Government Spending Effects in a Small Open Economy*

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April 16, 2010
Third Year Paper(Incomplete)

Abstract
In this paper, the government consumption spending effects in a small open economy(SOE), are explored. In a standard active monetary/passive fiscal policy(AM/PF) regime, openness reduces the effectiveness of the government spending on output through the terms of trade decrease. Consumption decreases less than in the closed economy, but this effect is so weak that the output does not rise more than that of the closed economy. In a passive monetary/active fiscal policy(PM/AF) regime, openness induces little change on output. With the existence of the terms of trade, openness causes less decrease in the primary surplus(by revaluating the government spending in CPI terms). Households feel wealthier than in the closed economy. This effect is compensated by the negative effects of the terms of trade(decrease) on output.

In the standard AM/PF regime, an increase in the government spending does not increae output and inflation more, compared with the closed economy. In contrast, in the PM/AF regime, the difference between open and closed economies are small in terms of output and inflation.

Keywords: Government Consumption Spending, Small Open Economy, Monetary Policy, Fiscal Policy
JEL Classification: E52; E62

*I am grateful to my advisors, Eric Leeper, Brian Peterson, Todd Walker and Bulent Guler for helpful comments and suggestions.
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1 Introduction

**Background** By mid 2009 policy interest rates were seem to be the minimum level in most central banks. During this downturn every country projected fiscal stimulus package. The packages differ in their compositions of tax cut and public sector deficit. Naturally economists became more interested in the effect of government spending or tax cut.

On the empirical evidence, some researchers found that government spending shock increases private consumption. Monacelli and Perotti (2006) investigated four OECD countries (U.S., U.K., Canada and Australia) using structural VAR. In all four countries they found that a rise in government spending tends to induce a real exchange rate depreciation and a trade balance deficit. Also they observed private consumption to rise in response to a government spending shock. Contrary to this empirical findings standard New Keynesian model shows private consumption decrease and a real exchange rate appreciation. With complete asset markets, consumption comoves positively with the real exchange rate.

The key mechanism to show a negative private consumption is the negative wealth effect of a positive government spending shock. (Monacelli and Perotti (2008)) To resolve this problem, Monacelli and Perotti (2006) introduces non-separable preferences. Botman et al. (2006) developed an overlapping generations model with distortionary taxes and rule-of-thumb consumers. In a closed economy, Bilbiie (2009) also used non-separable preferences. Galí et al. (2007) incorporates the non-Ricardian consumer behavior by including the rule-of-thumb consumers.

However, the empirical estimates of fiscal multiplier (including government spending multiplier) are so dispersed over a broad range. (See Spilimbergo et al. (2009)). Recent work by Coenen et al. (2009) reveals several causes behind these diverse estimates. They found out that the magnitude of fiscal multipliers depends, among other things, on the size of leakages into saving and imports and on the response of monetary policy to the fiscal actions. Using seven structural models, they argue that with accommodative monetary policy, government spending multiplier is larger and exceeds 1 on impact. Their paper implies that openness reduces the effects of government spending.

**Research Questions** In this paper, I investigate the government spending effects focusing on the degree of openness and policy interaction between monetary and fiscal policy. The basic questions are: How does the degree of openness affect the effectiveness of government spending?

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They argue that structural models are better suited for analyzing the interaction among factors that affect the efficacy of fiscal stimulus than the reduced-form models used in much of the literature.
spending in a small open economy?; How does the interaction between monetary and fiscal policy affects the effectiveness of government spending?; What governs the effects of government spending on consumption in a small open economy?; How large is the government spending multiplier in a small open economy?; With a distortionary taxes how does the government spending effect change?; Does the composition of government spending with domestic and foreign goods matter?

I explore the above questions without changing preferences or assuming imperfect asset market. Here the different policy regime between monetary and fiscal policy will be considered. Especially active monetary/passive fiscal and passive monetary/active fiscal regime will be studied. (Leeper (1991) and Kim (2003))


There is large literature focusing on optimal monetary policy in a small open economy. Most of them assume that Ricardian equivalence holds, that is, government budget constraint is omitted. In this line of research, Galí and Monacelli (2005) show that a small open economy can have the canonical New Keynesian representation which is isomorphic to the closed economy. In their model, prices are staggered in domestic production but not in import sector. Monetary policy is executed to react to the domestic inflation. Following the essential features of the model of Galí and Monacelli (2005), Unalmis (2008) concludes that the fiscal policy rule (tax policy rule) that is commonly used in the closed economy models might not work well in the open economy models. Galí and Monacelli (2005) and Unalmis (2008) focus only on the forward looking price setting behavior of domestic producers. In their models, the import inflation is determined by the law of one price.\(^2\) Monacelli (2005), Liu (2006) investigate optimal monetary policy introducing incomplete pass-through in the import inflation.\(^3\)

Meanwhile, only a few researchers like Garcia and Restrepo (2007), explicitly include the

\(^2\)If import price is not set in a forward looking manner, CPI based policy rule might not work well.

\(^3\)This incomplete pass-through assumption is not an ad hoc assumption. (Devereux and Yetman (2002))
government budget constraint. To introduce the government budget constraint, they assume no perfect pass-through in import sector. They analyze the effects of government spending in three different financing policies. In a closed economy, Kim (2003) explores the interaction between monetary policy and fiscal policy, introducing the government budget constraint.

**Model** The baseline model is a small open economy with infinitely lived agents. Only traded goods exist and there are two categories of goods, i.e. import goods and export goods. A continuum of domestic firms produces a differentiated good and a continuum of importing firms operate as price setters in the local retail market. Prices are sticky and firms set price in a forward looking manner.\(^4\) Firms’ production function is linear in labor. Household’s utility from consumption and leisure is separable and of CRRA form. Government spending is assumed to be unproductive and composed of only domestic goods. Complete asset market and international risk sharing (IRS) is assumed to hold.\(^5\) Uncovered interest rate parity condition is assumed to hold. The law of one price (LOP) is assumed to hold in domestic goods and but does not hold in imported goods. In this baseline model, the interest rate rule is set in terms of the CPI inflation. Tax policy is set to react to the past government debt.

In a New Keynesian closed economy model with Ricardian equivalence, an increase in unproductive government spending has intratemporal substitution effect, intertemporal substitution effect, and wealth effect. (See Davig and Leeper (2009).) In a small open economy (SOE) with active monetary and passive fiscal policy regime, the effects of government spending on output and consumption are quantitatively different.

Intratemporal Substitution effects becomes smaller when the terms of trade falls. This is because a decrease in the terms of trade causes an expenditure switching effects towards foreign goods. The real wage rises less, and consumption rises less. Over all demand increases less than in the closed economy.

Intertemporal Substitution effects becomes different due to the amplifying factor, \(\mu_\psi\). When prices are sticky, an increase in government purchases gradually raises the price level. Since inflation is a weighted average of domestic price and import price, inflation rises less than in the closed economy. When the expected inflation increases, monetary authority raises the interest rate sharply. Even though amplified by the real interest rate differential, a

\(^4\)To avoid indeterminacy problem and to consider government budget constraint explicitly, forward looking price setting behavior of importing firms is assumed following Monacelli (2005), Liu (2006) and Garcia and Restrepo (2007).

\(^5\)As Schmitt-Grohe and Uribe (2003) point out, this assumption is a purely technical matter in a small open economy (SOE).
smaller real interest rate increase dominates. This causes consumption to decrease less than in the closed economy.

Negative wealth effects are affected by the terms of trade. When the government spending increases agents expect higher present value of taxes. But the terms of trade changes the present value of the government spending. When the government spending is revaluated in CPI terms, the terms of trade effect kicks in. When compared with the close economy, the present value of surplus decreases with the fall of the terms of trade. (Agents expect an increase in the present value taxes.) Thus negative wealth effect is more severe than in the closed economy. This effect causes consumption decrease more than in the closed economy.

Based on this basic open economy dynamics, I will introduce different policy combinations and analyze the effects of government spending. There are possible extensions to the baseline model. I investigate these extensions separately. The first is the case when government expenditure is composed of both domestic and foreign goods. The second is the case that income tax exists.

In the future, capital goods can be introduced and accordingly, public capital can play an important role. Non-traded goods can be considered. Or the overlapping-generations model like Botman et al. (2006) can be used to analyze the different regimes.

Section 2 describes the baseline model in detail and the following results. Section 3 introduces income tax. Section 4 extends the model to the case that the government spending consists of both domestic goods and foreign goods. Then concluding remarks follow.

2 Government Spending Effect in SOE with Incomplete Pass-through

2.1 Baseline Model

The baseline model is a small open economy with infinitely lived agents. A small open economy(SOE) is one of a continuum of small open economies making up the world economy. Only traded goods exist and are classified into two types of goods, i.e. import goods and export goods. Uncovered interest rate parity(UIP) condition holds. Domestic households have a full access to a complete set of contingent claims traded internationally. The law

\(^{6}\)Asterisk(*) is attached to the variables of the world economy.
of one price (LOP) is assumed to hold in the domestic goods and but does not hold in the imported goods. By monopolistic competition, domestic producers can set their prices. By assumption, import retailers set their prices.

Key structural equations are similar to those proposed by Galí and Monacelli (2005), Monacelli (2005) and Liu (2006). The transportation cost or distribution technology is introduced to the import sector following Devereux and Yetman (2002).

### 2.1.1 Households’ Problem

A represent household seeks to maximize the present value utility

\[
\max_{C_t, N_t, M_t, B_t} E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\omega}}{1+\omega} + \delta \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-\kappa} \right]
\]

(1)

with \(0 < \beta < 1, \omega > 0, \kappa > 0, \chi > 0, \delta > 0\). \(\beta\) denotes the rate of time preference. \(\sigma\) is the inverse elasticity of intertemporal substitution and \(\omega\) is the inverse elasticity of labor supply. \(N_t\) represents hours of labor. Consumption, \(C_t\), is a composite consumption index of domestically produced goods (henceforth domestic goods) and foreign goods

\[
C_t \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}
\]

(2)

where \(\alpha \in [0, 1]\) is the import ratio and can be interpreted as the degree of openness. \(\eta\) is the elasticity of substitution between domestic goods and imported goods. The aggregate consumption indexes of domestic goods and imported goods are given by

\[
C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j)^{\frac{1}{\theta-1}} dj \right)^{\frac{\theta}{\theta-1}} \quad (\theta > 1)
\]

and

\[
C_{F,t} \equiv \left( \int_0^1 C_{F,t}(j)^{\frac{1}{\theta-1}} dj \right)^{\frac{\theta}{\theta-1}}.
\]

The elasticity of substitution between differentiated goods, \(\theta\), is assumed to be the same in all countries. Household’s...
budget constraint is represented by
\[
\int_0^1 P_{H,t}(j)C_{H,t}(j)\,dj + \int_0^1 P_{F,t}(j)C_{F,t}(j)\,dj + T_t + M_t + E_t Q_{t,t+1}B_t \leq M_{t-1} + B_{t-1} + W_t N_t + \Pi_t^n.
\]  
(3)

\(P_{H,t}(j)\) and \(P_{F,t}(j)\) denote the price of domestic goods \(j\) and imported goods \(j\). \(Q_{t,t+1}\) is the stochastic discount factor for one-period-ahead nominal payoffs relevant to the domestic households. By assumption households have a full access to a complete set of contingent claims which is traded internationally. \(B_t\) is nominal payoff of holding portfolio including stock, private bonds(\(B^P_t\)) and government bonds(\(B^G_t\)). \(W_t\) is the nominal wage and \(\Pi_t^n (\equiv P_{H,t}\Pi^n_H + P_{F,t}\Pi^n_F)\) is the nominal income from firm’s profits. The optimal allocation of any given expenditure within each category of goods yields the demand functions,

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t.
\]

\(4\)

\(P_t\) denotes the CPI which can be written by

\[
P_t \equiv \left[ (1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.
\]

\(5\)

Then total consumption expenditure is given by \(P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t\).

The representative household maximizes equation (1) subject to the budget constraint

\[
C_t + \frac{M_t}{P_t} + E_t \left[ Q_{t,t+1} \frac{B_t}{P_t} \right] + \tau_t \leq \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + \frac{\Pi_t^n}{P_t}
\]

\(6\)

\(P_{H,t}\) and \(P_{F,t}\) can be represented as

\[
P_{H,t} \equiv \left( \int_0^1 P_{H,t}(j)^{1-\eta} \, dj \right)^{\frac{1}{1-\eta}}, \quad P_{F,t} \equiv \left( \int_0^1 P_{F,t}(j)^{1-\eta} \, dj \right)^{\frac{1}{1-\eta}}
\]
Solving the household’s problem gives first order conditions,

\[ \chi C_t^\sigma N_t^\omega = \frac{W_t}{P_t} \]  

(7)

\[ Q_{t,t+1} \equiv \beta \left( \frac{C_t}{C_{t+1}} \right)^\sigma \frac{P_t}{P_{t+1}} \]  

(8)

\[ E_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\sigma \frac{P_t}{P_{t+1}} \right] = \frac{1}{R_t} \]  

(9)

\[ \frac{M_t}{P_t} = \delta^k \left( 1 - \frac{1}{R_t} \right)^{-\frac{1}{\sigma}} C_t^{\frac{\sigma}{\sigma}} \]  

(10)

Equation (7) represents intratemporal optimality condition. This condition can be interpreted as a competitive labor supply schedule which determines the quantity of labor supplied as a function of the real wage, given the marginal utility of consumption. To see the implication of equation (8), note that the following relation must hold for the optimizing household in the small open economy:

\[ \frac{V_{t,t+1} C_t^{-\sigma}}{P_t} = \xi_{t,t+1} \beta C_{t+1}^{-\sigma} \frac{1}{P_{t+1}} \]

where \( V_{t,t+1} \) is the period \( t \) price of an Arrow security, that is, a one-period security that yields one unit of domestic currency if a specific state of nature is realized in period \( t + 1 \). \( \xi_{t,t+1} \) is the probability of that state of nature being realized in \( t + 1 \). \( \)\( \) Galí (2008)) The left hand side(LHS) captures the utility loss resulting from the purchase of the Arrow security and the right hand side(RHS) measures the expected one-period-ahead utility gain from additional consumption by the security payoff. By defining the one-period stochastic discount factor as \( Q_{t,t+1} = \frac{V_{t,t+1} C_t^{-\sigma}}{\xi_{t,t+1}} \), equation (8) can be derived.

Necessary and sufficient conditions for household optimization are equations (7) - (10) and the transversality condition(TVC)

\[ \lim_{T \rightarrow \infty} E_t \left[ q_{t,T} \frac{A_T}{P_T} \right] = 0 \]  

(11)

\footnote{Under complete markets, the price of a one-period asset(or portfolio) yielding a random payoff, \( B_{t+1} \), must be given by \( \sum V_{t,t+1} B_{t+1} \) where the sum is over all possible \( t + 1 \) states. Equivalently, that price can be written as \( E_t \left\{ \frac{V_{t,t+1}}{\xi_{t,t+1}} B_{t+1} \right\} \).}
where

\[ A_t = B_t + M_t, \quad q_{t+1} = Q_{t+1} \frac{P_{t+1}}{P_t}. \]

Log linearizing equations (7), (9) and (10) gives

\[ \sigma \hat{c}_t + \omega \hat{n}_t = \hat{w}_t - p_t \quad (12) \]

\[ \hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{r}_t - E_t \pi_{t+1}) \quad (13) \]

\[ \hat{m}_t = -\frac{1}{\kappa} \frac{\beta}{1 - \beta} \hat{r}_t + \frac{\sigma}{\kappa} \hat{c}_t. \quad (14) \]

2.1.2 Government Consumption Spending

In this paper, the government spending implies only consumption spending, i.e. government investment spending is omitted. In addition, it is assumed that government spending does not create utility.\(^\text{10}\) Public goods are assumed to be composed only of domestically produced goods.\(^\text{11}\) These public goods are measured in terms of domestic price index. It is assumed that public goods have the same structure as private consumption goods. Demand for government consumption spending is given by

\[ G_t = \left( \int_0^1 G_t(j)^{\frac{\theta - 1}{\theta}} dj \right)^{\frac{\theta}{\theta - 1}} \quad (15) \]

\[ G_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta} G_t. \quad (16) \]

2.1.3 Domestic Inflation, CPI Inflation, Real Exchange Rate(RER) and Terms of Trade(TOT)

Unlike the closed economy, CPI is a weighted average of domestic price and import price level (in terms of domestic currency) as in equation (5). In addition, the (effective) terms of trade(TOT) is defined by the ratio between import price and domestic price levels in

\(^{10}\)Since household cannot choose the government spending, whether government spending yields utility or not, doesn’t change the results of this paper. In this reason, the case of productive government spending is excluded. However, it is possible to use a direct complementarity between private and public consumption in the utility function.(See Monacelli and Perotti (2006) and Bouakez and Rebei (2007))

\(^{11}\)This latter assumption is relaxed in Section 4.
domestic currency terms: \(^{12}\)

\[ S_t = \frac{P_{F,t}}{P_{H,t}}. \]  

(17)

Using the CPI inflation equation (5), one can link the overall price level with the domestic price as

\[
P_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha (p_{F,t} - p_{H,t})
\]

\[ = p_{H,t} + \alpha \hat{s}_t. \]

Hence the domestic inflation, the terms of trade (TOT) and the CPI inflation are related according to

\[
\pi_t = \pi_{H,t} + \alpha \Delta \hat{s}_t \quad \text{or} \quad \Delta \hat{s}_t = \pi_{F,t} - \pi_{H,t}.
\]  

(18)

These equations hold always regardless of complete pass-through assumption. The preceding equations imply that the difference between the CPI and the domestic inflation is proportional to the change in the terms of trade (TOT). This proportionality depends on the degree of openness \(\alpha\).

It is assumed that the law of one price (LOP) holds in the export sector, but need not hold in the import sector. By assumption a small open economy (SOE) has a negligible portion of the world economy, hence it cannot affect the world price. But still domestic producers have some market power by producing differentiated goods. For the import sector, incomplete pass-through is allowed. This implies that the law of one price (LOP) need not hold at all time in the import sector. As Galí and Monacelli (2005) point out, Campa and Goldberg (2005) estimate import pass-through elasticities\(^ {13}\) for a range of OECD countries. They find that first, the degree of pass-through elasticities is partial in the short run, and gradually complete in the long run. Second, the sensitivity of prices to exchange rate movements is much larger at wholesale import stage than at the consumer stage.

Assuming incomplete pass-through in the retailer level, we define the (effective) real exchange rate (RER), \(Q_t\), and the law of one price (LOP) gap, \(\Psi_{F,t}\), as

\[
Q_t = \frac{\varepsilon_t P_t^*}{P_t}, \quad \Psi_{F,t} = \frac{\varepsilon_t P_t^*}{P_{F,t}}
\]  

(19)

\(^{12}\)In the traditional trade theory, the reverse of this definition has been widely used. Compared with the traditional definition, it has the advantage that the real exchange rate (RER) and the terms of trade (TOT) show the same direction.

\(^{13}\)The exchange rate pass-through is defined as the elasticity of the import price to the exchange rate in the text book.
where $P_t^*$ is world price of world good. $\varepsilon_t$ denotes the nominal exchange rate. The real exchange rate (RER) measures the CPIs between home and world.\(^{14}\) The law of one price (LOP) gap measures the deviation of the import price from the import price when the law of one price (LOP) holds. The law of one price (LOP) gap works as a wedge between the world price and the domestic price of imported goods. Combining the definitions in equation (19), we derive the relationship between the law of one price (LOP) gap and the real exchange rate (RER)\(^ {15}\)

\[
\hat{q}_t = \psi_{F,t} + (1 - \alpha)\hat{s}_t. \tag{20}
\]

Equation (20) implies that the law of one price (LOP) gap is proportional to the real exchange rate (RER) and inversely related to the degree of international competitiveness, $s_t$.\(^ {16}\)

To explain the source of incomplete pass-through, the transportation cost (or distribution technology) is introduced following Devereux and Yetman (2002). When import prices are flexible, import prices in domestic currency terms equal to the foreign prices multiplied by the nominal exchange rate and the transportation costs: \(^ {17}\)

\[
P_{F,t} = P_t^*\varepsilon_t D_t \quad \text{or} \quad p_{F,t} = p_t^* + \hat{e}_t + \hat{d}_t. \tag{21}
\]

### 2.1.4 International Risk Sharing

Under complete international markets and perfect capital mobility, the expected return of risk free bonds in domestic currency terms, must be the same as the expected domestic

\(^{14}\)Since the portion of small open economy (SOE) is negligible, world economy has only CPI inflation, $P_t^*$.  

\(^{15}\)This relation is followed by

\[
\hat{q}_t = \hat{e}_t + p_t^* - p_t = \psi_{F,t} + p_{F,t} - p_t = \psi_{F,t} + (1 - \alpha)\hat{s}_t. 
\]

\(^{16}\)An increase in TOT, $\hat{s}_t$ implies depreciation (worsening), and a decrease in TOT, appreciation (improvement). Given a consumption, a TOT depreciation increases domestic output by inducing an expenditure switching effect towards domestic goods.  

\(^{17}\)When import prices are flexible, 

\[
\psi_{F,t} = -\hat{d}_t.
\]

\(\frac{1}{D_t}\) is the law of one price (LOP) gap when the import price is flexible. By introducing the transportation cost, we can easily extend the concept of marginal cost to the import sector. Galí and Monacelli (2005) do not introduce the transportation cost. In their paper, the law of one price (LOP) gap equals zero, when import price is flexible.
currency return from foreign bonds. This implies

$$\beta \left( \frac{C_t}{C_{t+1}} \right)^\sigma \frac{P_t}{P_{t+1}} = Q_{t,t+1} = \beta \left( \frac{C^*_t}{C^*_{t+1}} \right)^\sigma \frac{P^*_t}{P^*_{t+1}} \frac{\varepsilon_t}{\varepsilon_{t+1}}. \quad (22)$$

Combining the definition of the stochastic discount factor (SDF) and equation (22), together with the definition of real exchange rate (RER), gives

$$C_t = \vartheta C^*_t Q_t^{\frac{1}{\sigma}} \quad (23)$$

where $\vartheta$ is a constant which depend on initial conditions regarding relative net asset positions. Assume the initial conditions are symmetric ($\vartheta = 1$). Then

$$C_t = C^*_t Q_t^{\frac{1}{\sigma}}. \quad (24)$$

First order approximation around a symmetric steady state ($C = C^*, Q = 1$) yields

$$\hat{c}_t = \hat{c}^*_t + \frac{1}{\sigma} \hat{q}_t = \hat{c}^*_t + \frac{1}{\sigma} [ (1 - \alpha) \hat{s}_t + \psi_{F,t} ] .$$

Complete market assumption gives a simple relationship which links domestic consumption, world consumption and the real exchange rate (RER). A risk sharing (arbitrage) condition ties the ratio of marginal utility of consumption across countries to the real exchange rate (RER). Note that the deviation from the law of one price (LOP), by affecting the movements of the real exchange rate (RER), affects also the relative consumption baskets.

### 2.1.5 Uncovered Interest Parity (UIP) Condition

Uncovered interest rate parity (UIP) condition implies

$$E_t \left[ Q_{t,t+1} \left\{ R_t - R^*_t \frac{\varepsilon_t}{\varepsilon_{t+1}} \right\} \right] = 0. \quad (25)$$
The uncovered interest rate parity (UIP) condition can be represented in terms of the real exchange rate \( E_t \Delta \hat{q}_{t+1} = \hat{r}_t - E_t \pi_{t+1} - \hat{r}^*_t + E_t \pi^*_{t+1} \) (26)

where \( \Delta \hat{q}_{t+1} = \Delta \psi_{F,t+1} + (1 - \alpha) \Delta \delta_{t+1} \). The uncovered interest rate parity (UIP) condition is not an additional equilibrium condition because this condition can be derived by combining the consumption Euler-equations for both home and world economy with the risk sharing condition. Equation (26) implies that if domestic real interest rate increases relative to the world real interest rate, then the real exchange rate (RER) appreciates (decreases) or the real exchange rate (RER) depreciation (increase) is expected.

### 2.1.6 Domestic Production Firms’ Problem

In the domestic production sector, it is assumed that a continuum of identical monopolistically competitive firms exists. The \( j \)th firm produces a good, \( Y_t(j) \), using a linear technology

\[
Y_t(j) = A_t N_t(j)
\]

where \( a_t = \log A_t \). \( a_t \) follows an AR(1) process which describes the firm specific productivity. In a symmetric equilibrium, first order approximation\(^{19}\) gives

\[
\hat{y}_t = a_t + \hat{n}_t.
\]

Domestic producers set their prices to maximize their expected profit. Following Calvo (1983), each firm resets its price with probability, \( 1 - \varphi \), in a given period, independent of the last adjustment time. \( \varphi \) becomes a natural index of price stickiness. \( P^H_t(j) \) denotes the price level which optimizing firms set each period. An optimizing firm in period \( t \) seeks to maximize

\[
E_t \Delta \hat{q}_{t+1} = \hat{r}_t - \hat{r}^*_t
\]

\(^{18}\)This UIP condition can be represented in terms of nominal exchange rate as

\[
E_t \Delta \hat{\epsilon}_{t+1} = \hat{r}_t - \hat{r}^*_t
\]

\(^{19}\)Demand of a differentiated goods is given by \( Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta} Y_t \). Equating supply and demand gives

\[
A_t N_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta} Y_t.
\]

Integrating over all firms gives

\[
A_t \int_0^1 N_t(j) dj = Y_t \int_0^1 \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta} dj.
\]

With first order approximation, we abstract the price dispersion. Then one can derive equation (28).
the present value of its profit stream subject to the demand constraint:

$$\max_{P_{H,t}^N(j)} \quad E_t \sum_{i=0}^\infty \varphi^i Q_{t,t+i} \left[ (P_{H,t}^N(j) - MC^m_{t+i}) \right] Y_{t+i}(j)$$

$$\text{s.t.} \quad Y_{t+i}(j) \leq \left( \frac{P_{H,t}^N(j)}{P_{H,t+k}} \right)^{-\theta} Y_{t+i}$$

where $Q_{t,t+i} = \beta^i \left( \frac{C_t}{C_{t+i}} \right)^{\sigma} \frac{P_t}{P_{t+i}}$ and $MC^m_t = \frac{W_t}{A_t}$. $MC^m_t$ denotes the nominal marginal cost. First order condition (FOC) of this problem yields

$$P_{H,t}^N(j) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^\infty \beta^i \Lambda_{t,t+1}^i \varphi^i MC^m_{t+i} Y_{t+i}(j)}{E_t \sum_{i=0}^\infty \beta^i \Lambda_{t,t+1}^i \varphi^i Y_{t+i}(j)}$$

where $\Lambda_{t,t+1}^i = \left( \frac{C_t}{C_{t+i}} \right)^{\sigma} \frac{P_t}{P_{t+i}}$. Imposing symmetric equilibrium and log linearizing around zero inflation steady state gives

$$P_{H,t}^N = p_{H,t-1} + E_t \sum_{i=0}^\infty (\varphi \beta)^i \pi_{H,t+i} + (1 - \beta \varphi) E_t \sum_{i=0}^\infty (\varphi \beta)^i \hat{mc}_{t+i}.$$  \hspace{1cm} (32)

where $\hat{mc}_t$ denote log deviation of the real marginal costs. Equation (32) implies that firms set their prices according to the future discounted sum of domestic inflation and deviation of real marginal cost. Aggregate domestic price level evolves according to

$$P_{H,t} = \left[ (1 - \varphi)(P_{H,t}^N)^{1-\theta} + \varphi(P_{H,t-1})^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$  \hspace{1cm} (33)

Equation (33) clarifies that inflation arises from price resetting firms which choose a price that differs from the average price in the previous period. Combining equation (32) and log-linearized equation (33), gives the domestic inflation equation,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda \hat{mc}_t$$  \hspace{1cm} (34)

where $\lambda \equiv \frac{(1-\varphi)(1-\varphi \beta)}{\varphi}$ is strictly decreasing in the index of price stickiness $\varphi$. 

13
2.1.7 Import Retailers’ Problem

Import retailers also set their prices to maximize their expected profit. Following Calvo, $1 - \phi$ firms adjust their prices. An optimizing firm in period $t$ seeks to maximize the present value of its profit stream subject to the demand constraint:

$$\max_{P_{F,t}(j)} E_t \sum_{i=0}^{\infty} \varphi^i Q_{t,t+i}(P_{F,t}^{New}(j) - \varepsilon_{t+i} P_{t+i}^* D_{t+i}) C_{F,t+i}(j)$$

subject to:

$$C_{F,t+i}(j) \leq \left( \frac{P_{F,t}^{New}(j)}{P_{F,t+i}} \right)^{-\theta} C_{F,t+i}$$

where $D_t$ represents transportation costs or distribution technology. $C_{F,t}$ denotes the demand on imported goods. First order condition (FOC) of this problem yields

$$P_{F,t}^{New}(j) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+1} \varphi^i (\varepsilon_{t+i} P_{t+i}^* D_{t+i}) C_{F,t+i}(j)}{E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+1} \varphi^i C_{F,t+i}(j)}$$

where $\Lambda_{t,t+i} = \left( \frac{C_t}{C_{t+i}} \right) \beta \frac{P_t}{P_{t+i}}$. The parameter $\varphi$ governs the degree of exchange rate pass-through. In the case $\varphi = 0$, equation (37) reduces to a simple law of one price (LOP) equation (21), i.e. $p_{F,t} = \hat{e}_t + p_t^* + \hat{d}_t$. Imposing symmetric equilibrium and log linearizing around zero inflation steady state, gives

$$p_{F,t}^{New} = (1 - \beta \varphi) E_t \sum_{i=0}^{\infty} (\varphi \beta)^i (\hat{e}_{t+i} + p_{t+i}^* + \hat{d}_{t+i})$$

$$= (1 - \beta \varphi) E_t \sum_{i=0}^{\infty} (\varphi \beta)^i (\hat{q}_{t+i} - (1 - \alpha) \hat{s}_{t+i} + \hat{d}_{t+i} + p_{F,t+i}).$$

Equation (38) implies that import retailers are concerned with the future path of the import inflation as well as the law of one price (LOP) gap. Combining equation (38) and the import

$^{20}$ $\varphi$ need not be the same as in the domestic production sector. However, the estimation of Liu (2006) shows similar value of $\varphi$ in both domestic producers and import retailers. For simplicity, $\varphi$ is assumed same as in the domestic production sector.
inflation equation yields
\[
\pi_{F,t} = \beta E_t \pi_{F,t+1} + \lambda (\hat{q}_t - (1 - \alpha) \hat{s}_t + \hat{d}_t).
\]
\[
(39)
\]

2.1.8 Policies and Government Budget Constraint

The government converts private consumption goods into the government consumption goods one-for-one. These government purchases follow an AR(1) process. The government issues domestic currency \(M_t\). The government does not clear the budget period by period. Instead it issues one-period nominal government debt, \(B_t^G\), which pays the gross nominal interest rate of \(R_t\). Government debt is assumed to be composed of only domestic bonds. Lump-sum tax(or transfer) reacts to the lagged debt. This tax policy rule is specified by
\[
\hat{\tau}_t = \gamma \hat{b}_{t-1} + \epsilon_t^\tau.
\]
\[
(40)
\]
The government budget constraint can be represented by
\[
\frac{P_{H,t}}{P_t} G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t^G}{P_t} - \frac{R_t B_{t-1}^G}{P_t}.
\]
\[
(41)
\]
First order approximation\(^{22}\) yields
\[
\hat{b}_t + \hat{m}_t + \left( \frac{m}{b} + \frac{1}{\beta} \right) \pi_t + \left( \frac{G}{b} + \frac{1}{\beta} - 1 \right) \hat{r}_t - \frac{G}{b} \hat{q}_t + \frac{G}{b} \alpha \hat{s}_t = \frac{1}{\beta} \hat{b}_{t-1} + \frac{m}{b} \hat{m}_{t-1} + \frac{1}{\beta} \hat{r}_{t-1}.
\]
\[
(42)
\]
One difference from the closed economy is the existence of price adjustment, that is, the terms of trade(TOT). Since government purchases consist of only domestic goods, we need to represent government purchases in terms of CPI price level.
Monetary policy rule is given by
\[
\hat{r}_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \epsilon_t^r.
\]
\[
(43)
\]
where \(\phi_\pi\) and \(\phi_y\) are non-negative coefficients determined by the monetary authority. \(\phi_\pi\) and \(\phi_y\) describe the strength of the interest rate responses to the inflation and the output gap. \(\phi_\pi > 1\) implies that monetary authority raises the nominal interest rate more than
\[21\] \(R_t\) denotes one-period risk-free interest rate.
\[22\] For simplicity, \(b_t\) is used for government bonds.
one-for-one to the CPI inflation increase.

2.1.9 Aggregate Resource Constraint(ARC) and Competitive Equilibrium

Aggregate Resource Constraint

Aggregate resource constraint(ARC) implies

\[ Y_t = C_{H,t} + C^*_{H,t} + G_t \]  (44)

First order approximation around symmetric steady state\((S = 1, Q = 1)\) yields

\[ \hat{y}_t = (1 - g) \left((1 - \alpha)\hat{c}_{H,t} + \alpha\hat{c}^*_{H,t}\right) + g\hat{g}_t \]  (45)

where

\[
\begin{align*}
\hat{c}_{H,t} &= -\eta(p_{H,t} - p_t) + \hat{c}_t = \alpha \eta \hat{s}_t + \hat{c}_t \\
\hat{c}^*_{H,t} &= -\eta(p_{H,t} - \hat{e}_t - p^*_t) + \hat{c}^*_t = \eta \hat{s}_t + \eta \psi_F, t + \hat{c}^*_t.
\end{align*}
\]  (46) (47)

In equations (46) and (47), there are two things to note. First, an increase (or depreciation, worsening) in the terms of trade(TOT), \( \hat{s}_t \), will cause domestic consumers to substitute out of foreign goods into domestic goods for a given level of consumption. The size of substitution depends on the elasticity of substitution between foreign and domestic goods, \( \eta \), and the degree of openness, \( \alpha \). An increase in the terms of trade(TOT), \( s_t \), causes foreign consumers to substitute out of foreign goods into domestic goods. Second, the deviation from the law of one price(LOP) also affects the demand of foreign consumers on domestic goods and thus domestic demand. This is because, in an incomplete pass-through economy, the movement of the terms of trade(TOT) does not fully reflect the relative price change between domestic and imported goods.

23From the individual resource constraint

\[ Y_t(j) = C_{H,t}(j) + C^*_{H,t}(j) + G_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\theta} \left( C_{H,t} + C^*_{H,t} + G_t \right). \]

24Domestic consumption goods which are produced domestically are \( C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \). Foreign consumption goods which are produced domestically are \( C^*_{H,t} = \alpha \left( \frac{P_{H,t}}{P^*_t} \epsilon_t \right)^{-\eta} C^*_t \). In the strict sense as in Gali (2008), we need to introduce identical small open economies, i's, \( C^*_{H,t} = \alpha \int_0^1 \left( \frac{P_{H,t}}{P^*_t} \epsilon_t \right)^{-\eta} C^*_t \) d\( i \).

25This is equivalent to an increase in domestic competitiveness in the world market.
Combining the above goods market equilibrium conditions with international risk sharing condition gives

\[
\hat{y}_t = (1 - g) \left[ \hat{c}_t + \frac{1}{\sigma} (\mu_s - (1 - \alpha)) \hat{s}_t + \frac{1}{\sigma} (\mu_\psi - 1) \psi_{F,t} \right] + g \cdot \hat{g}_t \tag{48}
\]

\[
\hat{y}_t = (1 - g) \left[ \hat{c}^*_t + \frac{1}{\sigma} \mu_s \hat{s}_t + \frac{1}{\sigma} \mu_\psi \psi_{F,t} \right] + g \cdot \hat{g}_t \tag{49}
\]

where \( \mu_s \equiv \alpha (2 - \alpha) (\sigma \eta - 1) + 1 \) and \( \mu_\psi \equiv \alpha (\sigma \eta - 1) + 1 \). \( \mu_s \) and \( \mu_\psi \) denote the elasticity of relative output to the terms of trade (TOT) and the law of one price (LOP) gap respectively. Similarly one can get the world resource constraint

\[
\hat{y}_t^* = (1 - g) \hat{c}^*_t + g \hat{g}_t^*. \tag{50}
\]

Combining the world resource constraint (50) and the goods market equilibrium equation (49) yields

\[
\frac{1}{1 - g} \hat{y}_t = \frac{\mu_\psi}{\sigma} \psi_{F,t} + \frac{\mu_s}{\sigma} \hat{s}_t + \frac{1}{1 - g} \hat{y}_t^* - \frac{g}{1 - g} \hat{g}_t^* + \frac{g}{1 - g} \hat{g}_t.
\]

**Competitive Equilibrium** A stationary competitive equilibrium in this economy consists of a sequence of prices, \( \{P_t, P^*_t, P_{H,t}, P_{F,t}, W_t, \varepsilon_t, S_t, Q_t, \Psi_{F,t}\} \), and sequences of allocations, \( \{C_t, N_t, M_t, B_t\} \) for home households, \( \{C^*_t, C^*_{F,t}\} \) for home final goods producers, and \( \{Y_t, N_t\} \) for home intermediate good producers, along with a sequence of nominal interest rates, \( \{R_t\} \), determined by the monetary policy rules and a sequence of tax rates, \( \{\tau_t\} \), determined by the fiscal policy rules, such that (i) taking prices and the monetary and fiscal policy rules as given, the households’ allocations solve their utility maximization problems; (ii) taking prices and the monetary and fiscal policy rules as given, the producers’ and importers’ allocations solve their profit maximization problems; (iii) markets for final goods, intermediate goods, labor, money and bonds all clear.

**Trade Balance** Trade balance can be represented in terms of domestic output by

\[
NX_t = \frac{1}{Y} (Y_t - \frac{P_t}{P_{H,t}} C_t - G_t). \tag{51}
\]
First order approximation gives
\[
\tilde{nx}_t = \hat{y}_t - (1 - g)(p_t - p_{H,t} + \hat{c}_t) - g \cdot \hat{g}_t \\
= \hat{y}_t - (1 - g)\hat{c}_t - (1 - g)\alpha \hat{s}_t - g \cdot \hat{g}_t.
\] (52)

Note that the sign of the net export is ambigous and depends on the relative size of the parameters, \(\sigma\) and \(\eta\). (Refer to equation (48).)

2.1.10 Natural Level in an Open Economy

Unlike the closed economy there are two sectors (export and import sectors) in this small open economy (SOE). Natural level of the economy is defined as the output and the law of one price (LOP) gap when prices are flexible in both domestic and import sectors.

To define the natural level, firstly one can represent real marginal cost in terms of the output and the law of one price (LOP) gap:
\[
\tilde{mc}_t = \hat{w}_t - p_{H,t} - a_t \\
= \hat{w}_t - p_t + p_t - p_{H,t} - a_t \\
= \sigma \hat{c}_t + \omega \hat{n}_t + \alpha \hat{s}_t - a_t \\
= \kappa_y \hat{y}_t + \kappa_\psi \psi_{F,t} - \frac{g}{1 - g} \frac{1}{\mu_s} \hat{y}_t + \frac{\sigma}{1 - g} \frac{\mu_s - 1}{\mu_s} \hat{y}_t^* - \frac{g}{1 - g} \frac{\mu_s - 1}{\mu_s} \hat{y}_t^* - (1 + \omega) a_t 
\] (53)

where \(\kappa_y \equiv \frac{\sigma}{1 - g} \frac{1}{\mu_s} + \omega\) and \(\kappa_\psi \equiv 1 - \frac{\mu_s}{\mu_s}\). Third line shows that in a small open economy (SOE), real marginal cost depends also on the evolution of the terms of trade (TOT). A real depreciation (i.e. an increase in \(\hat{s}_t\)) induces a higher real marginal cost by increasing the product wage for any given level of consumption and output. Equation (53) implies that a change in domestic output has an effect on marginal cost through its impact on employment (\(\omega\)), and wealth effects (\(\sigma\)) on labor supply resulting from their impact on domestic consumption. \(\mu_s\) captures contracting effects through the terms of trade (TOT).

Similarly, marginal cost\(^{26}\) in the import sector can be represented in terms of the law of one price (LOP) gap:
\[
\tilde{mc}_t^F = \hat{q}_t - (1 - \alpha)\hat{s}_t + \hat{d}_t \\
= \psi_{F,t} + \hat{d}_t
\] (54)

\(^{26}\)In fact, this is not a marginal cost. For the simplicity this name is attached.
A natural level is defined as the state where domestic price and import price are both flexible. That is, in a natural level, $\hat{m}c_t = \hat{m}c^F_t = 0$. Then we can derive the natural output level, $\hat{y}_t^n$, and the natural law of one price (LOP) gap $\psi^F_{F,t}$. Using the natural level, marginal costs can be represented in terms of output gap and the law of one price (LOP) gap (from natural level):

$$\hat{m}c_t = \kappa_y \hat{y}_t + \kappa\psi_{F,t}$$
$$\hat{m}c^F_t = \hat{\psi}_{F,t}$$

where $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$ and $\tilde{\psi}_{F,t} = \psi_{F,t} - \psi^F_{F,t}$.

### 2.1.11 Dynamic System

For the intuition and tractability, we construct the dynamic system of the baseline model as simple as possible. The system consists of three equations, omitting the equilibrium government budget equation. Even though we are interested in the effects of the government spending, the government spending is omitted for a moment.

#### Consumption-Euler Equation

Combining household optimality condition with goods market equilibrium condition and the law of one price (LOP) gap definition yields

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{\mu_\psi}{\sigma} (\hat{r}_t - E_t \hat{\pi}_{t+1}) - \alpha \eta E_t \Delta \hat{s}_{t+1} + \frac{1}{\sigma} (\mu_\psi - 1)(\hat{r}_{t}^* - E_t \hat{\pi}_{t+1}^*)$$

where $\mu_\psi \equiv \alpha (\sigma \eta - 1) + 1$. World interest rate is given exogenously. $\mu_\psi$ captures the effect of the real interest rate change to the overall demand through the real exchange rate (RER) movement. In a complete pass-through economy, all the changes in the real interest rate differential between domestic and world economy are represented by the terms of trade (TOT), $\hat{s}_t$. Then $\mu_\psi = 1$. In this model, incomplete pass-through assumption is introduced. The ef-

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27 Corresponding natural values are represented by

$$\psi^F_{F,t} = -\hat{d}_t$$
$$\hat{y}_t^n = \frac{\kappa \hat{\psi}}{\kappa_y} \hat{d}_t + \frac{1}{\kappa_y} \left[ \frac{g \cdot \sigma \cdot \mu_s - 1}{1 - g \cdot \mu_s - 1} \hat{y}_t^* + \frac{g \cdot \sigma \cdot \mu_s - 1}{1 - g \cdot \mu_s - 1} \hat{y}_t^* + (1 + \omega)a_t \right].$$

28 For the complete description of the model, refer to the Appendix A
effects of real interest rate differential are partially passed through to the terms of trade (TOT). \(\mu_\psi\) captures the effects of relative real exchange rate (RER) change (through real interest rate differential) which are not passed through to the terms of trade (TOT). With standard parameter values, \(\mu_\psi > 1\). This implies that incomplete pass-through amplifies the real interest rate effects compared with the closed economy.

Note that an increase in openness raises the sensitivity of domestic demand. As the above consumption-Euler equation shows, it raises demand directly through the terms of trade (TOT). Indirectly, openness increases the magnitude of the amplification through the real interest rate differential. In this way, an increase in openness change the movement of private consumption.

**Goods Market Equilibrium Equation** Simplified goods market equilibrium condition reads

\[ \hat{y}_t = \hat{y}_t^* + \frac{\mu_\psi}{\sigma} \psi F_t + \frac{\mu_s}{\sigma} \hat{s}_t. \]

**Inflation Equation** Using two inflation equations (domestic inflation and import inflation) one can represent CPI inflation\(^{29}\) as

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda (\kappa_y \hat{y}_t + \kappa_\psi \hat{\psi}_F_t) \]

(55)

The terms of trade (TOT) equation\(^{30}\) reads

\[ \Delta \hat{s}_t = \beta E_t \Delta \hat{s}_{t+1} + \lambda (-\kappa_y \hat{y}_t + (1 - \kappa_\psi) \hat{\psi}_F_t). \]

(56)

For the stationarity of the terms of trade (TOT), see Galí (2008).\(^{31}\)

\(^{29}\)For the CPI inflation equation, we use the definition of inflation, that is, we average two inflation equations, \[ \pi_H,t = \beta E_t \pi_{H,t+1} + \lambda (\kappa_y \hat{y}_t + \kappa_\psi \hat{\psi}_F_t) \]
\[ \pi_F,t = \beta E_t \pi_{F,t+1} + \lambda \hat{\psi}_F,t. \]

\(^{30}\)For the terms of trade (TOT) equation, we subtracted the domestic inflation equation from the import inflation equation.

\(^{31}\)Galí (2008) argues that even though the nominal exchange rate and the domestic inflation are non-stationary, the terms of trade (TOT) can be stationary. In a steady state, \[ \hat{s}_t = p_F,t - p_H,t = \varepsilon_t p^*_t - p_H,t. \]
Monetary Policy Rule

Monetary policy rule is given by

\[ \hat{r}_t = \phi_x \pi_t. \]

Five Equation System

This basic system has five endogenous variables and three forecast errors.

\[
\begin{bmatrix}
1 & \frac{\mu_\psi}{\sigma} & \alpha \eta & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & 0 & \beta & 0 & 0 \\
1 & 0 & -\frac{\mu_s}{\sigma} & -\frac{\mu_\psi}{\sigma} & 0 \\
0 & 0 & 1 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\hat{y}_t \\
\pi_t \\
\Delta \hat{s}_t \\
\psi_{F,t} \\
\hat{s}_t
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{\mu_\psi}{\sigma} \phi_\pi & 0 & 0 & 0 \\
-\lambda \hat{\kappa}_y & 1 & 0 & -\lambda \hat{\kappa}_\psi & 0 \\
-\lambda \kappa_y & 0 & 1 & -\lambda (1 - \kappa_\psi) & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\hat{y}_{t-1} \\
\pi_{t-1} \\
\Delta \hat{s}_{t-1} \\
\psi_{F,t-1} \\
\hat{s}_{t-1}
\end{bmatrix}
\]

+ forecast errors and exogenous terms.

These five equations with policy rules consist of core dynamics. In the standard closed economy, goods market equilibrium condition is substituted into the consumption-Euler equation. Here, in addition to the substitution into the consumption-Euler equation, we need goods market equilibrium equation to construct the basic dynamic system. The interest rate policy reacts to inflation and output gap, not to the law of one price(LOP) gap. Inflation and output gap are determined by a forward looking manner, but the law of one price(LOP) gap is not. The law of one price(LOP) gap is determined by the goods market equilibrium condition.

2.2 Calibration and Impulse response Analysis

Calibration

Deep parameter values are standard. Following Galí and Monacelli (2005), \( \omega = 3, \mu = 1.2, \theta = 6, \varphi = 0.75, \beta = 0.99, r = 0.4, \alpha = 0.4, S = 1. \) From Kim (2003), \( G = g = 0.2, Y = 1, \frac{G}{b} = 0.07, \frac{m}{b} = 0.2, \tau^I = 0.25, \rho_y = \rho_d = \rho_s = \rho_r = \rho_t = 0.8. \) As in Garcia and Restrepo (2007), \( \alpha_G = 0.1. \) We set \( \sigma = 1.1 \) and \( \eta = 1.1 \) to satisfy the small open economy(SOE) parameter condition, \( \sigma \eta > 1. \)

AM/PF in a Closed Economy

In a closed economy, the government spending affects the economy through intratemporal substitution effects, intertemporal substitution effects and negative wealth effects. It is assumed that monetary policy is active(henceforth AM) in the
sense that monetary authority raises the interest rate more than one-for-one to the increase in inflation. Fiscal policy is assumed to be passive (henceforth PF) in the sense that fiscal authority adjusts the government debt to settle the budget constraint.

Intratemporal Substitution effects work as follows. The rise in the government spending raises overall demand. Then labor demand increases and real wage rises. As the real wage rises households work harder substituting consumption for leisure. Labor supply increases upwards along supply curve. Higher real wages increase firm’s marginal costs and cause them to raise prices.

Intertemporal Substitution effects work through the real interest rate. When prices are sticky, an increase in government purchases gradually raises the price level. When the expected inflation increases, monetary authority raises the interest rate sharply. The real interest rate rises due to sticky prices. The intertemporal price of consumption changes and agents postpone consumption.

There are also negative wealth effects. When the government spending increases, agents expect higher present value of taxes. Higher taxes create a negative wealth effect which increases the supply of labor when leisure is a normal good. Households reduce their consumption path because consumption is also a normal good.

**Real Interest Rate Differential in a Small Open Economy (SOE)** In a small open economy (SOE), the effects of government spending on output and consumption are quantitatively differ because of the terms of trade (TOT) and the real exchange rate (RER) movement.\(^{32}\) The difference comes directly from the movement of the terms of trade (TOT) by expenditure switching effect. Indirectly, domestic real interest rate effects are amplified by the amount, \(\psi_{F,t}\). These two effects are represented in the consumption-Euler equation,

\[
\hat{y}_t = E_t \hat{y}_{t+1} - \frac{\mu_s}{\sigma} (\hat{r}_t - E_t \pi_{t+1}) - \alpha \eta E_t \Delta s_{t+1} + \frac{1}{\sigma} (\mu_s - 1) \left( \hat{r}_t^* - E_t \pi_{t+1}^* \right).
\]

To explore the difference, first we turn to the uncovered interest rate (UIP) condition. When domestic real interest rate rises and uncovered interest rate parity (UIP) condition holds, the increase in the differential between domestic and world real interest rate causes the real exchange rate (RER) to decrease or the expected real exchange rate (RER) to increase.

The total effects of real exchange rate movements work through the terms of trade (TOT)

\(^{32}\)The real exchange rate (RER) works through the real interest rate differential between home and world economy.
and the law of one price (LOP) gap.\textsuperscript{33} First, the increase in the terms of trade (TOT) induces expenditure switching effects towards domestic goods.\textsuperscript{34} An increase (or deprecation, worsening) in the terms of trade (TOT), $s_t$, will cause domestic consumers to substitute out of foreign goods into domestic goods for a given level of consumption. An increase in the terms of trade (TOT), $s_t$, causes foreign consumers to substitute out of foreign goods into domestically produced goods. Second, an increase in the deviation from the law of one price (LOP) also raises the demand of foreign consumers on domestic goods and therefore domestic demand. This is because, in an incomplete pass-through economy, the movement of the terms of trade (TOT) does not fully reflect the relative price difference between domestic and imported goods. If complete pass-through is assumed, the terms of trade (TOT) fully represents the difference between domestic goods price and imported goods price.

Note that in the consumption-Euler equation, the effect of the terms of trade (TOT) is separated from the effect of real interest differential (through real exchange rate (RER)). The terms of trade (TOT) and amplifying factor $\mu_\psi$, represent the price difference between home and world economy. The terms of trade (TOT) is determined by the forward looking domestic producers and import retailers. But the terms of trade (TOT) does not fully reflect the relative price between home and world economy. The real interest rate differential affects the economy by some portion ($\mu_\psi$)\textsuperscript{35} of the deviation from the law of one price (LOP) gap. Since $\mu_\psi > 1$ with standard parameter values in a small open economy (SOE), $\mu_\psi$ amplifies the effects of domestic real interest rate movement.

**AM/ PF in a Small Open Economy** In a small open economy (SOE) with AM/PF regime, the effects of government spending on output and consumption are quantitatively different.

Intratemporal Substitution effects becomes smaller when the terms of trade (TOT) falls. This is because a decrease in the terms of trade (TOT) causes an expenditure switching effects towards foreign goods. The real wage rises less, and consumption rises less. Over all demand increases less than in the closed economy.

Intertemporal Substitution effects becomes larger due to the amplifying factor, $\mu_\psi$. When prices are sticky, an increase in government purchases gradually raises the price level. Since inflation is a weighted average of domestic price and import price, inflation rises less than in...

\textsuperscript{33}Refer to the uncovered interest rate parity (UIP) condition and the definition of the law of one price (LOP) gap, $E_t \Delta q_{t+1} = E_t \Delta \psi_{F,t+1} + (1 - \alpha) \Delta s_{t+1} = \hat{r}_t - E_t \pi_{t+1} - \hat{r}_t^* + E_t \pi_{t+1}^*$

\textsuperscript{34}Refer to the equations (46) and (47).

\textsuperscript{35} $\mu_\psi$ represents the elasticity of relative domestic output (demand) to the law of one price (LOP) gap.
the closed economy. When the expected inflation increases, monetary authority raises the interest rate sharply. Even though amplified by the real interest rate differential, a smaller increase in the real interest rate dominates. This causes consumption to decrease less than in the closed economy.

Negative wealth effects are affected by the terms of trade (TOT). When the government spending increases agents expect higher present value of taxes. But the terms of trade (TOT) changes the present value of the government spending. When the government spending is reevaluated in CPI terms, the terms of trade (TOT) effect kicks in. When compared with the close economy, the present value of surplus decreases with the fall of the terms of trade (TOT). Thus negative wealth effect is more severe than in the closed economy. This effect causes consumption to decrease more than in the closed economy.

**PM/AF in a Small Open Economy** We turn to the regime where monetary policy is passive and fiscal policy is active (PM/AF). The main channel of closed economy works in the same manner. One difference is that the terms of trade (TOT) effects appears in the equilibrium government budget equation.

In a closed economy, a decrease in the present value of current and future primary budget surpluses makes the real value of government liabilities exceed the present discounted value. Households feel wealthier and consume more. This pushes up the price level.

To compare with the closed economy case, first we consider the government budget constraint in a small open economy (SOE):

\[
\frac{M_{t-1} + R_{t-1}B_{t-1}^G}{P_t} = \tau_t - \frac{P_{H,t}}{P_t} G_t + \left( 1 - \frac{1}{R_t} \right) \frac{M_t}{P_t} + \frac{1}{R_t} \frac{P_{t+1}}{P_t} \left( \frac{M_t + R_tB_{t+1}^G}{P_{t+1}} \right)
\]

For the explanation, we rewrite the government budget constraint by imposing the transversality condition and equilibrium condition:

\[
\frac{M_{t-1} + R_{t-1}B_{t-1}^G}{P_t} = \sum_{i=0}^{\infty} \beta^i E_t \frac{U''(C_{t+i})}{U'(C_t)} \left[ \tau_{t+i} - \frac{1}{(1 - \alpha) + \alpha \beta_t^{1-\eta}} G_{t+i} + \left( 1 - \frac{1}{R_{t+i}} \right) \frac{M_{t+i}}{P_{t+i}} \right]
\]

As in the closed economy, real value of current government liabilities equals the present value of expected current and future primary budget surpluses (adjusted by the terms of trade (TOT)), plus the governments’s interest saved on the part of its liabilities that households are willing to hold in monetary form. An increase in the nominal value of government liabilities or a decrease in the present value of primary budget surpluses makes the real value
of government liabilities exceed the present discounted value. Then households convert the
government liabilities into current consumption. This raises the aggregate demand and price
level.

Openness works through the movement of the terms of trade (TOT). Consider the case
that the terms of trade (TOT) falls. When an economy becomes more open, the same gov-
ernment spending causes less decrease in primary surplus. Thus the real value of government
liabilities differs as openness increases.

Intertemporal Substitution effects might not contribute to increase consumption more
than in the closed economy. When there is a government spending shock, inflation rises.
Since CPI is a weighted average, inflation rises less than in the closed economy. When
monetary policy is passive, the real interest rate falls due to sticky prices. The real interest
rate decrease less, but amplified by the real interest rate differential effects, $\mu_\psi$. Overall
effect is ambiguous.

**Government Spending Effects with Different Degrees of Openness and Different
Policy Regimes** In figure 1, the upper two rows represent the regime of active monetary
policy($\phi_\pi = 1.5/1.1$) and passive fiscal policy($\gamma_b = 0.3$). The lower two rows represent the
regime of passive monetary policy($\phi_\pi = 0.9/0.5$) and active fiscal policy($\gamma_b = 0.1$).

In an AM/PF regime, a decrease in the terms of trade (TOT) dominate. Since the
terms of trade (TOT) falls, overall government spending effects on output becomes less when
compared with the closed economy. This output decrease is compensated a little bit by a
less decrease in consumption demand.

Consumption decreases a little less compared with the closed economy. First let’s condier
the intertemporal substitution effects. Given the same increase in demand, more open econ-
omy has less impacts on CPI inflation. Therefore the real interest rate rises less than the
closed economy. This causes consumption decrease less. Since this smaller negative effect
is amplified by the real interest rate differential between home and world, overall effect on
consumption is ambiguous. However, it seems that a smaller increase in the real interest
rate dominates the amplifying effect. This causes consumption to decrease less than in the
closed economy. The size of intratemporal substitution effects changes because of the size of
output (or demand). The terms of trade (TOT) effects cause less increasing demand and less
rising real wage. This makes consumption to increase less than in the closed economy. The
negative wealth effect is also affected by the terms of trade (TOT). When the government
spending increases, agents expect higher present value of taxes. But the terms of trade (TOT)
changes the present value of the government spending. The negative wealth effect is more
Figure 1: Government Spending Shock with Different Openness (Note: Solid line represents nearly closed economy, $\alpha = 0.001$. Dashed line represents the open economy with openness, $\alpha = 0.1$. Dash-Dotted line represents the standard open economy with openness, $\alpha = 0.4$)
severe than in the closed economy. This effect causes consumption increase less than in the closed economy.

Overall, consumption decrease less due to the intertemporal substitution effects. Except some periods in the beginning, this less decreasing amount does not overcome the less increasing demand induced by the fall in the terms of trade(TOT), i.e. expenditure switching effects. Note that consumption is composed of domestically produced goods and imported goods. In a small open economy, consumption increase does not cause the increase in demand one-for-one. Therefore the output increases less than in the closed economy.

In a PM/AF regime(3rd and 4th rows), even though the terms of trade decreases, the boosting effect of consumption on overall demand is bigger(opposite sign) than in the AM/PF regime. The effects of the government spending on output is quite closer to the closed economy. The positive wealth effects compensate the terms of trade(TOT) effects. This implies that consumption increase more than in the closed economy case. When the terms of trade(TOT) falls, the government spending leads to less decrease in primary surplus than in the closed economy. This effect causes consumption to rise more. Intertemporal substitution effects has a positive effect on consumption. Inflation rises less than in the closed economy. Since monetary policy is passive, the real interest rate falls due to sticky prices. Its effect is amplified by the real interest rate differential effects.

In sum, when openness is concerned, a AM/PF regime shows less increasing output and inflation with more open economy. In a PM/AF regime, the differences in output and inflation between open and closed economies are smaller than the AM/PF regime.

Government Spending Effects on various economic variables in a Small Open Economy(SOE) Figure 2 represents the case when $\alpha = 0.4$, that is, the standard small open economy(SOE). The terms of trade(TOT) decreases more in the PM/AF regime. Unlike the AM/PF regime, consumption rises in the PM/AF regime, mainly through the revaluating the present value of government spending. International risk sharing arbitrage condition ties the ratio of marginal utilities(MU) of consumption across countries to the real exchange rate(RER). The real exchange rate(RER) shows similar path to consumption. Since the terms of trade(TOT) decreases, the net export deteriorates. In a PM/AF regime, the real exchange rate(RER) increases and this effects makes the net export to decrease less compared with the AM/PF regime. In an AM/PF regime, the decline in private consumption tends to cause an improvement of the trade balance(the absorption effect). The real exchange rate(RER) appreciation and a decrease in the terms of trade(TOT) also causes a switching effects towards foreign goods.
Figure 2: Government Spending Shock Effects (Note: Solid line represents the AM/PF regime with $\phi_\pi = 1.1$ and $\gamma_b = 0.3$. Dashed line represents the PM/AF regime with $\phi_\pi = 0.5$ and $\gamma_b = 0.1$.)
Figure 3: Monetary Policy Shock, World Interest Rate Shock and Productivity Shock (Note: Solid line represents the AM/PF regime with $\phi_\pi = 1.1$ and $\gamma_b = 0.3$. Dashed line represents the PM/AF regime with $\phi_\pi = 0.5$ and $\gamma_b = 0.1$.)

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The Effects of Various Shocks in a Small Open Economy

In figure 3, the response of the economy to the various shocks are represented. The first column represents the response to the monetary policy shock. The second and third column represent a world interest rate shock and a productivity shock. In an AM/PF regime, a monetary policy shock has a contraction effects. This negative shock decreases demand. Labor demand decreases and real wage falls. Then households work less substituting leisure for consumption. In a sticky price economy, an upward interest rate shock causes real interest rate rise. A decrease in consumption is amplified by the real interest rate differential. An increase in the interest rate causes negative wealth effects. Households expect net present value of expected primary budget surpluses. In a PM/AF regime, households feel wealthier and consume more. The economy boosts up. Compared with the government spending shock, the initial increase in consumption is less. Since there is a positive interest rate shock, intertemporal substitution effect doesn’t work much to boost the economy.

To a productivity shock, different regimes do not make a significant difference. This figure confirms the typical RBC results that a persistent technology shock increases consumption. A productivity shock raises the marginal product of labor and real wage. As real wage rises households work harder substituting consumption for leisure. At the same time the increase in consumption lowers the marginal utility of income and reduces work effort (intertemporal substitution effect in labor supply) reflecting positive wealth effects.

3 Introducing Income Tax

We introduce income tax($\tau^I_t$) at the consumer level (stockholder), considering no tax on firms’ undistributed profits. This changes the household’s budget constraint as

$$C_t + \frac{M_t}{P_t} + E_t \left[ Q_{t,t+1} \frac{B_t}{P_t} \right] + \tau_t \leq (1 - \tau^I_t) \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} + (1 - \tau^I_t) \frac{\Pi^a_t}{P_t}$$

where $\Pi^a_t$ denotes the nominal profits from holding shares of domestic production firms and import retailers. Corresponding first order condition (FOC) for labor includes the income taxes:

$$\chi C_t^\sigma N_t^\sigma = (1 - \tau^I_t) \frac{W_t}{P_t}$$

Real marginal cost is affected by the income taxes because income taxes distort the labor
Figure 4: Government Spending Shock When the Income Tax Exists (Note: Solid line represents the AM/PF regime with $\phi_\pi = 1.1$, $\gamma_b = 0.3$ and $\tau^f = 0$. Dash-dotted line represents the AM/PF regime with $\phi_\pi = 1.1$, $\gamma_b = -1.5$ and $\tau^f = 0.25$. Dashed line represents the PM/AF regime with $\phi_\pi = 0.5$, $\gamma_b = 0.1$ and $\tau^f = 0$. Dotted line represents the PM/AF regime with $\phi_\pi = 0.5$, $\gamma_b = -1.0$ and $\tau^f = 0.25$.)
supply decision of households. Natural level also changes because of the real marginal cost.

The introduction of income taxes changes government budget constraint. The government budget constraint becomes

\[ \frac{P_{H,t} G_t}{P_t} = \tau_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} - \frac{R_{t-1} B_{t-1}}{P_t} + \tau_t^f W_t N_t + \tau_t^f \Pi^g_t. \]

Unlike the closed economy, real profits come from both domestic producers and import retailers. First order approximation yields

\[
\frac{b}{Y} \hat{b}_t + m \hat{m}_t + \left( \frac{m}{Y} - \frac{1}{\beta} \frac{b}{Y} \right) \pi_t + \left( \frac{G}{Y} + \frac{b}{Y} \left( \frac{1}{\beta} - 1 \right) - \tau_t^f \right) \hat{\tau}_t - \frac{G}{Y} \hat{g}_t + \frac{G}{Y} \alpha \hat{s}_t \\
+ \tau_t^f (\hat{\tau}_t^f + \alpha \hat{s}_t + \hat{y}_t - \frac{C}{Y} \alpha \hat{d}_t) = \frac{1}{\beta} \frac{b}{Y} \hat{b}_{t-1} + m \frac{1}{Y} \hat{m}_{t-1} + \frac{1}{\beta} \frac{b}{Y} \hat{\tau}_{t-1}
\]

We compare the economy where income taxes exist with the economy where income taxes do not exist. Figure 4 represents the results of this comparison. Note that the existence of income taxes doesn’t affect the economy when monetary policy is active and fiscal policy is passive.

In a closed economy with a PM/AF regime(Kim (2003)), consumption and output increase less when income taxes exist. When there is no income taxes, an increase in the government spending raises consumption demand and overall demand by decreasing the value of current and future budget surpluses. When the income tax is introduced, an increase in overall demand raises net income tax level, which partially offsets the initial fall in the present value. This causes the effectiveness of government spending to be slightly weaker than the case with no income tax.

In an open economy with a PM/AF regime, the terms of trade(TOT) decreases slightly more, when the income taxes exist. This makes the overall effects on output and inflation quite similar to no income tax case.

\[36\] real marginal cost is represented by

\[ \hat{m}_t = \kappa_y \hat{y}_t + \kappa_\psi \psi_{F,t} - \frac{g}{1 - g} \mu_s - \frac{1}{g} \mu_s \hat{y}_t + \frac{\sigma}{1 - g} \mu_s - \frac{1}{1 - g} \mu_s \hat{y}_t^* - \frac{g}{1 - g} \mu_s - \frac{1}{1 - g} \mu_s \hat{y}_t^* + \frac{\tau_t^f}{1 - \tau_t^f} \hat{\tau}_t - (1 + \omega) a_t \]

where \( \kappa_y = \frac{\sigma}{1 - g} \frac{1}{\mu_s} + \omega \) and \( \kappa_\psi = 1 - \frac{\mu_s}{\mu_s} \).

\[37\] In the strict sense, the profits from import retailers should be included in GDP. Garcia and Restrepo (2007) introduce import production process and make clear the category of GDP. However, for simplicity we assume that GDP reflects only the domestic goods production.
4 Introducing Foreign Goods to Government Spending

In this section we assume that the government consumption is composed of both domestic and imported goods. This implies that same amount of government spending has less effects on domestic production due to the slightly lower ratio of domestic goods in government consumption. We assume that the elasticity of substitution in government consumption is same as the private consumption. Instead, a different import ratio is assumed. ($\alpha_G = 0.1$)

Then composite index for government spending can be represented by

$$G_t \equiv \left[ (1 - \alpha_G)^{\frac{1}{\eta}} G_H^{\frac{\eta-1}{\eta}} + \frac{1}{\alpha_G} G_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Now, the government consumption of domestic goods is

$$G_H,t = (1 - \alpha_G) \left( \frac{P_H,t}{P_G,t} \right)^{-\eta} G_t.$$

Similarly, we consider the government consumption of imported goods. Corresponding price index for government consumption is

$$P_G,t = \left[ (1 - \alpha_G) P_H^{1-\eta} + \alpha_G P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

In the government budget constraint, only the price index for government consumption is different from the baseline model. The government price index reflects the composition of government consumption. The government budget constraint is represented by

$$\frac{P_G,t}{P_t} G_t = \tau_t + \frac{M_t - M_{t-1}}{P_t} + \frac{B_t}{P_t} - \frac{R_{t-1} B_{t-1}}{P_t}.$$

First order approximation yields

$$\hat{g}_t + \frac{m}{b} \hat{m}_t + \left( \frac{m}{b} - \frac{1}{\beta} \right) \hat{\pi}_t + \left( \frac{G}{b} + \frac{1}{\beta} - 1 \right) \hat{\tau}_t - \frac{G}{b} \hat{g}_t + \frac{G}{b} (\alpha - \alpha_G) \hat{s}_t = \frac{1}{\beta} \hat{b}_{t-1} + \frac{m}{b} \hat{m}_{t-1} + \frac{1}{\beta} \hat{r}_{t-1}.$$

The introduction of imported goods to the government spending changes the domes-
Figure 5: Government Spending Shock When Consisting of Domestic and Foreign Goods
(Note: Solid line represents the AM/PF regime with $\phi_\pi = 1.1$, $\gamma_b = 0.3$ and $g_H = 0.2$. Dash-dotted line represents the AM/PF regime with $\phi_\pi = 1.1$, $\gamma_b = 0.3$ and $g_H = 0.18$. Dashed line represents the PM/AF regime with $\phi_\pi = 0.5$, $\gamma_b = 0.1$ and $g_H = 0.2$. Dotted line represents the PM/AF regime with $\phi_\pi = 0.5$, $\gamma_b = 0.1$ and $g_H = 0.18.$)
tic government spending-GDP ratio. The ratio decreases from 0.20 (= g) to 0.18 (= g_H), Consumption-Euler equation\textsuperscript{39} and goods market equilibrium condition\textsuperscript{40} include the change in the composition of government spending. Trade balance is measured in terms of domestic output

\begin{equation}
NX_t = \frac{1}{Y_t} (Y_t - \frac{P_t}{P_{H,t}} C_t - \frac{P_{G,t}}{P_{H,t}} G_{H,t}).
\end{equation}

The marginal cost of the production firm also changes, reflecting the smaller portion of government spending in domestic goods.

Figure 5 shows the impulse-response analysis result. In a PM/AF regime, the government spending effect causes a less decrease in the net present value primary budget surplus than the case that government spending is composed of only domestic goods. Consumption increases slightly more than that of the baseline model. In a AM/PF regime, intertemporal substitution effect dominates with low inflation. Consumption decreases less compared with baseline model. (This implies less decreasing consumption.)

In both regimes, the effects on output are negligibly small. Demand boosting effect of government spending is smaller than that of baseline model, since government spending has small portion of domestic goods. This smaller boosting effect is compensated by the a slight larger(or less smaller) consumption demand.

5 Concluding remarks

In a small open economy(SOE) with active monetary and passive fiscal policy regime, the effects of government spending on output and consumption are quantitatively different from the closed economy counterpart.

In an AM/PF regime, the effects of terms of trade(TOT) dominate on the output. Since the terms of trade(TOT) falls, overall government spending effects on output becomes less

\begin{equation}
\frac{1}{1-g_H} \dot{y}_t = \frac{1}{1-g_H} E_t \dot{y}_{t+1} - \frac{\mu_{\psi}}{\sigma} (\hat{r}_t - E_t \pi_{t+1}) - Z_1 E_t \Delta \hat{s}_{t+1} + \frac{1}{\sigma} (\mu_{\psi} - 1)(\hat{r}^*_t - E_t \pi^*_{t+1}) - \frac{g_H}{1-g_H} E_t \Delta \hat{g}_{t+1}
\end{equation}

where $Z_1 \equiv \alpha \eta - \frac{g_H}{1-g_H} \alpha C \eta$ and $g_H = g \cdot (1 - \alpha_C)$.

\begin{equation}
\frac{1}{1-g_H} \dot{y}_t = \frac{\mu_{\psi}}{\sigma} \psi_{F,t} + \left( \frac{\mu_s}{\sigma} \frac{g_H}{1-g_H} \alpha C \eta \right) \dot{s}_t + \frac{1}{1-g_H} \dot{y}^*_t - \frac{g_H}{1-g_H} \dot{g}^*_t + \frac{g_H}{1-g_H} \dot{g}_t.
\end{equation}

\textsuperscript{39}Corresponding consumption-Euler equation becomes

\begin{equation}
\frac{1}{1-g_H} \dot{y}_t = \frac{1}{1-g_H} E_t \dot{y}_{t+1} - \frac{\mu_{\psi}}{\sigma} (\hat{r}_t - E_t \pi_{t+1}) - Z_1 E_t \Delta \hat{s}_{t+1} + \frac{1}{\sigma} (\mu_{\psi} - 1)(\hat{r}^*_t - E_t \pi^*_{t+1}) - \frac{g_H}{1-g_H} E_t \Delta \hat{g}_{t+1}
\end{equation}

\textsuperscript{40}Corresponding goods market equilibrium condition becomes

\begin{equation}
\frac{1}{1-g_H} \dot{y}_t = \frac{\mu_{\psi}}{\sigma} \psi_{F,t} + \left( \frac{\mu_s}{\sigma} \frac{g_H}{1-g_H} \alpha C \eta \right) \dot{s}_t + \frac{1}{1-g_H} \dot{y}^*_t - \frac{g_H}{1-g_H} \dot{g}^*_t + \frac{g_H}{1-g_H} \dot{g}_t.
\end{equation}
when compared with the closed economy.

Consumption decreases a little less compared with the closed economy. First the intertemporal substitution effect becomes larger. Given the same increase in demand, more open economy has less impact on CPI inflation. Therefore the real interest rate rises less than the closed economy. This makes consumption to decrease less. Since this smaller negative effect is amplified by the real interest rate differential between home and world, overall effect on consumption is ambiguous. However, it seems that a smaller real interest rate increase dominates. This causes consumption to decrease less than in the closed economy. Second, the size of intratemporal substitution effects changes because of the size of output (or demand). The terms of trade (TOT) effects causes less increasing demand and less rising real wage. This causes consumption increases less than in the closed economy. Third, the negative wealth effect is also affected by the terms of trade (TOT). When the government spending increases agents expect higher present value of taxes. But the terms of trade (TOT) changes the present value of the government spending. The negative wealth effect is more severe than in the closed economy. This effect causes consumption increase less than in the closed economy.

Overall, consumption decrease less due to the intertemporal substitution effects. But this amount does not overcome the less increasing demand by the fall in the terms of trade (TOT), i.e. expenditure switching effects. Note that consumption is composed of domestically produced goods and imported goods. In a small open economy (SOE), consumption increase does not cause the increase in demand one-for-one. Therefore the output increases less than in the closed economy.

In a PM/AF regime, even though the terms of trade (TOT) decreases, the boosting effect of consumption on overall demand is bigger than in the AM/PF regime. The effects of the government spending on output is quite closer to the closed economy. The terms of trade (TOT) effects and positive wealth effects compete.

When the terms of trade (TOT) falls, the government spending causes less decrease in primary surplus than in the closed economy. This effect makes consumption to rise more. Intertemporal Substitution effects has a positive effect on consumption. Inflation rises less than in the closed economy. Since monetary policy is passive, the real interest rate falls due to sticky prices. Its effect is amplified by the real interest rate differential effects. Then consumption increase more than in the closed economy case. In a passive monetary and active fiscal policy regime, openness works through the movement of the terms of trade (TOT). When the terms of trade (TOT) falls and an economy becomes more open, the same gov-
ernment spending induces less decrease in primary surples. The real value of government liabilities differs as openness increases. In a small open economy (SOE), PM/AF regime shows similar or slightly higher effects on output compared with the closed economy.

In sum, in a AM/PF regime, in a small open economy (SOE), the government spending does not increase output and inflation more, compared with the closed economy. In contrast, in a PM/AF regime, the difference between open and closed economies are small.
Appendices

A  Linearized System of the Baseline Model

Consumption Euler equation\textsuperscript{41}:

\[
\frac{1}{1-g} \hat{y}_t = \frac{1}{1-g} E_t \tilde{y}_{t+1} - \frac{\mu_\psi}{\sigma} (\hat{r}_t - E_t \hat{\pi}_{t+1} - \hat{\pi}_t) - \alpha \eta E_t \Delta \hat{s}_{t+1} + \frac{1}{\sigma} \left( \mu - 1 \right) \left( \hat{\pi}_t^* - E_t \hat{\pi}_{t+1}^* \right) - \frac{g}{1-g} E_t \Delta \hat{g}_{t+1}
\]

Goods market equilibrium condition\textsuperscript{42}:

\[
\frac{1}{1-g} \tilde{y}_t = \frac{\mu_\psi}{\sigma} \hat{\psi}_{F,t} + \frac{\mu_s}{\sigma} \hat{s}_t + \frac{1}{1-g} \hat{y}_t^* - \frac{g}{1-g} \hat{y}_t^* + \frac{g}{1-g} \hat{y}_t + \bar{\rho}_t
\]

CPI inflation:

\[
\pi_t = \beta E_t \pi_{t+1} + \lambda (\kappa_y \hat{y}_t + \kappa_\psi \hat{\psi}_{F,t}) + \epsilon_t^S
\]

Terms of trade:

\[
\Delta \hat{s}_t = \beta E_t \Delta \hat{s}_{t+1} + \lambda (-\kappa_y \hat{y}_t + (1 - \kappa_\psi) \hat{\psi}_{F,t}).
\]

Government budget constraint:

\[
\hat{b}_t + \frac{m}{b} \hat{m}_t + \left( \frac{m}{b} + 1 - \beta \right) \pi_t + \left( \frac{G}{b} + 1 - \beta \right) \hat{r}_t - \frac{G}{b} \hat{g}_t + \frac{G}{b} \alpha \hat{s}_t = \beta \hat{b}_{t-1} + \frac{m}{b} \hat{m}_{t-1} + \frac{1}{\beta} \hat{r}_{t-1}.
\]

Fiscal policy rule:

\[
\hat{r}_t = \gamma_b \hat{b}_{t-1} + \epsilon_t^r.
\]

\textsuperscript{41}The natural rate of interest, \( \pi_t \), is

\[
\pi_t = \frac{\sigma}{\mu_\psi} \frac{1}{1-g} \left[ \frac{\kappa_\psi}{\kappa_y} (1 - \rho_d) \hat{d}_t + \frac{1}{\kappa_y} \left[ \frac{g}{1-g} \frac{\sigma}{\mu_s} (1 - \rho_g) \hat{g}_t - \frac{\sigma}{1-g} \frac{\mu_s - 1}{\mu_s} (1 - \rho_g) \hat{g}_t^* \right] \right].
\]

\textsuperscript{42}\( \bar{\rho}_t \) can be represented by

\[
\bar{\rho}_t = -\frac{1}{1-g} \left[ \left( \frac{\kappa_\psi}{\kappa_y} + (1-g) \frac{\mu_\psi}{\sigma} \right) \hat{d}_t + \frac{1}{\kappa_y} \left[ \frac{g}{1-g} \frac{\sigma}{\mu_s} \hat{g}_t - \frac{\sigma}{1-g} \frac{\mu_s - 1}{\mu_s} \hat{g}_t^* + \frac{g}{1-g} \frac{\mu_s - 1}{\mu_s} \hat{g}_t^* + (1+\omega) (1-\rho_a) \right] \right].
\]
Monetary policy rule:
\[ \hat{r}_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \epsilon_t^R. \]

LOP gap:
\[ \psi_{F,t} = \hat{q}_t - (1 - \alpha) \hat{s}_t, \]

Money stock equation:
\[ \hat{m}_t = -\frac{1}{\kappa} \frac{\beta}{1 - \beta} \hat{r}_t + \frac{1}{\kappa} \hat{c}_t. \]

Government spending shock:
\[ \hat{g}_t = \rho_g \hat{g}_{t-1} + u^g_t. \]

World economy shock shock:
\[ \hat{y}^*_t = \rho_y \hat{y}^*_{t-1} + u^{y^*}_t. \]

World government spending shock:
\[ \hat{g}^*_t = \rho_{g^*} \hat{g}^*_{t-1} + u^{g^*}_t. \]

World real interest rate shock:
\[ (r^*_t - E_t \pi^*_{t+1}) = \rho_{r^*} (r^*_{t-1} - \pi^*_t) + u^{r^*_t}. \]

Productivity shock:
\[ \epsilon^a_t = \rho_a \epsilon^a_{t-1} + u^a_t. \]

Distribution technology shock:
\[ \epsilon^d_t = \rho_d \epsilon^d_{t-1} + u^d_t. \]

Supply shock:
\[ \epsilon^s_t = \rho_s \epsilon^s_{t-1} + u^s_t. \]

MP policy shock:
\[ \epsilon^r_t = \rho_r \epsilon^r_{t-1} + u^r_t. \]

Lump-sum tax shock:
\[ \epsilon^\tau_t = \rho_\tau \epsilon^\tau_{t-1} + u^\tau_t. \]

Income tax shock:
\[ \epsilon^{\tau I}_t = \rho_{\tau I} \epsilon^{\tau I}_{t-1} + u^{\tau I}_t. \]
References


