Testing the Expectations Hypothesis in Continuous-Time
Joonyoung Hur\textsuperscript{1}

Abstract

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\textsuperscript{1}Department of Economics, Indiana University
1. Introduction

The term structure of interest rates has been one of the most debated issues in financial economics. In particular, the expectations hypothesis has played a central role in the analysis of the term structure of interest rates. Introduced by Fisher (1986) more than a century ago, the expectations hypothesis has become a standard framework for explaining how yields of different maturities are related. Specifically, the expectations hypothesis of the term structure of interest rates posits that the long-term rate is determined by the market’s expectation for the short-term rate over the holding period of the long-term asset, plus a constant risk premium. In addition, this statement is equivalent to the statement that expected excess returns are time invariant. Hence, if the expectation theory provides a proper description of the term structure, then the expected future yields of short maturity bonds will be the determining force of the current long maturity bond yields. On the contrary, if the expectations hypothesis is not an accurate explanation for the term structure, then predictable changes in future excess return, thus the term premia, will heavily affect the movement of yield curve.

Numerous studies have investigated the expectations hypothesis. A plethora of literature, however, has presented empirical failure of the expectations hypothesis in which the failure attributes to the fact that risk premia in bond returns are time varying. To account for the time-varying bond risk premia, previous research used the information in the term structure. For example, Fama and Bliss (1987) document that risk premia in Treasury bond returns vary reliably through time and can be forecasted using the information in forward rates. Recently, Cochrane and Piazzesi (2005) recasts the evidence of variation through time in risk premia by using a linear combination of forward rates. One of the more troubling results is found in Campbell and Shiller (1991) based on the information in yields of different maturities. In their influential work, they found not only that the expectations hypothesis is rejected but also the slope of the term structure almost always forecasts the wrong direction for the short-term change in the long-term rate, which constitutes a part of the Campbell and Shiller paradox. Similar evidence of variation through time in risk premia is given by Shiller, Campbell, and Schoenholtz (1983), Engel, Lilien, and Robins (1987), Roberds and Whiteman (1999), Dai and Singleton (2002), and others.

Deviating from the most intuitive single equation tests such as Fama and Bliss (1987) and Campbell and Shiller (1991), substantial research has been devoted to developing a better methodology in several directions, among them, multivariate tests. Proposed by Campbell and Shiller (1987) and Bekaert and Hodrick (2001), this methodology tests the restrictions implied by the expectations hypothesis on a general vector autoregression (VAR) representation of the short-term and long-term rates. Since this test can be applied to a VAR with more than two interest rates, or that includes other economic variables, a VAR test can encompass a wider array of alternative hypotheses than single equation tests. However, a caveat of VAR tests is that they are more difficult to employ than the single equation tests that generate the stylized facts in term structure, such as the Campbell and Shiller paradox. Hence, single equation tests and their variation are still a parsimonious way to test the expectations hypothesis and are frequently employed. For example, Dai and Singleton (2002) uses the single equation test combined with the vast class of affine term structure
models.

Much less attention has been given to the frequency of tests in that the expectations hypothesis has been tested only in the discrete-time framework with low-frequency data. However, there are several reasons to analyze bond market features using a continuous-time model. First of all, discrete-time modeling approaches overlook the high-frequency nature of the U.S. Treasury market. The Treasury market clears continuously in time and is extremely liquid. These bond market features wipe out arbitrage opportunities almost. Therefore, no arbitrage restrictions on the bond market seem to be necessary. Since this restriction is expected hold continuously in time, it seems that continuous-time models are more relevant to analyze the Treasury bond market. In general, selecting the relevant modeling frequency is crucial to empirical studies. For example, suppose we examine a relationship which holds continuously in time or on a daily basis. This does not imply that results hold over a monthly or quarterly basis. A mere treatment on this aspect may induce potential model misspecification problems. In this sense, we believe our consideration of continuous-time models reduce the gap between the nature of the Treasury market and the modeling frequency. Secondly, as Longstaff (1990) addressed, testing the expectations hypothesis depends heavily upon the period over which bond returns are measured. In particular, an aggregation of bond returns over an interval longer than the expectations hypothesis is assumed to hold may amplify the time variability of risk premia, thus making usual expectation hypothesis tests invalid. Continuous-time models resolve this problem by considering instantaneous bond returns instead of annually accumulated returns.

This paper takes up the single equation test approach in continuous-time. In particular, we test the Fama-Bliss and Campbell-Shiller regressions in a continuous-time setup. A virtue of using single equation models to test the expectations hypothesis lies in its simplicity. The models give a clear-cut criterion of the validity of the expectations hypothesis by interpreting the magnitude of the slope coefficients of the regressions. In addition, continuous-time modeling provides a new view to understand these regression tests under no arbitrage restrictions.

For the estimation of our model, we use the martingale regression based on time change developed by Park (2009) for inference on continuous time conditional mean models. This method is quite simple and intuitive in that it identifies the true parameter value simply by imposing the martingale condition for the error process. The martingale condition on error processes can be easily handled by the celebrated Dambis-Dubins-Schwarz theorem. The intuition of the theorem is that if we read a continuous martingale by a time clock which elapses in inverse proportion to its quadratic variation instead of one which advances in physical time, then the continuous martingale reduces to a standard Brownian motion.

The martingale regression is particularly attractive because it concentrates on estimating the conditional mean part without specifying the conditional variance part. The conditional variance part of many asset pricing models usually have quite complicated features and, therefore, invoke a trade-off between estimating the conditional mean part and the conditional variance part. There is no such problem with the martingale regression. Another virtue of the martingale regression method is that, despite its simple estimation scheme, it allows for the presence of a wide variety of both deterministic and stochastic volatilities in the error process of asset pricing models. Sufficient research reports that time-varying and
stochastic volatilities are intrinsic components of financial data. By using the martingale regression, we can easily handle such features observed in bond market data.

The remainder of the paper is organized as follows. Section 2 briefly reviews the two tests of the expectations hypothesis, the Fama-Bliss and Campbell-Shiller regressions. Section 3 derives the continuous-time counterpart of each regression and account for a new view to understand the models under no arbitrage restrictions. Section 4 accounts for the statistical underpinnings of our econometric methodologies. Section 5 explains our empirical procedure more detail. Section 6 displays and discusses our main results. A comparison to the discrete-time model results are also addressed. Section 7 does a simulation study to verify a robustness of our results. Then we conclude in Section 8.

2. Motivating Regressions

2.1 Notation

We denote $P_t^{(\tau)}$ as the price of $\tau$-year discount bond at time $t$. Furthermore, we define $y_t^{(\tau)}$ as the log yield of maturity $\tau$-year zero coupon bond at $t$. Then, the log yield is given as

$$y_t^{(\tau)} = -\frac{1}{\tau} \log P_t^{(\tau)}.$$

We define the log holding period return from buying an $\tau$-year bond at time $t$ and selling it as $\tau - 1$ year bond at time $t + 1$ as

$$hpr_{t\rightarrow t+1}^{(\tau)} = \log P_{t+1}^{(\tau-1)} - \log P_t^{(\tau)},$$

and we write the log forward rate at time $t$ for loans between time $t + \tau - 1$ and $t + \tau$ as

$$f_t^{(\tau-1\rightarrow\tau)} = \log P_t^{(\tau-1)} - \log P_t^{(\tau)}.$$

In continuous time, we can define the instantaneous short rate $r_t$ by

$$r_t = \lim_{h \to 0} y_t^{(h)}.$$

We write the instantaneous holding period return of $\tau$-year bonds at $t$ as

$$hpr_t^{(\tau)} = \frac{dP_t^{(\tau)}}{P} - \frac{1}{P} \frac{\partial P_t^{(\tau)}}{\partial \tau} dt.$$

We define the instantaneous forward rate at $t$

$$f_t^{(\tau)} = -\frac{1}{P} \frac{\partial P_t^{(\tau)}}{\partial \tau}.$$

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2Throughout this paper, time subscript $t$ and maturity superscript $\tau$ are sometimes omitted intentionally when their presence is considered to be obvious.
2.2 Classical Regressions on Expectations Hypothesis Testing

Many previous literatures use the regression test to check up the validity of the expectations hypothesis in practice. Among them, Campbell and Shiller (1991) and Fama and Bliss (1987) are the two most influential works in the field of the expectations hypothesis testing. Campbell and Shiller (1991) investigated the movement of the longer-term bond over the life of the shorter-term bond. In particular, they regressed the change in future yields against the yield spreads of the same maturity proportioned by the maturity difference of the spreads. The expectations hypothesis states that, when the yield spread is high, the future long-term yield should rise to compensate the capital loss of holding the longer-term bond. Then this stylized fact insures that the slope coefficient of the long-term regression equals 1—its theoretical value if the expectations hypothesis holds. However, the empirical result gives quite opposite evidence of the expectations hypothesis, in that, the coefficients are negative for almost all maturities. This finding constitutes a part of so called ‘Campbell and Shiller’s paradox’ and is referred to the presence of the time-varying risk premium which invokes the empirical failure of the expectation hypothesis.

To derive the regression test, we consider the following decomposition of the change in yields into holding period returns and yield spreads,

\[ y_{t+1}^{(\tau-1)} - y_t^{(\tau)} = \left( -\frac{1}{\tau - 1} \right) \left( hpr_{t \rightarrow t+1}^{(\tau)} - y_t^{(1)} \right) + \left( \frac{1}{\tau - 1} \right) \left( y_t^{(\tau)} - y_t^{(1)} \right). \]

By definition, the expectation conditional on the information set up to time t of the first term of the right-hand side becomes a risk premium. In a modern setting, the risk premium is a function of second- and higher-order conditional moments of the stochastic discount factor (or pricing kernel). If these moments do not vary over time, then risk premiums will be constant, the expectations hypothesis will hold, and changes in long-term bond will result only from the gap between long- and short-bond yields interpreted a slope component of the current term structure. The movements of long-term bonds are unpredictable at time t, so that they can be interpreted as a white noise error term. Equation (1) then leads naturally to the “long-term regression” of Campbell and Shiller (1991):

\[ y_{t+1}^{(\tau-1)} - y_t^{(\tau)} = \alpha_\tau + \beta_\tau \left( \frac{1}{\tau - 1} \right) \left( y_t^{(\tau)} - y_t^{(1)} \right) + \epsilon_{\tau,t}, \]

where \( \alpha_\tau \) and \( \beta_\tau \) are maturity-specific regression intercept and slope coefficients. As was explained above, under the expectations hypothesis, the estimated slope coefficient \( \beta_\tau \) will equal unity. Moreover, deviations from the expectations hypothesis will push the slope coefficient away from one. As Mankiw and Miron (1986) pointed out, the presence of a time-varying risk premium can drive the estimated slope coefficient \( \beta_\tau \) to zero or even to negative values.

We now turn to another interpretation of the expectations hypothesis test. The validity of the expectations hypothesis is equivalent to the statement that bond returns should not be predictable. In the context of the bond return predictability, the Campbell-Shiller regression tests the predictive power of yield spreads to bond excess returns. No forecasting power
implies the risk premia to be constant over time, therefore, the expectations hypothesis holds. We can deduce this relationship from Equation (1).

$$hpr_{t \rightarrow t+1}^{(\tau)} - y_t^{(1)} = \alpha^{\prime} + \beta^{\prime} \left( y_t^{(\tau)} - y_t^{(1)} \right) + \epsilon_{\tau,t}.$$  \tag{3}

and the expectations hypothesis implies $\beta^{\prime} = 0$. Note that those two regression tests, (2) and (3), contain the same amount of information in the context of the expectations hypothesis testing in which the slope coefficients are linked by the relationship $\beta^{\prime} = 1 - \beta$. A negative estimate $\beta$ of the original Campbell-Shiller long-term regression is equivalent to a positive $\beta^{\prime}$ that exceeds one which corresponds to a strong positive relationship between yield spreads and excess returns on long bonds. Hence, the presence of a time-varying risk premium implies the high predictive power of the yield spread on the excess returns and vice versa.

The latter interpretation of the Campbell-Shiller long-term regression is closely related to the Fama-Bliss return predictability regression. The only difference between them is the variable by which excess bond returns are explained. Instead of yield spreads, Fama and Bliss (1987) examined the forecasting power in forward rates on the same maturity excess return and provided an evidence against the expectations hypothesis in long-term bonds. The derivation of the regression formula is quite similar to that of the Campbell-Shiller, thus is omitted. The predictability test of expected excess returns based on forward-spot spreads is given by

$$hpr_{t \rightarrow t+1}^{(\tau)} - y_t^{(1)} = \alpha + \beta \left( f_t^{(\tau-1 \rightarrow \tau)} - y_t^{(1)} \right) + \epsilon_{\tau,t}.$$  \tag{4}

Under the expectations hypothesis, the null hypothesis is given as $\beta = 0$.

3. Continuous-Time Counterparts of Regression Tests

3.1 Derivation of Continuous-Time Regression Tests

To test the expectations hypothesis in a continuous-time, this subsection derives the continuous-time counterpart of the Campbell-Shiller and Fama-Bliss regressions. In particular, we focus on the derivation of the Campbell-Shiller regression. If we interpret those two regressions in the bond return predictability framework, as in (3) and (4), the only difference between the two regressions is the regressor used in each model. Hence, deriving the Fama-Bliss continuous-time counterpart is a simple modification of the Campbell-Shiller model.

Deriving the continuous-time formula begins with the bond yield decomposition described in (1). Since (1) is a theoretical relationship held by the definitions of bond prices, the decomposition is valid for any time interval. Hence, for a short time interval $h \leq \tau$, we can write the equation (1) as

$$y_{t+h}^{(\tau)} - y_t^{(\tau)} = \left( -\frac{1}{\tau-h} \right) \left( hpr_t^{(\tau)} - y_t^{(h)} \right) + \left( \frac{h}{\tau-h} \right) \left( y_t^{(\tau)} - y_t^{(h)} \right).$$
As $h \downarrow 0$, above equation becomes

$$\frac{dy_t(\tau)}{dt} - \frac{\partial y_t(\tau)}{\partial \tau} dt = \left( -\frac{1}{\tau} \right) \left( hpr_t(\tau) - r_t dt \right) + \left( \frac{1}{\tau} \right) \left( y_t(\tau) - r_t \right) dt.$$  \hspace{1cm} (5)

Equation (5) corresponds to the decomposition of yield changes of $\tau$-maturity bonds within an infinitesimally small amount of time. By comparing (1) and (5), one may deduce that the continuous-time Campbell-Shiller formula also admits an interpretation in a return predictability framework. It examines the predictive power of the slope factor in term structure on instantaneous excess holding period returns. In this sense, the continuous-time Campbell-Shiller model is in line with its discrete-time formula. This fact will be explained more formally below.

In continuous time, a plethora of research models the price of risky assets as diffusions. For an earlier theoretical work on the setting, refer to Cox, Ingersoll, and Ross (1981). To consider bonds of various maturities, we adapt to this approach in which the bond price of each maturity is determined by its maturity specific drift and diffusion functions. In particular, suppose, for standard, one-dimensional Brownian motions $W_t$, the price process of $\tau$-maturity bond $P_t(\tau)$ follows

$$\frac{dP_t(\tau)}{P} = \mu_t(\tau) dt + \sigma_t(\tau) dW_t.$$  \hspace{1cm} (6)

The Campbell-Shiller regression of the excess one-period return on a $\tau$-period bond onto the yield spread between $\tau$-year bonds and short-rates can be written as

$$\log P_{t+1}(\tau-1) - \log P_t(\tau) - y_t(1) = \alpha_\tau + \beta_\tau \left( y_t(\tau) - y_t(1) \right) + \epsilon_{\tau,t}. \hspace{1cm} (7)$$

For a small time interval $h$, rewriting (7) gives

$$\log P_{t+h}(\tau-h) - \log P_t(\tau) - y_t(h) h = \alpha_\tau h + \beta_\tau \left( y_t(\tau) - y_t(h) \right) h + \epsilon_{\tau,t}. \hspace{1cm} (8)$$

By taking the limit of (8) and using a simple application of Ito’s formula, the continuous-time formula of the Campbell-Shiller regression to testing the expectations hypothesis for $\tau$-year maturity bonds is given as

$$\frac{dP_t(\tau)}{P} = \left[ \frac{1}{P} \frac{\partial P_t(\tau)}{\partial \tau} + r_t + \frac{1}{2} \left( \sigma_t(\tau)^2 + \alpha_\tau + (1 - \beta_\tau) \left( y_t(\tau) - r_t \right) \right) \right] dt + \sigma_t(\tau) dW_t, \hspace{1cm} (9)$$

where the null hypothesis is given as $\beta_\tau = 1$.

Equation (9) provides a characterization of the continuous-time expectations hypothesis testing based on the Campbell-Shiller model. In particular, subtracting $\left( \frac{1}{P} \frac{\partial P_t(\tau)}{\partial \tau} + r_t \right) dt$ from the both sides of (9) makes the left-hand side of the equation the excess expected rate of return over the next instant of time associated with holding the discount bond maturing at $t + \tau$. Hence, this continuous-time formula examines the bond return predictability of the slope factor in term structure over an infinitesimally small amount of time.
Note that the drift function of (9) has \( \left( \sigma_t^{(\tau)} \right)^2 / 2 \) which is not observed in the discrete-time formula and only appears in continuous-time models. In the equilibrium setting, Cox, Ingersoll and Ross (CIR, 1981) used this term as a characterization variable for a set of mutually exclusive expectations hypotheses. The term \( \left( \sigma_t^{(\tau)} \right)^2 / 2 \) may correspond to CIR’s unbiased expectations hypothesis since we consider the expectations hypothesis formulated in logs. However, we consider a different version of the expectations hypothesis from that of CIR’s in that we allow constant risk premia. As Campbell (1986) showed, there is no incompatibility issue among the expectations hypothesis statements we consider. Simply, the term is a direct consequence of using logarithmic bond prices that are modeled as diffusion processes. We refer to the term as a “continuous-time adjustment”.

To see why (9) is indeed a testing formula for the expectations hypothesis, we derive a continuous-time form of the expectations hypothesis. The expectations hypothesis states that bond yields \( y_t^{(\tau)} \) are expected values of average future short rates, i.e.,

\[
y_t^{(\tau)} = \frac{h}{\tau} \mathbb{E}_t \left[ y_t^{(h)} + y_{t+h}^{(h)} + \ldots + y_{t+\tau-h}^{(h)} \right] + C_{\tau}, \tag{10}
\]

where \( \mathbb{E}_t \) denotes a mathematical expectation conditioning on information up to time \( t \) and \( C_{\tau} \) represents a constant term premium that depends only upon maturities, but not upon time. From (10) and the definition of yields, we have

\[
\mathbb{E}_t \left[ \log p_t^{(\tau-h)} - \log p_t^{(\tau)} \right] = y_t^{(h)} h + C', \tag{11}
\]

where \( C' \) is a linear combination of \( C_{\tau} \) and \( C_{\tau+h} \). When \( h \downarrow 0 \), (10) becomes

\[
\mathbb{E}_t \left[ \frac{dP_t^{(\tau)}}{P} - \frac{1}{\tau} \frac{\partial P_t^{(\tau)}}{\partial \tau} - \frac{1}{2} \left( \sigma_t^{(\tau)} \right)^2 \right] = r_t dt + C'. \tag{12}
\]

Hence, testing (12) is equivalent to testing the expectations hypothesis in continuous-time.

Similarly, we formulate the continuous-time Fama-Bliss regression as follows.

\[
\frac{dP_t^{(\tau)}}{P} = \left[ \frac{1}{\tau} \frac{\partial P_t^{(\tau)}}{\partial \tau} + r_t + \frac{1}{2} \left( \sigma_t^{(\tau)} \right)^2 + \alpha_{\tau} + \beta_{\tau} \left( f_t^{(\tau)} - r_t \right) \right] dt + \sigma_t^{(\tau)} dW_t, \tag{13}
\]

and the expectations hypothesis implies \( \beta_{\tau} = 0 \). As was mentioned earlier, the Fama-Bliss regression uses forward-spot spreads, instead of yield spreads, to test the expectations hypothesis.

### 3.2 No Arbitrage Restriction

In general, the assumption of no arbitrage seems natural for bond yields. This is particularly valid because most bond markets are extremely liquid, and arbitrage opportunities are traded away immediately by large investment banks. Hence, no arbitrage restrictions are expected to hold continuously in time. In terms of modeling frequency, continuous-time models are more relevant to reflect the arbitrage free feature of the Treasury bond market.
Another virtue of the no arbitrage restriction is that it provides a new way to view the continuous-time formulae derived above. To look at this aspect, we derive the no-arbitrage formula under the bond price process given in (6) as follows.

The fundamental pricing equation for a zero coupon, default-free bond can be written as

\[
\mathbb{E}_t \left( \frac{dP_t^{(\tau)}}{P} \right) - \frac{1}{P} \frac{\partial P_t^{(\tau)}}{\partial \tau} dt = r_t dt - \mathbb{E}_t \left( \frac{d\lambda_t}{\lambda_t} \frac{dP_t^{(\tau)}}{P} \right),
\]

where \( \lambda_t \) is the stochastic discount factor (SDF). No arbitrage and market completeness imply that \( \lambda_t \) exists and is unique. This fact guarantees the existence of unique market price of risk, \( \eta_t \). Then Novikov’s condition suffices,

\[
\mathbb{E}_t \left[ \exp \left( \frac{1}{2} \int_0^T \eta_t^2 dt \right) \right] < \infty.
\]

When (15) applies, \( \lambda_t \) is given by

\[
\frac{d\lambda_t}{\lambda_t} = -r_t dt - \eta_t dW_t.
\]

Plugging the bond price process of previous subsection (6) and (16) into (14), we have

\[
\mu_t^{(\tau)} = \frac{1}{P} \frac{\partial P_t^{(\tau)}}{\partial \tau} + r_t + \sigma_t^{(\tau)} \eta_t.
\]

Then a bond price process that rules out arbitrage opportunities is given by

\[
\frac{dP_t^{(\tau)}}{P} = \left[ \frac{1}{P} \frac{\partial P_t^{(\tau)}}{\partial \tau} + r_t + \sigma_t^{(\tau)} \eta_t \right] dt + \sigma_t^{(\tau)} dW_t,
\]

where \( \sigma_t^{(\tau)} \eta_t \) represents the risk premia of holding \( \tau \)-maturity bonds. Note that this formula imposes cross-section restrictions between bonds of different maturities. To see this feature, we denote

\[
\nu_t^{(\tau)} = \mu_t^{(\tau)} - \frac{1}{P} \frac{\partial P_t^{(\tau)}}{\partial \tau}.
\]

Then for arbitrary maturity dates \( \tau_1, \tau_2 \), it follows that the condition

\[
\frac{\nu_t^{(\tau_1)} - r_t}{\sigma_t^{(\tau_1)}} = \frac{\nu_t^{(\tau_2)} - r_t}{\sigma_t^{(\tau_2)}} \equiv \eta_t,
\]

should hold to fulfill the absence of arbitrage. From (18), we deduce that the market price of risk \( \eta_t \) is defined as an adjusted Sharpe ratio where the adjustment is conducted by bond maturity reduction effects.

The no arbitrage bond pricing formula (17) nests the Campbell-Shiller continuous-time regression under the specific market price of risk setting. If the market price of risk process \( \eta_t \) is given as

\[
\eta_t = \frac{\alpha_t + (1 - \beta_t) \left( \frac{y_t^{(\tau)} - r_t}{\sigma_t^{(\tau)}} \right)}{\sigma_t^{(\tau)}},
\]

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and this quantity is invariant across maturity $\tau$, then the Campbell-Shiller continuous-time regression specifies the increase in expected instantaneous rate of return on a bond per an additional unit of risk to be proportional to the slope factor of term structure. Here we rule out the continuous-time adjustment term. This is justifiable since the term is a byproduct of using a diffusion price process and contributes nothing to the risk dynamics of holding long-term bonds.

Similarly, the Fama-Bliss continuous-time regression models the market price of risk process as

$$ \eta_t = \frac{\alpha_\tau + \beta_\tau (f^{(\tau)}_t - r_t)}{\sigma^{(\tau)}_t}, $$

where the information in the slope factor is extracted by using forward-rates, instead of long-term yields.

### 3.3 Continuous-Time Estimation Formulae

We now turn to deriving the estimation formulae for the empirical methodology in our work, the martingale regression. The methodology requires to estimate the time change $(T_t)$. Since this is an empirical matter, however, we will discuss this in detail later in the empirical section. Throughout this subsection, we assume that the time change is known. Then, by definition of the instantaneous holding period return, the integral of (9) for the random intervals $[T_{(i-1)\Delta}, T_{i\Delta}]$ can be written as

$$ \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} hpr^{(\tau)}_s - \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} r_s ds - \frac{1}{2} \Delta = \alpha_\tau (T_{i\Delta} - T_{(i-1)\Delta}) + (1 - \beta_\tau) \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} (y^{(\tau)}_s - r_s) ds + \varepsilon_{T_{i-1},i}, \tag{19} $$

where $\varepsilon_{T_{i-1},i} \equiv \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} \sigma^{(\tau)}_s dW_s$. Note that the third term of the left-hand side of (19) is a direct consequence of the time change definition,

$$ \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} \left( \sigma^{(\tau)}_t \right)^2 dt = \Delta. $$

More detail on this feature will be explained in the following section.

Similarly, from (13), the martingale regression formula of the Fama-Bliss regression becomes

$$ \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} hpr^{(\tau)}_s - \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} r_s ds - \frac{1}{2} \Delta = \alpha_\tau (T_{i\Delta} - T_{(i-1)\Delta}) + \beta_\tau \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} (f^{(\tau)}_s - r_s) ds + \varepsilon_{T_{i-1},i}, \tag{20} $$

where $\varepsilon_{T_{i-1},i}$ is defined as above.
4. Econometric Methodology: Martingale Estimation of Continuous-Time Regressions

This section explains how to specify and estimate the continuous time counterpart of the Campbell and Shiller regression. As was derived earlier, the continuous time Campbell-Shiller regression is given as

\[
\frac{dP_t(\tau)}{P_t} - \frac{1}{P_t} \frac{\partial P_t(\tau)}{\partial \tau} dt - r_t dt - \frac{1}{2} \left( \sigma_t^{(\tau)} \right)^2 dt = \left[ \alpha_\tau + (1 - \beta_\tau) (y_t^{(\tau)} - r_t) \right] dt + \sigma_t^{(\tau)} dW_t. \tag{21}
\]

Note that the econometric methodology explained below is applicable to each maturity \( \tau \)-year bond in the same context. Hence, for the sake of simplicity, we suppress the superscript \( \tau \), indicating bond maturities, throughout this subsection.

We define \((X_t)\) to be the instantaneous excess holding period return of maturity \( \tau \)-year bonds with the continuous time adjustment term \( \sigma_t^2/2 \), which is basically the left-hand side variable of expression (21), as

\[
dX_t = \frac{dP_t}{P_t} - \frac{1}{P_t} \frac{\partial P_t}{\partial \tau} dt - r_t dt - \frac{1}{2} (\sigma_t^2) dt = hpr_t - r_t dt - \frac{1}{2} (\sigma_t^2) dt.
\]

Then, we may rewrite (21) as

\[
X_t = A_t(\theta) + U_t, \tag{22}
\]

where \( \theta = (\alpha, \beta) \), \( dA_t = [\alpha + (1 - \beta) (y_t - r_t)] dt \) and \( dU_t = \sigma_t dW_t. \)

Notice that the process \((X_t)\) is a semimartingale with a bounded variation component \((A_t)\) and martingale component \((U_t)\). Specifically, the error process \((U_t)\) of (22) is a continuous martingale with respect to the filtration \((F_t)\), to which the Brownian motion \((W_t)\) is adapted. Our estimation methodology uses this stylized fact in the error process to identify the unknown coefficients, \((\alpha, \beta)\). We formally explain below the theoretical and implementational framework to estimate the parameter \((\alpha, \beta)\) of the model given above.

Park (2009) has recently developed a general methodology to estimate the continuous time conditional mean model by using an identification condition of the martingale error process. This estimation method is possible due to the celebrated theorem by Dambis, Dubins and Schwarz, henceforth denoted DDS theorem. To introduce the DDS theorem, we denote \([U]_t\) to be the quadratic variation of \((U_t)\) which is given by

\[
[U]_t \equiv \lim_{d_t \rightarrow 0} \sum_{k=1}^{n} (U_{t_k} - U_{t_{k-1}})^2,
\]

where \(d_t\) is the mesh of the partition \(0 \equiv t_0 < \ldots < t_n \equiv t\) of the interval \([0,t]\). We assume that \([U]_t \rightarrow \infty\) a.s. as \(t \rightarrow \infty\). Then, the time change \((T_t)\) is defined as

\[
T_t = \inf\{s \geq 0 | [U]_s > t\}. \tag{23}
\]

\(^3\)Applying this econometric procedure to the Fama-Bliss continuous time counterpart is trivial.
The DDS theorem proves that if $U_t$ is a continuous martingale, then there exists a standard Brownian motion $B$ which satisfies $U_t = B_{T_t}$, or equivalently,

$$U_{T_t} = B_t.$$ 

The Brownian motion $(B_t)$ is often referred to as the DDS Brownian motion. The intuition of the DDS theorem is that if we read a continuous martingale by the time clock which elapses in inverse proportion to its quadratic variation instead of physical time, then the continuous martingale reduces to a standard Brownian motion. In this paper, we apply this time change method to the continuous time Campbell and Shiller regression. Then, (22) can be written as

$$X_{T_i} = A_{T_i}(\theta) + U_{T_i} = A_{T_i}(\theta) + B_t. \tag{24}$$

After this time change, the error process of (24) becomes a standard Brownian motion. The martingale estimation method proposed by Park (2009) uses this stylized fact and estimates parameters $\theta = (\alpha, \beta)$ that make the time-changed error process best approximate the standard Brownian motion. Since the bounded component contributes nothing to calculate the quadratic variation of semimartingale processes, no prior information on the unknown parameters $(\alpha, \beta)$ is required to obtain the time change $(T_i)$. A consequence of this fact is that the quadratic variation of the error process $\left(\{U_t\}_t\right)$ equals to the quadratic variation of the excess holding period return process $\left(\{X_t\}_t\right)$, i.e., $d[U]_t = d[X]_t = d[P]_t/P^2_t$.

An implementation of the methodology requires us to fix $\Delta > 0$ which is the increment of the quadratic variation between time interval $[T_{(i-1)\Delta}, T_{i\Delta}]$. Since the choice of $\Delta$ is purely an empirical issue, however, it will be explained in the next section. For the time being, $\Delta$ is assumed to be known. Then, it follows from (22) that

$$X_{T_i\Delta} - X_{T_{(i-1)\Delta}} = \alpha \left( T_{i\Delta} - T_{(i-1)\Delta} \right) + (1 - \beta) \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} (y_s - r_s) \, ds + \left( U_{T_i\Delta} - U_{T_{(i-1)\Delta}} \right), \tag{25}$$

where $\left( U_{T_i\Delta} - U_{T_{(i-1)\Delta}} \right)$ are independent and identically distributed as $N(0, \Delta)$ for $i = 1, \ldots, N$, due to the DDS theorem.

To estimate the parameter $\theta = (\alpha, \beta) \in \Theta$, we define $z_i(\theta)$ to be the normalized increments of the error process $(U_t)$ given by

$$z_i(\theta) = \frac{1}{\sqrt{\Delta}} \left[ X_{T_i\Delta} - X_{T_{(i-1)\Delta}} - \alpha \left( T_{i\Delta} - T_{(i-1)\Delta} \right) - (1 - \beta) \int_{T_{(i-1)\Delta}}^{T_{i\Delta}} (y_s - r_s) \, ds \right]. \tag{26}$$

Moreover, we let

$$z_i^d(\theta) = (z_i(\theta), z_{i-1}(\theta), \ldots, z_{i-d+1}(\theta)),$$

be the d-dimensional vector consisting of d-adjacent samples starting from $i = 1, \ldots, N - d + 1$, and denote $\Phi_N(\cdot, \theta)$ the empirical distribution of $(z_i^d(\theta))$ for each $\theta \in \Theta$. In the paper, we use the martingale estimation method with $d = 1$.

The criterion function $Q_N$ of parameter $\theta$ is defined as

$$Q_N(\theta) = \int_{-\infty}^{\infty} [\Phi_N(x, \theta) - \Phi(x)]^2 \, d\Phi(x),$$

11
where $\Phi$ is the distribution function of the $d$-dimensional multivariate standard normal random vector. The martingale estimator $\hat{\theta}_N$ of the parameter $\theta$ is then defined as the minimizer of the criterion function $Q_N$, i.e.,

$$\hat{\theta}_N = \arg\min_{\theta \in \Theta} Q_N(\theta).$$

The martingale estimator is therefore a minimum-distance estimator in the sense that it finds the parameters in which the empirical distribution of the errors is as close as possible to the standard normal multivariate distribution. The distance here uses the Cramer-von Mises (CvM) distance.

As was shown in Park (2009), the methodology can be easily implemented and all the theoretical results continue to hold for discretely sampled observations, as long as the sampling intervals are sufficiently small relative to the time horizon of the samples. For our empirical analysis, we use daily observations over almost fifty years. The necessary modifications required to deal with discretely observed samples are largely trivial and obvious. To obtain the time change, for instance, we use the realized variance of the instantaneous excess hold period returns $(Y_t)$,

$$dY_t = h_{pr_t} - r_t dt,$$

instead of its quadratic variation $\langle [Y]_t \rangle$, if $(Y_t)$ is observed at intervals of length $\delta > 0$, which is a day in our case, over time horizon $[0, T]$ with $T = n\delta$, where $n$ is the size of samples collected at the daily frequency. In our setup, we require $n >> N$ so that we have sufficient daily observations in each of the time change intervals $[T(i-1)\Delta, T_i\Delta]$ for $i = 1, \ldots, N$.

Finally, we may readily allow for the existence of a jump component in our model (22). Indeed, we may easily deal with the presence of discrete jumps in our methodology, simply by discarding the observations of $(Y_t)$ for days believed to have jumps. In our empirical studies, we use the test developed recently by Lee and Mykland (2008) to detect jump days. Although it is well known that the jumps are frequently observed for many intra-day samples, it appears that jumps are infrequent for samples with daily or lower frequency observations. We detect some evidence of jumps in our daily observations, but their number is relatively small compared to the sampling span of the data.

5. Empirical Procedure

5.1 Data

We use the Gürkaynak, Sack, and Wright (2006) zero-coupon daily treasury yields and forwards data of maturity 1-, 2-, 3-, 4-, 5- and 7-year bonds. Bond returns and spreads associated with the maturity we consider are calculated from the data. For bond returns, we calculate the excess returns on holding Treasury bonds over the risk free rate of return over the sample period July 1961 through June 2009. Specifically, we calculate holding period returns of each maturity bond by summing up continuously compounded logarithmic returns and negative of instantaneous forward rates adjusted to the daily level by dividing
by 250, the number of average trading days in a year. For the risk free rate of return, we use three-month treasury bill rates, also divided by 250 for the daily adjustment, taken from the Federal Reserve Board of Governors. Since the daily series on the three-month T-bill rates can be considered as a risk free return from today to tomorrow, we calculate the daily excess holding returns on the bond by subtracting yesterday’s T-bill rate from today’s holding period return on the bond. For the right-hand side variables of the continuous time Campbell-Shiller and Fama-Bliss regressions, we use long-short yield and forward-short spreads of the same maturity structure, respectively. These variables are also adjusted to the daily level by the same procedure described above. We report the descriptive statistics on the data in Table 1.

Now, we address a potential issue in using Gürkaynak, Sack, and Wright (GSW, henceforth) dataset for our research. As explained by Cochrane and Piazzesi (2008), examining bond return predictability based on the GSW dataset is irrelevant in some cases since the yields in the dataset are smoothed across maturities by the Svensson fitted function. Specifically, a multicollinearity problem occurs when using the data for the purpose of the Cochrane and Piazzesi (2008) excess return regression. This fact does harm on their regression particularly because they investigate a forecasting power of higher order factors which is not included in the usual “level”, “slope”, and “curvature” factors by adapting at least five forward rates of different maturities. Even small amounts of smoothing can hamper the performance of the forecasting regressions by losing accumulated measurement errors through the channel of multiple forward rates. Our univariate model is much less contaminated by such a problem.

5.2 Implementation of Econometric Methodology

Our martingale estimation is based on the bond holding period returns in volatility time, not in usual calendar time. In order to calculate the required time change \( T_\Delta \) for \( i = 1, \ldots, N \), we need to set the level \( \Delta \) of increments for the quadratic variation of the error process. The choice of \( \Delta \) is not important theoretically because the asymptotic theory of the martingale estimator does not depend on \( \Delta \). However, it may matter in finite samples and thus we formally specify the \( \Delta \) setting criterion below.

The \( \Delta \) setting strategy consists of the following steps. First, in establishing the \( \Delta \) setting rule, we construct an admissible \( \Delta \) range by discerning maximum and minimum values of \( \Delta \) suitable for our estimation procedure. Once the total number of observations is fixed, there is an inverse relationship between the size of \( \Delta \) and the number of time changed samples. Thus it is tempting to select \( \Delta \) as small as possible to get as many time changed samples as possible. On the other hand, however, too small a \( \Delta \) value makes the time changed error process deviate from an i.i.d. normal distribution, and thus hampers the performance of the martingale estimator. In practice, we gauge \( \Delta \) by using the number of days to be included to measure the average volatility defined to be \( K \). More formally, \( \Delta \) is calculated as

\[
\Delta = TQV \times \frac{K}{n},
\]

where TQV is the total realized quadratic variation and \( n \) is the total number of samples. We interpret this \( \Delta \) to be the average quadratic variation within \( K \) trading days. Our
martingale estimation results for various ∆s indicates that the ∆s calculated by the average quadratic variation with K less than 60 suffer from the non-normality problem. On the other hand, ∆s associated with more than 200 volatility days yield insufficient data for the martingale estimation. Thus ∆s with K between 60 and 200 constitute the range of admissible ∆s for all maturity bonds we consider in our work.

Then we propose the following method to refine the ∆ set. We choose ∆ that minimizes the standard error of the martingale estimator among the admissible ∆s. In the statistical sense, therefore, the selected ∆ achieves the optimality of the martingale estimator. This ∆ selection criterion allows ∆ to vary for different maturities. Although this setting slightly deviates from the Litterman and Scheinkman (1991) finding that the three common principal components explain over 96% of the total variation in yield changes, our empirical result demonstrates that considering idiosyncratic factors helps our estimator to attain optimality.

To calculate the standard error of the estimator, we use the block bootstrap methods of block size $N^{1/3}$ which is shown to be optimal by Hall et al. (1995). Our estimation result indicates that the minimum standard error criterion is met when the average quadratic variations of the days are between 150 and 180. The number of days for the average quadratic variation varies by maturity, but they are in the range given above for all maturities.

In the next step, we calculate the estimate $(T^\delta_t)$ of the time change $(T_t)$ as

$$T^\delta_{i\Delta} = \arg\min_{t \geq T^\delta_{(i-1)\Delta}} \{ [P^\delta_t > i\Delta] \}.$$  

for $i = 1, \ldots, N$, in a recursive manner starting from $T^\delta_0 \equiv 0$. Once we fix ∆ and estimate $T^\delta_t$, we may now collect the estimation sample $(z_\delta^i(\theta))$ for each $\theta \in \Theta$ based on $T^\delta_t$ correspondingly as $(z_i(\theta))$ in (22). Of course, we replace all integral values in defining $(Z^\delta_t(\theta))$ by their corresponding Riemann sums using daily observations and estimate $(Z^\delta_{T^\delta_t(\theta)})$ by $(Z^\delta_{T^\delta_t(\theta)})$ in computing $(z_i(\theta))$. As discussed earlier, we discard jump observations in this preliminary estimation sample and also delete all repeated observations to estimate our model parameters.

6. Empirical Results

In the empirical analysis, we obtain the parameter estimate of $(\alpha_\tau, \beta_\tau)$ of Campbell-Shiller and Fama-Bliss models respectively, based on the continuous-time modeling method. We estimate the model by the martingale regression and compare the estimated parameters with that from the OLS-GMM discrete-time model.

6.1 Baseline Estimation: OLS-GMM Results

For a baseline estimation, we tabulate the OLS-GMM estimation result by using the Gürkaynak, Sack, and Wright (2006) data in Table 2. Since the discrete-time estimation is based on monthly data, a monthly yield dataset is generated from the GSW daily dataset by selecting the end-of-month yields. Panel A and B present the Campbell-Shiller and Fama-Bliss result respectively. These estimates are similar and representative of the literature.
In particular, the estimates in panel A are uniformly negative and decrease steadily as the maturity of the long rate increases. Despite differences in the sample range, it captures the pattern of the estimates in Campbell and Shiller (1991) constituting a term structure paradox. The standard errors of the $\beta_\tau$ coefficients indicate that the expectations hypothesis can be statistically rejected for each of the four regressions over our full sample. Note that the standard errors are based on the Hansen-Hodrick correction for serial correlation due to data overlaps. Similarly, the Fama-Bliss excess return regression results for our extended sample period in panel B confirm the rejection of the expectations hypothesis. The estimates are uniformly positive and statistically different from zero. The excess returns on bonds are predictable, thus the expectations hypothesis is rejected.

6.2 Martingale Regression Results

Table 3 displays the summary statistics of the Campbell-Shiller continuous-time model with the full sample, from July 1961 to June 2009. Panel A shows the estimation results when jumps are not considered and excluded. To gauge the impact of the presence of jumps, we also report the estimates of the estimation results under the 1% and 5% Lee-Mykland jump tests respectively in panel B and C. The last column of each panel reports Cramer-von Mises (CvM) distance accounted for in the previous section.

One finding from the results is that the presence of jumps do not affect the estimation results of continuous-time models. The consideration of the presence of jumps in return processes does not change the stylized facts of the results. Moreover, even if jumps are considered, our results are robust to either significant level.

More importantly, however, when using the continuous-time model with this new estimation technique, the estimates we obtained are incompatible with that of discrete-time models. The $\beta_\tau$ coefficient are positive except for the 7-year maturity bond and declines as maturity increases. In addition, the $\beta_\tau$ coefficients are not statistically different from unity, providing firm evidence of the expectations hypothesis.

We observe similar results in the case of the continuous-time Fama-Bliss estimation presented in Table 4. The $\beta_\tau$ estimates are not statistically zero and, thus the bond returns are not predictable. Thus we cannot reject the expectations hypothesis based on the data. Note that, similar to the Campbell-Shiller case, the Fama-Bliss continuous-time estimation results are not affected by the presence of jumps.

We interpret these results from both continuous-time regressions as confirming the expectation hypothesis. However, these results are quite contradictory to what the standard OLS-GMM tests conclude. Our next task is to shed more light on this issue.

6.3 Does Sample Period Matter?

Before proceeding, we investigate subperiod results of the continuous-time expectations hypothesis testing. There have been several pieces of research that point out the difference in the performance of the expectations hypothesis in different time period. Mankiw and Miron (1986) examine the validity of the expectations hypothesis for various periods and find that the Federal Reserve’s interest rate stabilizing policy affects the empirical evidence
of the expectation hypothesis. A recent work by Rudebusch and Wu (2007) finds a regression
evidence of a shift in the term structure. By estimating the discrete-time Campbell and
Shiller regression, they show that the expectations hypothesis is rejected with the yield data
from 1970 to 1987, while it is not rejected for all maturities between two- and five-year with
the data after 1988. Based on this stylized fact, they conclude that there is a structural shift
in the U.S. term structure in the middle or late 1980s. These findings urge us to investigate
the subperiod analysis of the expectations hypothesis test.

We split the yield data into two subperiods, before 1987 and after 1988. This crite-
rian focuses on the appointment of the Fed Chairman Alan Greenspan and commensurate
changes in monetary policy. Since monetary policy heavily affects term structure, especially
through its short-end channel, changing behavior of the monetary policy authority can be
a reasonable guideline in differentiating one subperiod from the other. In addition, our
first sample period, 1961-1987, includes a period of historically high interest rate volatility
associated with the monetary experiment of the Federal Reserve in the late 1970s. This will
also draw a clear line between the first and latter samples.

Table 5 reports the full sample and subsample estimates of the Campbell-Shiller regres-
sion based on the OLS-GMM and MGE estimation methods. The numbers in parentheses
stand for the standard errors. From the statistics presented, we know both the discrete-
and continuous-time models reject the expectations hypothesis for the first period, while
they cannot reject the hypothesis for latter period. This result reconfirms the previous
finding that these two periods are best viewed as different regimes. We may have to note
that, however, the degree of rejection of the expectations hypothesis is quite different. For
the first subsample, it is clear that the OLS-GMM $\beta_\tau$ estimates locate around $\pm 2$ standard
error bands, while the MGE estimates are placed just outside of $\pm 1$ standard error bounds
of the estimator for all maturities to be considered. A similar pattern of the estimates is
observed in the latter subsample. The $\beta_\tau$ estimates of the MGE always give stronger results
in favor of the expectations hypothesis. The same pattern can be observed in the subsample
estimates of the continuous-time Fama-Bliss regression in Table 6.

Therefore, the empirical results on the expectations hypothesis test depend upon the
period we consider. Accounting for the potential sources of this difference might be a cru-
cial work to understand term structure behavior, but is beyond the scope of our research.
Rather, we focus more on how to explain the gap between the OLS-GMM and MGE esti-
mates in testing the expectations hypothesis. As was addressed, regardless of the sample
period, the estimates of $\beta_\tau$ from the continuous-time models are always more supportive of
the expectations hypothesis than that of the discrete-time models. To account for the gap
between results of two different frequency models, the next section conducts a simulation
study.

7. A Simulation Study

The results in the previous section is quite puzzling. While the estimated OLS-GMM pa-
rameters reject the null, the MGE estimators cannot reject the null and give favorable
results to the expectations hypothesis using full sample data, 1961-2009. Though it is not
as severe as for the full sample results, the slope estimators of MGE regressions are systematically above (below) that of the discrete-time Campbell-Shiller (Fama-Bliss) regression for the subperiods.

We posit that these features are not indeed contradictory, but can be interpreted as a systematic over rejection of discrete-time regression approaches in testing the expectations hypothesis. We arrive at this conjecture by investigating whether the discrete-time results can be restored from the continuous-time model and its estimated parameters. For this purpose, we simulate yield data sets corresponding to the full sample period based on the continuous-time models. The estimates of the discrete- and continuous-time models show the largest gap when the full sample is considered. This is why we choose the full sample period for the simulation study. In this subsection, we account in more detail the simulation procedure of the Campbell-Shiller model. Every procedure described below is directly applicable to the Fama-Bliss model in the same way.

We first generate hypothetical yields corresponding to the full sample period by using the continuous-time Campbell-Shiller formula given as

\[ \frac{dP}{P} = \left[ -f_t^{(r)} + r_t + \frac{1}{2} \left( \sigma_t^{(r)} \right)^2 + \alpha_t + (1 - \beta_t) \left( y_t^{(r)} - r_t \right) \right] dt + \sigma_t^{(r)} dW_t. \]  

As was presented earlier, the estimated parameters of Equation (28) cannot reject the expectations hypothesis for the full sample. In generating data, we consider the sampling frequency \( \delta \) to be 1 day (\( \delta = 1/250 \)) for consistency in data frequency to our analysis. In order to simulate data from the model, we employ the Euler discretization scheme,

\[ \frac{\Delta P_{t\delta}}{P} = \mu_{(i-1)\delta}\delta + \sigma_{(i-1)\delta}\sqrt{\delta}\varepsilon_i, \]  

where

\[ \mu_{(i-1)\delta} = \left[ -f_{(i-1)\delta}^{(r)} + r_{(i-1)\delta} + \frac{1}{2} \left( \sigma_{(i-1)\delta}^{(r)} \right)^2 + \alpha_t + (1 - \beta_t) \left( y_{(i-1)\delta}^{(r)} - r_{(i-1)\delta} \right) \right], \]

and \( \varepsilon_i \) is an i.i.d. standard normal variable. Note that Equation (28) and (29) are dynamic (time-series) restrictions of the bond price movement. Once an initial bond price is given, a time-series of simulated bond prices can be generated by the rule described in (29). Specifically, simulating the diffusion part requires an estimation of the instantaneous volatility \( \sigma_{(i-1)\delta}^{(r)} \). Having daily data, the instantaneous volatility is directly estimable from the data. Here we use the nonparametric estimation of instantaneous volatility proposed by Kristensen (2010) to estimate the daily volatilities. In particular, we use the Gaussian kernel estimator and set the bandwidths for the kernel estimation based on the least squares cross-validation for the volatility analysis. For the drift part, we use the historical forward-, short-rates, and yields combined with the parameters estimated by the martingale regression in the previous section. In addition, the square of the instantaneous volatility term uses the estimated volatility obtained as above. A simulation of the drift and diffusion functions jointly determine the law of motion of bond prices in the given continuous-time model.
This procedure gives us a time series of generated bond price changes, \( \Delta P_t^{(\tau)} / P \), in which each of the observations in the series corresponds to the daily fluctuation. Basically, our aim is to generate bond yields, not bond prices itself. It is well known that bond yields and prices are related by the equation given as

\[
y_t^{(\tau)} = -\frac{1}{\tau} \log P_t^{(\tau)}.
\]  

(30)

By differentiating (30), we have

\[
dy_t^{(\tau)} = -\frac{1}{\tau} \frac{dP_t^{(\tau)}}{P}.
\]  

(31)

Then a discrete approximation of Equation (31) is to generate a time series of daily bond yields corresponding to our martingale estimation result, for every maturity \( \tau \). The actual bond yield of each maturity is used to generate the yield data.

The rest of the simulation scheme is quite straightforward. To evaluate the generated yields in the monthly Campbell-Shiller regression framework, we select the end-of-month yields from the generated yield data for maturities from 1- through 5-years. Then, we do the Campbell-Shiller regression by using the generated monthly data.

As noted earlier, the entire simulation scheme is directly applicable to the Fama-Bliss case. The only difference is found in the drift function in that we use historical forward-spot spreads instead of yield spreads.

We report the OLS estimation results based on 1,000 simulations in Table 7. Panel A and B represent the Campbell-Shiller and Fama-Bliss simulation results respectively. Despite the fact that the estimated parameters of the continuous-time models do not violate the expectations hypothesis, the OLS regressions with data generated from the continuous-time model recast the stylized facts of the OLS results based on the historical data. In particular, the means of slope coefficients \( \beta_\tau \) are distributed around the actual OLS parameter estimates. This fact can be observed not only in both regression models but for all maturities to be considered.

These results are insensitive to the choice of bandwidth of kernel estimators. Our simulation scheme involves the estimation of instantaneous volatilities by using the Nadaraya-Watson typed kernel estimator. In doing a kernel-based estimation, it is well known that the bandwidth choice is crucial to estimation results. However, though not reported, the mean and variance of the estimated parameters with simulated data are stable. This shows that our simulation results are quite robust to potential over- or under-smoothing of instantaneous volatilities.

These results show that the discrete-time OLS-based tests tend to generate results that are against the expectations hypothesis even if the true data generating process is consistent with the expectations hypothesis: estimates of \( \beta_{\tau MGE} \) are not nearly always statistically different from unity, while estimates of \( \beta_{\tau OLS} \) are nearly always statistically different from 1, in the Campbell-Shiller regression. Similar results are observed in the Fama-Bliss regression case.
8. Conclusion

We test the expectations hypothesis in a continuous-time setting. In particular, the continuous-time version of Campbell-Shiller and Fama-Bliss regressions are used for the estimation. This approach allows us to examine validity of the expectation hypothesis assumed to hold continuously in time. To estimate the models, we use a novel econometric approach—the martingale regression with time change—proposed by Park (2009).

We find plausible but unusual results based on the yield data spanned from 1961 to 2009. The continuous-time MGE estimates provide much more supportive results to the expectations hypothesis, while discrete-time OLS-based estimates display a strong rejection to the hypothesis for the same data period. For subperiods, the continuous-time estimates always give results in favor of the expectations hypothesis. Our simulation study finds that the discrete-time estimates tend to reject the expectations hypothesis severely even if the data is generated under the expectations hypothesis.

Note that the tendency of over rejection of the discrete-time OLS estimates are different from the small-sample bias in Bekaert et al. (1997). Rather, our findings suggest that the test frequency is crucial in examining the expectations hypothesis.
References


Park, J.Y. “Martingale regression and time change,” mimeographed, Department of Economics, Indiana University, 2009.


# Tables and Figures

Table 1: Summary Statistics of Daily Bond Excess Holding Period Returns

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<tr>
<th></th>
<th>1-year</th>
<th>2-year</th>
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<th>4-year</th>
<th>5-year</th>
<th>7-year</th>
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<tr>
<td>Mean (%)</td>
<td>0.003</td>
<td>0.005</td>
<td>0.006</td>
<td>0.006</td>
<td>0.007</td>
<td>0.007</td>
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<tr>
<td>Std. Dev. (%)</td>
<td>0.068</td>
<td>0.141</td>
<td>0.212</td>
<td>0.278</td>
<td>0.339</td>
<td>0.455</td>
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Table 2: OLS-GMM Regression Results

Panel A: Campbell-Shiller Regressions

<table>
<thead>
<tr>
<th>Maturity $\tau$</th>
<th>$\alpha_\tau$</th>
<th>s.e.$(\alpha_\tau)$</th>
<th>$\beta_\tau$</th>
<th>s.e.$(\beta_\tau)$</th>
<th>$R^2$</th>
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</thead>
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<tr>
<td>2</td>
<td>-0.001</td>
<td>0.003</td>
<td>-0.574</td>
<td>0.659</td>
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<td>0.002</td>
<td>-1.743</td>
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Panel B: Fama-Bliss Excess Return Regressions

<table>
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<tr>
<th>Maturity $\tau$</th>
<th>$\alpha_\tau$</th>
<th>s.e.$(\alpha_\tau)$</th>
<th>$\beta_\tau$</th>
<th>s.e.$(\beta_\tau)$</th>
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Note: The table presents the OLS-GMM regression results of discrete-time Campbell-Shiller and Fama-Bliss models respectively. Standard errors use Hansen-Hodrick correction for serial correlation due to data overlaps.
Table 3: Campbell-Shiller Martingale Regression Results

Panel A: Without Considering Jumps

<table>
<thead>
<tr>
<th>Maturity τ</th>
<th>α_τ</th>
<th>s.e.(α_τ)</th>
<th>β_τ</th>
<th>s.e.(β_τ)</th>
<th>CvM</th>
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<td>0.018</td>
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<td>0.283</td>
<td>1.116</td>
<td>0.017</td>
</tr>
<tr>
<td>5</td>
<td>0.008</td>
<td>0.016</td>
<td>0.004</td>
<td>1.036</td>
<td>0.017</td>
</tr>
<tr>
<td>7</td>
<td>0.002</td>
<td>0.020</td>
<td>-0.263</td>
<td>1.328</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Panel B: Eliminating Jump Observations Based on 1% LM Test

<table>
<thead>
<tr>
<th>Maturity τ</th>
<th>α_τ</th>
<th>s.e.(α_τ)</th>
<th>β_τ</th>
<th>s.e.(β_τ)</th>
<th>CvM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.008</td>
<td>0.005</td>
<td>0.732</td>
<td>0.810</td>
<td>0.028</td>
</tr>
<tr>
<td>2</td>
<td>0.008</td>
<td>0.009</td>
<td>0.515</td>
<td>0.966</td>
<td>0.013</td>
</tr>
<tr>
<td>3</td>
<td>0.008</td>
<td>0.012</td>
<td>0.247</td>
<td>1.124</td>
<td>0.019</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
<td>0.015</td>
<td>0.133</td>
<td>1.295</td>
<td>0.025</td>
</tr>
<tr>
<td>5</td>
<td>0.007</td>
<td>0.017</td>
<td>0.013</td>
<td>1.247</td>
<td>0.018</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
<td>0.019</td>
<td>-0.162</td>
<td>1.185</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Panel C: Eliminating Jump Observations Based on 5% LM Test

<table>
<thead>
<tr>
<th>Maturity τ</th>
<th>α_τ</th>
<th>s.e.(α_τ)</th>
<th>β_τ</th>
<th>s.e.(β_τ)</th>
<th>CvM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.008</td>
<td>0.005</td>
<td>0.899</td>
<td>0.966</td>
<td>0.034</td>
</tr>
<tr>
<td>2</td>
<td>0.010</td>
<td>0.007</td>
<td>0.734</td>
<td>0.887</td>
<td>0.032</td>
</tr>
<tr>
<td>3</td>
<td>0.010</td>
<td>0.011</td>
<td>0.475</td>
<td>1.048</td>
<td>0.018</td>
</tr>
<tr>
<td>4</td>
<td>0.006</td>
<td>0.019</td>
<td>0.162</td>
<td>1.384</td>
<td>0.034</td>
</tr>
<tr>
<td>5</td>
<td>0.007</td>
<td>0.020</td>
<td>0.186</td>
<td>1.277</td>
<td>0.025</td>
</tr>
<tr>
<td>7</td>
<td>-0.001</td>
<td>0.025</td>
<td>-0.375</td>
<td>1.360</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Note: The table presents the martingale regression results of the continuous-time Campbell-Shiller model. A block bootstrap method is used to calculate the standard errors of the estimators. To detect the presence of jumps (Panel B and C), we use the jump test developed by Lee and Mykland (2008).
Table 4: Fama-Bliss Martingale Regression Results

Panel A: Without Considering Jumps

<table>
<thead>
<tr>
<th>Maturity $\tau$</th>
<th>$\alpha$</th>
<th>s.e.$(\alpha)$</th>
<th>$\beta$</th>
<th>s.e.$(\beta)$</th>
<th>CvM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.009</td>
<td>0.005</td>
<td>0.054</td>
<td>0.472</td>
<td>0.027</td>
</tr>
<tr>
<td>2</td>
<td>0.011</td>
<td>0.010</td>
<td>0.192</td>
<td>0.763</td>
<td>0.024</td>
</tr>
<tr>
<td>3</td>
<td>0.011</td>
<td>0.012</td>
<td>0.271</td>
<td>0.598</td>
<td>0.019</td>
</tr>
<tr>
<td>4</td>
<td>0.010</td>
<td>0.012</td>
<td>0.556</td>
<td>0.720</td>
<td>0.014</td>
</tr>
<tr>
<td>5</td>
<td>0.010</td>
<td>0.018</td>
<td>0.544</td>
<td>0.918</td>
<td>0.016</td>
</tr>
<tr>
<td>7</td>
<td>0.001</td>
<td>0.022</td>
<td>0.908</td>
<td>0.973</td>
<td>0.017</td>
</tr>
</tbody>
</table>

Panel B: Eliminating Jump Observations Based on 1% LM Test

<table>
<thead>
<tr>
<th>Maturity $\tau$</th>
<th>$\alpha$</th>
<th>s.e.$(\alpha)$</th>
<th>$\beta$</th>
<th>s.e.$(\beta)$</th>
<th>CvM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.008</td>
<td>0.005</td>
<td>0.187</td>
<td>0.599</td>
<td>0.027</td>
</tr>
<tr>
<td>2</td>
<td>0.008</td>
<td>0.008</td>
<td>0.291</td>
<td>0.610</td>
<td>0.013</td>
</tr>
<tr>
<td>3</td>
<td>0.012</td>
<td>0.013</td>
<td>0.221</td>
<td>0.784</td>
<td>0.019</td>
</tr>
<tr>
<td>4</td>
<td>0.008</td>
<td>0.014</td>
<td>0.432</td>
<td>0.850</td>
<td>0.026</td>
</tr>
<tr>
<td>5</td>
<td>0.010</td>
<td>0.017</td>
<td>0.475</td>
<td>0.889</td>
<td>0.019</td>
</tr>
<tr>
<td>7</td>
<td>0.004</td>
<td>0.020</td>
<td>0.621</td>
<td>0.878</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Panel C: Eliminating Jump Observations Based on 5% LM Test

<table>
<thead>
<tr>
<th>Maturity $\tau$</th>
<th>$\alpha$</th>
<th>s.e.$(\alpha)$</th>
<th>$\beta$</th>
<th>s.e.$(\beta)$</th>
<th>CvM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.008</td>
<td>0.004</td>
<td>0.078</td>
<td>0.630</td>
<td>0.034</td>
</tr>
<tr>
<td>2</td>
<td>0.009</td>
<td>0.008</td>
<td>0.246</td>
<td>0.701</td>
<td>0.031</td>
</tr>
<tr>
<td>3</td>
<td>0.011</td>
<td>0.011</td>
<td>0.338</td>
<td>0.787</td>
<td>0.018</td>
</tr>
<tr>
<td>4</td>
<td>0.009</td>
<td>0.019</td>
<td>0.392</td>
<td>1.054</td>
<td>0.035</td>
</tr>
<tr>
<td>5</td>
<td>0.010</td>
<td>0.022</td>
<td>0.394</td>
<td>1.031</td>
<td>0.028</td>
</tr>
<tr>
<td>7</td>
<td>0.004</td>
<td>0.023</td>
<td>0.793</td>
<td>1.026</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Note: The table presents the martingale regression results of the continuous-time Fama-Bliss model. A block bootstrap method is used to calculate the standard errors of the estimators. To detect the presence of jumps (Panel B and C), we use the jump test developed by Lee and Mykland (2008).
Table 5: $\beta_r$ Coefficients of Campbell-Shiller Regressions with Different Sample Period

**Panel A: OLS-GMM**

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Full sample: July 1961 to June 2009</th>
<th>Subsample A: July 1961 to December 1987</th>
<th>Subsample B: January 1988 to June 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.891 (0.833)</td>
<td>-0.085 (1.229)</td>
<td>0.867 (0.935)</td>
</tr>
<tr>
<td>3</td>
<td>0.696 (1.028)</td>
<td>-1.081 (1.229)</td>
<td>0.631 (0.903)</td>
</tr>
<tr>
<td>4</td>
<td>0.513 (0.866)</td>
<td>-1.009 (1.143)</td>
<td>0.540 (0.995)</td>
</tr>
<tr>
<td>5</td>
<td>0.283 (1.116)</td>
<td>-1.023 (1.419)</td>
<td>0.637 (1.012)</td>
</tr>
<tr>
<td>7</td>
<td>0.004 (1.036)</td>
<td>-1.203 (1.691)</td>
<td>0.223 (1.419)</td>
</tr>
<tr>
<td></td>
<td>-0.263 (1.328)</td>
<td>-1.398 (1.955)</td>
<td>0.119 (1.057)</td>
</tr>
</tbody>
</table>

**Panel B: Martingale Regression**

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Full sample: July 1961 to June 2009</th>
<th>Subsample A: July 1961 to December 1987</th>
<th>Subsample B: January 1988 to June 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.891 (0.833)</td>
<td>-0.085 (1.229)</td>
<td>0.867 (0.935)</td>
</tr>
<tr>
<td>2</td>
<td>0.696 (1.028)</td>
<td>-1.081 (1.229)</td>
<td>0.631 (0.903)</td>
</tr>
<tr>
<td>3</td>
<td>0.513 (0.866)</td>
<td>-1.009 (1.143)</td>
<td>0.540 (0.995)</td>
</tr>
<tr>
<td>4</td>
<td>0.283 (1.116)</td>
<td>-1.023 (1.419)</td>
<td>0.637 (1.012)</td>
</tr>
<tr>
<td>5</td>
<td>0.004 (1.036)</td>
<td>-1.203 (1.691)</td>
<td>0.223 (1.419)</td>
</tr>
<tr>
<td>7</td>
<td>-0.263 (1.328)</td>
<td>-1.398 (1.955)</td>
<td>0.119 (1.057)</td>
</tr>
</tbody>
</table>

Note: The table presents the full- and subsample estimation results of the Campbell-Shiller $\beta_r$ coefficient. Panel A and B display the discrete- and continuous-time results respectively. Standard errors are provided in parentheses. Jump observations are not excluded for Panel B.
Table 6: $\beta_r$ Coefficients of Fama-Bliss Regressions with Different Sample Period

**Panel A: OLS-GMM**

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample: July 1961 to June 2009</td>
<td>0.787</td>
<td>0.959</td>
<td>1.130</td>
<td>1.303</td>
<td>1.623</td>
</tr>
<tr>
<td>(0.330)</td>
<td>(0.426)</td>
<td>(0.502)</td>
<td>(0.562)</td>
<td>(0.650)</td>
<td></td>
</tr>
<tr>
<td>Subsample A: July 1961 to December 1987</td>
<td>1.082</td>
<td>1.285</td>
<td>1.491</td>
<td>1.719</td>
<td>2.181</td>
</tr>
<tr>
<td>(0.383)</td>
<td>(0.541)</td>
<td>(0.731)</td>
<td>(0.932)</td>
<td>(1.322)</td>
<td></td>
</tr>
<tr>
<td>Subsample B: January 1988 to June 2009</td>
<td>0.158</td>
<td>0.339</td>
<td>0.466</td>
<td>0.567</td>
<td>0.728</td>
</tr>
<tr>
<td>(0.511)</td>
<td>(0.590)</td>
<td>(0.617)</td>
<td>(0.624)</td>
<td>(0.627)</td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Martingale Regression**

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample: July 1961 to June 2009</td>
<td>0.054</td>
<td>0.192</td>
<td>0.271</td>
<td>0.556</td>
<td>0.544</td>
<td>0.908</td>
</tr>
<tr>
<td>(0.472)</td>
<td>(0.763)</td>
<td>(0.598)</td>
<td>(0.720)</td>
<td>(0.918)</td>
<td>(0.973)</td>
<td></td>
</tr>
<tr>
<td>Subsample A: July 1961 to December 1987</td>
<td>1.079</td>
<td>2.084</td>
<td>2.233</td>
<td>2.424</td>
<td>1.893</td>
<td>2.331</td>
</tr>
<tr>
<td>(1.057)</td>
<td>(0.767)</td>
<td>(1.094)</td>
<td>(1.451)</td>
<td>(1.136)</td>
<td>(1.382)</td>
<td></td>
</tr>
<tr>
<td>Subsample B: January 1988 to June 2009</td>
<td>0.094</td>
<td>0.211</td>
<td>0.271</td>
<td>0.219</td>
<td>0.377</td>
<td>0.611</td>
</tr>
<tr>
<td>(0.716)</td>
<td>(0.499)</td>
<td>(0.622)</td>
<td>(0.736)</td>
<td>(1.053)</td>
<td>(0.860)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The table presents the full- and subsample estimation results of the Fama-Bliss $\beta_r$ coefficient. Panel A and B display the discrete- and continuous-time results respectively. Standard errors are provided in parentheses. Jump observations are not excluded for Panel B.
Table 7: Summary Statistics for OLS Regression with Simulated Yields

**Panel A: Campbell-Shiller Regression with Simulated Yield Data**

<table>
<thead>
<tr>
<th>Maturity $\tau$</th>
<th>Actual $\alpha$</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Simulation $\alpha$</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Actual $\beta$</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Simulation $\beta$</th>
<th>Mean</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.001</td>
<td>-0.003</td>
<td>0.003</td>
<td>-0.574</td>
<td>-0.594</td>
<td>0.190</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.004</td>
<td>0.008</td>
<td>-0.840</td>
<td>-0.797</td>
<td>0.199</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.008</td>
<td>0.006</td>
<td>-1.084</td>
<td>-1.072</td>
<td>0.248</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
<td>0.010</td>
<td>0.005</td>
<td>-1.315</td>
<td>-1.429</td>
<td>0.271</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Fama-Bliss Regression with Simulated Yield Data**

<table>
<thead>
<tr>
<th>Maturity $\tau$</th>
<th>Actual $\alpha$</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Simulation $\alpha$</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Actual $\beta$</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Simulation $\beta$</th>
<th>Mean</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.001</td>
<td>0.003</td>
<td>0.002</td>
<td>0.787</td>
<td>0.855</td>
<td>0.058</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.001</td>
<td>-0.001</td>
<td>0.003</td>
<td>0.959</td>
<td>1.003</td>
<td>0.063</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>-0.002</td>
<td>0.003</td>
<td>1.130</td>
<td>1.066</td>
<td>0.069</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.002</td>
<td>-0.016</td>
<td>0.005</td>
<td>1.303</td>
<td>1.278</td>
<td>0.066</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The table reports the OLS regression results with simulated yield data. The data generating process uses the continuous-time Campbell-Shiller (for Panel A) and Fama-Bliss (for Panel B) models with the parameters estimated by the martingale regression displayed in Table 3 and 4. The column title “Actual” represents the OLS estimation results with the historical yield data as shown in Table 2. The number of simulations is 1,000.