the traditional condition where \( \tilde{\beta} = \beta \). In this case, monetary policy is able to pin down the price level independently of the fiscal authority, and, therefore, any bond financed increase in transfers must be financed through increases in future taxes, delivering the well-know property of Ricardian equivalence. With a positive probability of death, the dynamics are more complicated. In this case, the monetary authority is unable to pin down the price level independently of the fiscal authority. Although debt financed increases in transfers under passive fiscal policy will eventually bring the level of debt back to its steady-state level, \( b^* \), the
interim effects cause the monetary authority to temporarily lose control of inflation and, therefore, lead to increases in the price level. Thus, in the perpetual youth model, disturbances to transfers are only partially financed by increases in future taxes. The remainder of the financing is delivered by an increase in the inflation tax. Of course, this regime is not the only regime that will deliver a bounded equilibrium. I now consider the other case.

3.2.2 Passive Monetary and Active Fiscal Policy

As Davig and Leeper (2006, 2009) make clear, monetary and fiscal authorities do not continuously base policy on the same rules. Instead, as a consequence of both political and economic factors, policy fluctuates between active and passive regimes. Under active fiscal policy, the fiscal authority no longer bases tax policy on the size of government debt and instead sets taxes exogenously. Such a policy could be related to a wide variety of factors including re-election, stimulus, or even nearing the fiscal limit. The most recent evidence for such a regime occurred during the Bush tax cuts of 2001, when income and dividend taxes were slashed while the debt-to-GDP ratio was beginning to steadily rise.

Under passive monetary policy, the monetary authority no longer aggressively adjusts the nominal interest rate in response to changes in inflation, instead basing policy on other factors such as output stabilization. This policy most typically arises during economic downturns in an effort to curtail the severity of recessions. The best example of this policy was the monetary authority’s response to the financial crisis of 2007-2010, when rates were pegged near their lower bound while a multitude of unconventional techniques were applied to help alleviate the credit crunch and rescue failing financial institutions.

For analytical convenience, assume the monetary authority pegs the nominal interest rate at its steady-state level, $R^*$, while the fiscal authority fixes lump-sum taxes at a constant level, $\tau^*$. Under this policy mix, the linearized Fisher equation, (41), reduces to

$$E_t \hat{\pi}_{t+1} = -\lambda b_t,$$

so that expected inflation is set according to deviations of real government debt from its steady-state level. With a positive probability of death, an increase in debt causes expected inflation to fall. This result follows from the fact that under active fiscal policy, an increase in debt must be caused by a decrease in expected transfers (see (46)), which causes the expected price level and inflation to fall. This is in contrast with the results of the conventional infinite horizon setup, where expected inflation is not impacted by fiscal policy and is nailed down by the inflation target. In neither case, however, is the actual level of inflation pinned down by the monetary authority. That is, under a passive monetary policy, the monetary authority is unable to pin down the price level regardless of the probability of death.

In order to see how the price level gets nailed down, impose the active fiscal policy on the intertemporal equilibrium condition, (46), and substitute into the linearized government budget
characterizing the effects of entitlement reform in a perpetual youth model

Before Fiscal Limit | After Fiscal Limit
---|---
$t = 0, 1, \ldots, T - 1$ | $t = T, T + 1, \ldots$

| Tax Policy | $\hat{\tau}_t = \gamma b_{t-1}$ | $\hat{\tau}_t = 0$
| Monetary Policy | $\hat{R}_t = \alpha \tilde{\pi}_t$ | $\hat{R}_t = 0$

Table 1: Monetary and Fiscal Policy Rules Before and After the Date $T$ Fiscal Limit

constraint, \((42)\), to obtain

$$\hat{P}_t = \hat{B}_{t-1} + \frac{\beta}{1 + \lambda} \sum_{k=0}^{\infty} \beta^k E_t \hat{Z}_{t+k}.$$ \hspace{1cm} (48)

Given that transfers policy is set exogenously, at time $t$ the right-hand side of this expression is predetermined. Therefore, under this policy mix, fiscal policy delivers a unique price level, and given that this conclusion is consistent with the infinite horizon setup where $\lambda = 0$, the well-established fiscal theory of the price level is shown to be independent of the probability of death. However, when the probability of death is positive, the fiscal authority not only determines the price level and the level of inflation but also expected inflation, since the exogenous transfers process pins down the level of real bonds.

This section has examined price level determinacy within the context of the perpetual youth model by contrasting its results with the results under the conventional infinite horizon setup. Under both of these policy mixes the price level is uniquely determined, but the mechanisms are quite different from the infinite horizon setup. Under two very different policy responses, I now consider the impact of imposing a fiscal limit that is known to be reached at time $T$.

3.3 Fiscal Limit Impact

In the previous section, the fiscal authority had the option to raise taxes indefinitely in response to increases in the level of real debt. However, as was discussed above, there exists a limit to the level of taxes that are plausible both economically and politically. Using the same model considered in Section 3.2, this section considers the effects of reaching the fiscal limit by imposing a maximum tax rate, $\tau_{\text{max}}$, and assuming reform is not passed prior to date-$T$. The fiscal limit is guaranteed to be hit as transfers are assumed to follow an explosive path. Upon reaching the fiscal limit, the monetary authority abandons its inflation target and two policy options are considered—one where reform never occurs and one where the fiscal authority passes reform that places transfers on a stable trajectory indefinitely. Prior to reaching the fiscal limit, the fiscal authority passively adjusts taxes to the size of lagged real debt while the monetary authority aggressively fights inflation.

3.3.1 Monetary Policy Adjustment: No Reform

First consider the effect of the fiscal authority never passing reform. In this case, the monetary authority switches to a passive regime upon reaching the fiscal limit at a known date $T$ and is solely responsible for stabilizing debt. Under this policy response, government transfers are unaffected by the economy hitting the fiscal limit, following an explosive process that holds for all time periods.
and is given by
\[ \hat{Z}_t = \phi \hat{Z}_{t-1} + \epsilon_t, \quad (49) \]
where \( \phi > 1, \bar{\beta}_\phi < 1, |\rho| < 1, \) and \( \epsilon_t \sim N(0, \sigma^2_\epsilon). \) Monetary and fiscal policy rules before and after time \( T \) are summarized in Table 1. If monetary policy remained active after reaching the fiscal limit, neither authority would stabilize the increases in real debt that are caused by the explosive transfers process and debt would eventually explode. By switching to a passive monetary policy at time \( T, \) inflation is able to rise. This stabilizes debt as higher prices reduce the relative value of past debt while also increasing the amount of revenue from taxes.

In order to solve for the initial equilibrium level of real debt, use (46) and the tax process prior to the fiscal limit to write the initial stock of real bonds relative to target as
\[ \hat{b}_0 = \bar{\beta}_\gamma [\gamma \hat{\tau}_b - \hat{Z} E_0 \hat{Z}_1] + \bar{\beta} E_0 \hat{b}_1. \]
Solving for \( \hat{b}_0, \) iterating forward, and using (46) to substitute for the expected level of real bonds at time \( T - 1 \) then yields
\[ \hat{b}_0 = -E_0 \sum_{k=1}^{T-1} \left( \frac{\bar{\beta}}{1 - \gamma \bar{\beta} \hat{\tau}} \right)^k \hat{Z} \hat{Z}_k + \left[ \frac{1}{1 - \gamma \bar{\beta} \hat{\tau}} \right] E_0 \sum_{k=T}^{\infty} \bar{\beta}^k (\hat{\tau} \hat{\tau}_k - \hat{Z} \hat{Z}_k). \]
After substituting for the transfers processes, we obtain
\[ \hat{b}_0 = - \left[ \left( \frac{1}{1 - \gamma \bar{\beta} \hat{\tau}} \right)^{T-1} \left( \frac{\bar{\beta}_\phi}{1 - \beta \phi} \right)^T + \sum_{k=1}^{T-1} \left( \frac{\bar{\beta}_\phi}{1 - \gamma \bar{\beta} \hat{\tau}} \right)^k \right] \hat{Z} \hat{Z}_0. \quad (50) \]
Thus, after evaluating the geometric progression, date-0 equilibrium real debt can be represented as a function of the steady-state monetary and fiscal policy targets, \( \{R^*, \pi^*, b^*, \tau^*, z^*\}, \) and is given by
\[ \hat{b}_0 = \left( \frac{\bar{\beta}_\phi}{1 - \gamma \bar{\beta} \hat{\tau}} \right)^T \left( \frac{\gamma \bar{\beta} \hat{\tau}(1 - \gamma \bar{\beta} \hat{\tau})}{(1 - \beta \phi)(1 - \beta(\phi + \gamma \hat{\tau}))} \right) - \left( \frac{\bar{\beta}_\phi}{1 - \beta(\phi + \gamma \hat{\tau})} \right)^T \hat{Z} \hat{Z}_0. \quad (51) \]
From this result, it is evident that Ricardian equivalence does not hold, as two different tax sequences will alter the aggregate household’s intertemporal budget constraint and, therefore, its optimal consumption plan. It is important to note that this result is not an artifact of the presence of finitely-lived agents, since the probability of death only shows up through the discount factor, \( \bar{\beta}. \)

Two other results follow through from the infinite horizon setup. First, transfers are negatively related to the value of real government debt. This follows directly from the fact that higher transfer obligations increase demand for goods and drive up the price level, which reduces the discounted present value of net government income, as indicated by (46). Second, the strength of the fiscal authority’s response to increases in debt, \( \gamma, \) impacts the size of real government debt. Only in the
special case where agents are infinitely-lived and policy is permanent does this result break down. The main implication of finitely-lived agents is that monetary policy now affects the level of real government debt, as both the nominal interest rate and inflation targets are components of the discount factor, a condition that follows directly from the presence of real debt in the Fisher equation.

In order to determine the unique price level first evaluate the government budget constraint at time zero to obtain

$$\hat{P}_0 = \hat{B}_{-1} + \hat{R}_{-1} - \frac{\beta}{1 + \lambda} [\hat{b}_0 + \hat{\tau}_0 - \hat{Z}\hat{Z}_0].$$ (52)

Given initial conditions $R_{-1}, B_{-1} > 0$ and initial fiscal policies, the price level is pinned down by (51). Using the policy rules specified in Table 1, the complete trajectory of prices, real debt, and inflation prior to the fiscal limit can be recursively solved. After the fiscal limit, the economy evolves according the fixed regime considered in Section 3.2.2. Section 3.4 will simulate how the level of real government debt and inflation is affected by the presence of a fiscal limit.

### 3.3.2 Monetary Policy Adjustment: With Reform

Alternatively, the fiscal authority could delay reform until the economy reaches the fiscal limit. Under this policy response, the monetary authority is still forced to abandon its inflation target at the fiscal limit, since the fiscal authority is no longer able to stabilize debt by adjusting the tax rate to increases in debt. However, by passing reform, transfers are indefinitely placed on a stable path, drastically helping to reduce the growth rate of government debt and inflation. Assuming reform passes at time $T$, policy before and after the fiscal limit is summarized in Table 2.

In order to solve for the equilibrium, substitute for the pre- and post-reform transfers processes in (50) to obtain

$$\hat{b}_0 = -\left[\frac{1}{1 - \gamma\beta}\right]^{T-1} (\tilde{\beta}\rho)^T + \frac{\sum_{k=1}^{T-1} \left(\frac{\tilde{\beta}\phi}{1 - \gamma\beta}\right)^k}{1 + \lambda} \tilde{Z}\tilde{Z}_0.$$ 

Thus, after evaluating the geometric progression, date-0 equilibrium real debt is now given by

$$\hat{b}_0 = \left[\left(\frac{\tilde{\beta}}{1 - \gamma\beta T}\right)^T \left(\frac{\phi^T(1 - \gamma\beta T)}{1 - \beta(\phi + \gamma T)} - \rho^T(1 - \gamma\beta T)\right) - \left(\frac{\tilde{\beta}\phi}{1 - \beta(\phi + \gamma T)}\right)\right] \tilde{Z}\tilde{Z}_0$$ (53)
In this case, equilibrium real bonds are pinned down by the parameters governing both the stable and unstable transfers processes. However, the results of the previous section still hold—debt is negatively related to transfers and dependent on all policy targets. Once again, the price level is pinned down by combining the government budget constraint, (42), with the equilibrium value of government debt, (53), and the paths of real debt and inflation can be obtained recursively given initial conditions and the policies specified in Table 2.

3.4 Simulations

In order to obtain a clearer picture of how the presence of a fiscal limit and the various policy responses affect the trajectories of debt and inflation, it is useful to simulate the equilibrium. Figures 4 and 7 isolate the effect of a fiscal limit by comparing the fixed regime considered in Section 3.2.2 to the model where the active monetary/passive fiscal regime only prevails until the fiscal limit is hit at a known date \(T\), under the assumption that government transfers follow a stable process for all periods.\(^{15}\)

Figure 4 shows the trajectory of deviations of real debt from steady-state prior to the fiscal limit. Notice that as the fiscal limit approaches, the discrepancy between the regimes with and without a fiscal limit slowly disappears, and from time \(T\) onward the paths of real debt are identical. This follows directly from the fact that each agent’s decisions are governed by long-run policies, so that by the time the fiscal limit is reached, forward-looking agents have completely accounted for the known policy adjustment. Prior to the fiscal limit, however, the paths of real debt are quite different. The presence of a fiscal limit greatly increases the volatility of real debt, as the deviations from steady-state are roughly forty percent larger when compared to the fixed regime model.\(^{16}\) Unlike the conventional active monetary/passive fiscal regime, where a more responsive fiscal authority (higher \(\gamma\)) helps to keep real debt closer to its steady-state level, when the fiscal authority faces a fiscal limit, greater sensitivity leads to adverse outcomes. As Figure 5 makes clear, even though the economy follows the conventional policy mix (AM/PF) prior to the fiscal limit, a higher \(\gamma\) makes the level of real debt much more sensitive to changes in transfers, further increasing its volatility.

The probability of death parameter measures the size of the deviation from the more traditional Ricardian framework. Figure 6 illustrates the effect of increasing the probability of death parameter to \(\vartheta = 0.5\). Regardless of the presence of a fiscal limit, real debt becomes substantially more volatile as deviations from steady-state increase two-fold from the benchmark calibration. Moreover, the impact of the fiscal limit becomes even more pronounced, as the deviations from steady-state become twice as large as the fixed regime case. This result stems from the fact that an increase in \(\vartheta\), increases the gross return on debt, which increases the required backing and heightens the sensitivity of real debt to changes in government transfers.

\(^{15}\)The baseline calibration is as follows: Prior to the fiscal limit \(\alpha = 1.5\) and \(\gamma = 0.1\). Upon reaching the fiscal limit, both policy parameters are set to zero. The subjective discount factor, \(\beta\) is set in accordance with an annual rate of interest of 4 percent. I assume steady-state values of \(\tau^* = 0.19\), \(z^* = 0.17\), \(\pi^* = 1.02\), and \(b^*/Y^* = 0.5\). The parameters of the transfers process are \(\rho = 0.9\) and \(\sigma = 0.002\). Following Leith and Wren-Lewis (2000), the death parameter is set to 0.015. All figures are generated based on identical realizations of government transfers.

\(^{16}\)Although only one realization of the transfers process is considered, the qualitative results considered in this section are not sensitive to the seed.
Figure 4: Comparison of the trajectory of real debt under the fixed regime considered in Section 3.2.2 (red line) and the trajectory of real debt in the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$ (blue line). In this comparison, transfers policy is fixed and stable for all periods. The probability of death parameter is set to its baseline value, $\vartheta = 0.015$.

Figure 5: Effects on real debt from increasing the parameter governing the fiscal authority’s sensitivity to changes in real debt to $\gamma = 0.2$. All other parameters are identical to the baseline model. Trajectories correspond to the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$.

Figure 6: Effects on real debt from increasing the probability of death parameter to $\vartheta = 0.5$. All other parameters are identical to the baseline model. Trajectories correspond to the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$. 
Figure 7 presents the path of inflation relative to its steady-state prior to the fiscal limit. Given that inflation is heavily dependent on the level of real debt and the fact that the monetary authority is unable to respond to inflation after the fiscal limit is reached, the presence of a fiscal limit also greatly increases the volatility of inflation. Moreover, figures 8 and 9 show that the qualitative effects of increasing either the probability of death or the fiscal authority’s response to increases in real debt carry over to inflation. Although the effects of changing the fiscal policy parameter are relatively small, the effects of increasing the death parameter are quite stark. This results from the fact that both contemporaneous and lagged real debt affect the level inflation at any point, compounding the heightened sensitivity to fluctuations in government transfers.

As a means of comparison, Figure 7 also plots the path of expected inflation. Prior to reaching the fiscal limit, expected inflation is obtained by combining (41) with the simple Taylor rule given in (43) and is given by

$$E_t \hat{\pi}_{t+1} = \alpha \hat{\pi}_t - \lambda b_t,$$

(54)

In general, expected inflation is slightly more volatile than realized inflation, consistently underestimating the magnitude of the deviation of realized inflation from steady-state. Even though agents are constantly revising their expectations and reacting when realized inflation begins to adjust back toward steady-state, the countervailing forces from real debt cause agents to be systematically incorrect in their inflation forecast. How aggressively the monetary authority fights inflation has no impact on the trajectory of either real debt or realized inflation, as $\alpha$ fails to show up in either expression. The parameter $\alpha$ does, however, impact expected inflation. The greater the response of the monetary authority to increases in inflation, the more volatile is expected inflation as (54) shows.

As is the case with real debt, the trajectory of realized and expected inflation under the presence of a fiscal limit converges to the path followed by its fixed regime counterpart and from time time $T$ onward those paths are identical. When monetary policy relents on its commitment to fighting inflation upon reaching the fiscal limit, expected inflation no longer follows (54) and is instead given by (47). It is important to note that in the absence of a fiscal limit, when $\vartheta \neq 0$ and the economy is in a passive monetary/active fiscal regime indefinitely, expected inflation is still not pinned down by the inflation target, instead responding to changes in real debt. When the death parameter is small, however, any fluctuations from target are relatively unnoticeable and expected inflation is far more stable than in the model with a fiscal limit. Even though increasing the death parameter to $\vartheta = .5$ magnifies these effects and hinders the central bank’s ability to control expected inflation, the central bank is still much better able to control expected inflation when a fiscal limit is not attainable.

3.4.1 Effect of the Policy Options

The previous section highlighted the effects of imposing a fiscal limit by assuming that government transfers follow a stable process indefinitely. I now take seriously the fact that transfers are currently projected to follow an unstable path and examine the implications of the two policy options considered earlier—one where reform never occurs and one where the fiscal authority passes
CHARACTERIZING THE EFFECTS OF ENTITLEMENT REFORM IN A PERPETUAL YOUTH MODEL

Figure 7: Comparison of the trajectory of inflation under the fixed regime considered in Section 3.2.2 (red line) to the trajectory of real debt in the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$ (blue line). The green line plots expected inflation under the fiscal limit. In this comparison, transfers policy is fixed and stable for all periods.

Figure 8: Effects on inflation from increasing the parameter governing the fiscal authority’s sensitivity to changes in real debt to $\gamma = 0.2$. All other parameters are identical to the baseline model. Trajectories correspond to the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$.

Figure 9: Effects on inflation from increasing the probability of death parameter to $\vartheta = 0.5$. All other parameters are identical to the baseline model. Trajectories correspond to the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$.
Figure 10: Comparison of the trajectory of real debt under the fixed regime considered in Section 3.2.2 (red line) and the trajectory of real debt in the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$ (blue line). In this comparison, transfers policy is fixed and \textit{explosive} for all periods with persistence parameter $\phi = 1.03$. All remaining parameters are set to their baseline values.

Figure 11: Comparison of the trajectory of inflation under the fixed regime considered in Section 3.2.2 (red line) and the trajectory of real debt in the model where active monetary/passive fiscal policy only prevails until the fiscal limit is hit at date $T$ (blue line). The green line plots expected inflation under the fiscal limit. In this comparison, transfers policy is fixed and \textit{explosive} for all periods with persistence parameter $\phi = 1.03$. All remaining parameters are set to their baseline values.
Figure 12: Comparison of the trajectory of real debt under the assumption that no reform is passed before or after the fiscal limit is reached (blue line) to the trajectory of real debt under the assumption that the fiscal authority passes reform at the fiscal limit that places government transfers on a stable trajectory indefinitely (red line). In this comparison, if reform is not passed, transfers policy is fixed and explosive for all periods with persistence parameter $\phi = 1.03$. If the fiscal passes reform, the transfers process switches at time $T$ from explosive to stable with persistence parameters $\phi = 1.03$ and $\phi = 0.9$. Both of these policies operate under the assumption that the monetary policy switches from being active to passive at the fiscal limit All remaining parameters are set to their baseline values.

Figure 13: Comparison of the trajectory of inflation under the assumption that no reform is passed before or after the fiscal limit is reached (blue line) to the trajectory of real debt under the assumption that the fiscal authority passes reform at the fiscal limit that places government transfers on a stable trajectory indefinitely (red line). In this comparison, if reform is not passed, transfers policy is fixed and explosive for all periods with persistence parameter $\phi = 1.03$. If the fiscal passes reform, the transfers process switches at time $T$ from explosive to stable with persistence parameters $\phi = 1.03$ and $\phi = 0.9$. Both of these policies operate under the assumption that the monetary policy switches from being active to passive at the fiscal limit All remaining parameters are set to their baseline values.
reform that indefinitely places transfers on a stable trajectory immediately after the fiscal limit is hit.

Figures 10 and 11 illustrate the effect of the monetary authority switching to a passive regime upon reaching the fiscal limit at a known date $T$ when reform never takes place. Under this policy response, government transfers are unaffected by the economy hitting the fiscal limit, following an explosive process that holds for all time periods. Many of the results from the previous simulations carry over. The fiscal limit still increases the volatility of both real debt and inflation relative to the fixed regime setup; As the fiscal limit approaches, the trajectories of real debt and inflation converge to the fixed regime setup, following an identical path after reaching the fiscal limit; And, the effects of increasing the fiscal policy parameter, $\gamma$, or the death parameter, $\vartheta$, continue to imply negative consequences for economic stability.

Although the qualitative results are quite similar to the case where government transfers follow a stable process, the quantitative implications for the volatility of real debt, inflation, and expected inflation are immense. The deviations from steady-state are fifteen times greater when comparing the models with and without a fiscal limit. Moreover, the effect of imposing a fiscal limit is even more pronounced. As noted above, in the baseline model, the presence of a fiscal limit increases volatility by roughly forty percent. By comparison, when transfers are explosive, the presence of a fiscal limit increases volatility in excess of 500 percent from its fixed regime counterpart.

Alternatively, the fiscal authority could decide that the economic consequences of continuing to delay reform are too great and pass reform immediately after the fiscal limit. Although the monetary authority is still forced to adopt an interest rate peg at the fiscal limit, in this case, transfers switch to a stable process, drastically reducing its growth rate. Figures 12 and 13 compare the paths of real debt and inflation under these two fiscal policy responses. The results indicate that both real debt and inflation are far more stable prior to the fiscal limit, since a stable government transfers process from time $T$ onward reduces volatility by roughly 75 percent. Obviously the quantitative results are heavily dependent on the parameters governing the transfers process, but the qualitative result regarding volatility will hold true regardless of the parameter choices.

Even though both policies result in the monetary authority abandoning its goal of meeting its inflation target, after the fiscal limit is hit, these two policies have extremely different implications for the paths of real debt, realized inflation, and expected inflation. If reform never takes place, both the level of real debt and realized inflation remain extremely volatile as government transfers continue to follow an explosive path. The upside, however, is that as long as the probability of death is relatively small, expected inflation will be stable, hovering around the monetary authority’s inflation target. Alternatively, if reform is passed at the fiscal limit, real debt, realized inflation, and expected inflation become far less volatile, following a conventional path described by the fiscal theory of the price level.

\footnote{For this exercise, the persistence parameter is set to $\phi = 1.03$, which given the baseline calibration satisfies the necessary condition $\beta \phi < 1$.}
4 Conclusion

As the CBO makes clear, the U.S. is entering a period of heightened fiscal uncertainty. With little or no indication from lawmakers about how future policy may adjust, this paper takes a stand on how policy might unfold while taking seriously the reality that there exists a limit (political or economic) to the revenues that can be generated from taxes. The possibility of reaching the fiscal limit has always existed, but with entitlement obligations that are projected to place government transfers on an explosive path, this outcome is becoming increasingly probable.

By addressing the looming fiscal unbalance through entitlement reform that places transfers on a stable trajectory prior to the fiscal limit, the fiscal authority, in conjunction with a monetary authority that is committed to fighting inflation, is able to mitigate the degree of inflation volatility. If, however, reform is not passed, the economic consequences of reaching the fiscal limit are quite stark, as both the monetary and fiscal authority lose the ability to control inflation. Upon reaching the fiscal limit, taxes are no longer able to passively adjust to rising debt, and, assuming government default is not possible, the monetary authority is forced to abandon its pursuit of inflation in order to stabilize debt. Although the economic consequences of delayed action have been made clear, by passing reform at or shortly after the fiscal limit is reached, policymakers are able to drastically reduce both real debt and inflation volatility.
References


