Using Reserves to Finance the Increase in Government Infrastructure Investment in China

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Abstract

In this paper I analyze the impact of increasing government infrastructure investment by using existing large foreign exchange reserves in a fully optimizing model matching the Chinese economy. Even though the private investment falls in the nontradable sector, real expenditure and money balance decrease in the short run, the infrastructure investment greatly enhances the efficiency and productivity of the private sector, and gives rise to an entire sequence of productivity increases. In the long run, private investment is enhanced, total output, real expenditure and money balance rise to a higher level with no negative impact on the budget of the central government.

JEL classification: E6; H54.
Keywords: Infrastructure investment; Fiscal Policy.

1 Introduction

Investment in social infrastructure has long been considered important for large developing countries (LDCs). The infrastructure investment in LDCs is far below the average level of the industrial countries. The stock of infrastructure is an important input in the production process, raising the efficiency and productivity in the private sector, and ultimately giving rise to a higher output and living standards.

Financing the new infrastructure investment is a difficult task for LDCs, especially in poor, resource-constrained developing countries with fiscal deficits. A common way for financing the infrastructure investment is borrowing from abroad, or relying on aid programs. However, such financing creates dramatic monetary management problems for LDCs, making them reluctant to pursue these avenues. A large amount of literature is focusing on those issues. In contrast, those problems do not hold for China. Even though China is an LDC, the Chinese government holds huge amounts of public savings in the form of foreign exchange reserves. In July 2009,
China’s foreign exchange reserves topped $2 trillion for the first time. The composition of foreign exchange reserves is presently regarded as a state secret in China, so there is no detailed official report on it, but it is commonly estimated that 60% of the Chinese foreign reserves are in US dollars and bonds. According to a United States Treasury Department report, in October 2009, China held $798.9 billion in US Treasury bonds, $479.8 billion of long-term agency bonds, $24.5 billion of long-term corporate bonds and $100.8 billion in the US stock market, adding up to $1.4 trillion. The remainder is primarily invested in Japanese Yen, Euro, and British Pounds.

US dollar assets comprise a large portion of China’s foreign exchange reserves, and China does not have diversified channels to preserve the value of these reserves. The book value of some of these assets have fallen significantly since 2000 due to a devaluation of the US dollar. Moreover, the interest returns on those US treasury bonds have fallen over time. The interest rates for 1-Year Treasury bonds in 1990 were 7.4%, and fell to 5.8% in 2000, and the latest report from the Federal Reserve shows that the interest rates at 0.33% this February. Hence, the interest return on those foreign exchange reserves is low. On the contrary, the return on infrastructure investment is high. The empirical estimates for the long-term return on infrastructure investment are high in LDCs. China has a very thin stock of infrastructure, even though the Chinese government started to increase the investment in public infrastructure. A recent empirical estimation shows that infrastructure investment spending comprises only 6% of total investment before the stimulus package in 2008. There are too few highways, poor road conditions, urban congestion, lack of investment in education in the rural areas, etc. The insufficient infrastructure significantly affects the productivity and efficiency in the private sector. With a decreasing return from foreign assets holdings, and an increasing return from infrastructure investment, would it be a good idea for the Chinese government to increase its infrastructure investment by using the existing foreign exchange reserves?

In this paper, I develop an open economy model to address the issue of using foreign exchange reserves to increase the government infrastructure investment. I investigate the short term and long term effects of increasing government infrastructure on private investment in both tradable and nontradable sectors, aggregate investment, total expenditure, and real output in a fully articulated, optimizing model matching the Chinese economy. Under the predetermined exchange rate system, in the short run, large government infrastructure spending takes resources away from the private sector, investment in the nontradable sector decreases, real expenditure and money holdings fall, but real GDP still increases slightly due to the boom in the tradable sector. Large foreign exchange reserves finance the surge of infrastructure investment, so the government budget is balanced in both the short term and in the long term. Even though the infrastructure investment has contractionary effect on the nontradable sector, the long run effect is expansionary. Higher infrastructure stock enhances the productivity and efficiency in the private sector, investment in both sectors rises, real GDP, expenditure and money holdings are all reaching a higher level without raising the inflation. However, the short run contractionary effect on the nontradable sector shows that the Chinese government should combine some stimulus plans for the nontradable sector together with the increase in infrastructure investment to offset the effects of contraction. (Literature review will be added.)

The rest of the paper is organized as follows. Section 2 outlines the model and its main

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1 FRED: Federal Reserve Bank at St.Louis, Economic Data
2 Canning and Bennathan, World Bank Working Paper
characteristics. Section 3 solves the dynamic system. Section 4 describes how the model was calibrated, and provides a numerical examination of the transitional paths in response to an unanticipated, permanent increase in the infrastructure investment. Section 5 gives the sensitivity check, and section 6 concludes.

2 The Model

This paper focuses on the Chinese economy that produces a nontraded good and a composite traded good. There are no trade taxes and all world market prices equal unity, so domestic prices of traded goods are set by the exchange rate $e$. The exchange rate system is predetermined, so the government fixes the rate of change of the exchange rate, but not the level. Capital account is closed.

2.1 Technology and the Supply Side

Technology is identical in both tradable and nontradable sectors. Production functions take the CES functional form.

$$Q_n = aZ^\eta \left( b_1 K_n^{\sigma_n-1} L_n^{\sigma_n-1} \right)^{\frac{\sigma_n}{\sigma_n-1}}$$

$$Q_T = aZ^\eta \left( b_2 K_T^{\sigma_T-1} L_T^{\sigma_T-1} \right)^{\frac{\sigma_T}{\sigma_T-1}}$$

where $b_1$ and $b_2$ are share parameters, and $\sigma_i$ is the elasticity of substitution ($i = T, n$). $Q_i$ is the output in sector $i$. Both nontraded good and traded good are produced by capital $K$ and labor $L$. Government infrastructure stock $Z$ creates a positive externality for the production in both sectors, so the increase of infrastructure stocks can increase the productivities in general. Capital is mobile ex ante, but sector-specific ex post. Labor supply is inelastic.

Firms in both sectors are perfectly competitive. With perfect competition and CES technology, the unit cost functions are

$$\tilde{C_n} = \frac{C_n^\eta (w, r_n)}{aZ^\eta}$$

$$\tilde{C_T} = \frac{C_T^\eta (w, r_T)}{aZ^\eta}$$

where $C^i$ is the cost function for producing $aZ^\eta$ units of output. $w$ is the wage rate measured in units of the traded good, and is determined by basic market forces. $r_i$ is measured in units of the traded good, and is the capital rental in sector $i$. The sectoral demands for capital stocks are

$$K_n = \frac{C_n^\eta (w, r_n)Q_n}{aZ^\eta}$$

$$K_T = \frac{C_T^\eta (w, r_T)Q_T}{aZ^\eta}$$
Labor is not sector-specific, so labor is mobile across sectors. The sectoral labor demands are given by

\[ L_n = \frac{C_n(w, r_n)Q_n}{aZ^n} \]  

\[ L_T = \frac{C_T(w, r_T)Q_T}{aZ^n} \]  

Competitive firms earn zero profits in equilibrium, so the zero profit conditions are

\[ P_n = \frac{C_n(w, r_n)}{aZ^n} \]  

\[ 1 = \frac{C_T(w, r_T)}{aZ^n} \]  

where \( P_n \) is the relative price of nontraded good to traded good. The inverse of \( P_n \) is the real exchange rate.

### 2.2 Private Agent’s Optimization Problem

A representative agent derives utility from consumption of traded and nontraded goods, and from the liquidity services generated by holdings of domestic currency. The Chinese government has strict capital controls, which prevent the private agent from borrowing abroad or purchasing foreign assets. Domestic money and physical capital are the only vehicles available for wealth accumulation. Preferences take the CES-CRRA functional form

\[ U = \int_0^\infty \left[ \frac{C(C_n, C_T)^{1-1/\tau}}{1-1/\tau} + h\left(\frac{M}{P}\right)^{1-1/\tau} \right] e^{-\rho t} dt \]  

where \( C(C_n, C_T) = [k_0 C_T^{(\beta_c-1)/\beta_c} + k_1 C_n^{(\beta_c-1)/\beta_c}]^{\beta_c/(\beta_c-1)} \) is linearly homogeneous CES aggregator function, \( h \) is constant; \( k_0 \), and \( k_1 \) are share parameters; \( \rho \) is the pure time preference rate; \( \tau \) is the intertemporal elasticity of substitution; \( \beta_c \) is the elasticity of substitution between traded and nontraded consumption goods.

The private agent solves the optimization problem in two stages. In the first stage, \( C_n \) and \( C_T \) are chosen to maximize \( C(C_n, C_T) \) subject to the budget constraint \( P_nC_n + C_T = E \), where \( E \) is the total expenditure measured in units of the traded good. The optimal choices of \( C_n^* \) and \( C_T^* \) give the indirect utility function \( V(P_n, E) = C[C_n^*(P_n, E), C_T^*(P_n, E)] = \frac{(E/P_n)^{1-1/\tau}}{1-1/\tau} \), where \( c(P_n) = (k_0^{\beta_c} + k_1^{\beta_c} P_n^{1-\beta_c})^{1/(1-\beta_c)} \). Therefore, the exact consumer price index can be written as

\[ P = ec(P_n) \]  

The rate of inflation is

\[ \pi = \chi + \gamma \frac{\dot{P}_n}{P_n} \]  

where \( \pi = \frac{\dot{P}}{P} \) is the inflation rate; \( \chi = \frac{\dot{e}}{e} \) is the rate of currency depreciation, and is controlled by the government; \( \gamma = (k_0^{\beta_c} + k_1^{\beta_c} P_n^{1-\beta_c})^{-1} k_1^{\beta_c} P_n^{1-\beta_c} \) is the consumption share of the nontraded good.
In the second stage, the private agent chooses expenditure and money holdings to maximize

\[ \int_{0}^{\infty} \left[ V(P_n, E) + h\phi\left(\frac{m}{c(P_n)}\right) \right] e^{-\rho t} dt \quad (14) \]

subject to

\[ \dot{m} = r_T K_T + r_n K_n + w(L_n + L_T) + T - E - P_k \left[ I_n + \frac{v(\frac{I_n}{K_n} - \delta)^2 K_n}{2} + I_T + \frac{v(\frac{I_T}{K_T} - \delta)^2 K_T}{2} \right] - \chi m \quad (15) \]

\[ \dot{K}_n = I_n - \delta K_n \quad (16) \]

\[ \dot{K}_T = I_T - \delta K_T \quad (17) \]

where \( m = \frac{M}{e} \) is the money balance measured in units of the traded good; \( \phi\left(\frac{m}{c(P_n)}\right) = \frac{\left(\frac{m}{c(P_n)}\right)^{1-1/\tau}}{1-1/\tau} \); \( T \) is the lump-sum transfer from the government; \( I_i \) is the investment in sector \( i \); \( \delta \) is the depreciation rate; \( \frac{v(\frac{I_n}{K_n} - \delta)^2 K_n}{2} \) is the investment adjustment cost function; \( P_k \) is the supply price of capital, measured in units of the traded good. Physical capital is produced by combining 1 unit of an imported machine with \( b_n \) units of nontraded inputs, therefore

\[ P_k = 1 + b_n P_n \quad (18) \]

Equation (15) is the budget constraint, which states that the money holdings increase over time when income exceeds spending on consumption and investment; and equation (16) and (17) represent capital accumulations in the nontradable and tradable sectors, respectively.

The necessary conditions for an optimum consist of

\[ V_E(P_n, E) = \lambda_1 \quad (19) \]

\[ \lambda_2 = \lambda_1 P_k \left( 1 + v \left( \frac{I_n}{K_n} - \delta \right) \right) \quad (20) \]

\[ \lambda_3 = \lambda_1 P_k \left( 1 + v \left( \frac{I_T}{K_T} - \delta \right) \right) \quad (21) \]

\[ \dot{\lambda}_1 = (\rho + \chi) \lambda_1 - h\phi'\left(\frac{m}{c(P_n)}\right) \frac{1}{c(P_n)} \quad (22) \]

\[ \dot{\lambda}_2 = \lambda_2 (\rho + \delta) - \lambda_1 \left[ r_n - P_k \frac{v}{2} \left( \frac{I_n}{K_n} - \delta \right)^2 + P_k v (\frac{I_n}{K_n} - \delta) K_n \right] \quad (23) \]

\[^3\text{The adjustment cost function is necessary to have well defined sectoral investment functions. Without this convex adjustment costs, the solution to the private agent’s optimization problem is of the big-bang variety: investment is } +\infty \text{ in the sector where the capital rental is highest, and } -\infty \text{ in the other sector.}\]
\[ \dot{\lambda}_3 = \lambda_3 (\rho + \delta) - \lambda_1 \left[ r_T - P_k v \left( \frac{I_T}{K_T} - \delta \right)^2 + P_k v \left( \frac{I_T}{K_T} - \delta \right) \frac{I_T}{K_T} \right] \]  

(24)

where \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) are the multipliers attached to the constraints (15), (16) and (17). Equation (19) state that the marginal utility of consumption equals to the shadow price of wealth. Equation (20) and (23) define a Tobin’s q, \( \lambda_2/\lambda_1 P_k \left( 1 + v \left( \frac{I_n}{K_n} - \delta \right) \right) = \lambda_2/V_E P_k \left( 1 + v \left( \frac{I_n}{K_n} - \delta \right) \right) \) is the ratio of the demanded price of capital to its supply price in the nontradable sector; equation (21) and (24) define Tobin’s q in the tradable sector. Equation (22) is simply an Euler equation.

2.3 The Public Sector

The government comprises the fiscal authority and the monetary authority. The balance sheet of the Central Bank is \( \dot{M} = \dot{DC} + e \dot{R} \), so the changes of money supply depends on the domestic credits and the accumulation of foreign exchange reserves \( R \). The government’s flow constraint is given by

\[ \dot{m} = P_z I_z + \dot{T} - r \dot{R} - \chi m + \dot{\Gamma} \]  

(25)

where \( I_z \) is the government investment in infrastructure; \( P_z \) is the supply price of infrastructure, which is given by

\[ P_z = 1 + b_z P_n \]  

(26)

Similar to the private capital stock, the infrastructure is produced by combining 1 unit of an imported machine with \( b_z \) units of nontraded inputs. The rate of infrastructure accumulation is

\[ \dot{Z} = I_z - \delta Z \]  

(27)

As indicated by equation (25), the government has four sources of revenue: interest income on its international reserves; capital gain on its international reserves; money printing, and inflation tax. The government expenditure consists of lump-sum transfers and the cost of infrastructure investment. The government sells foreign exchange reserves to pay for the increase in net infrastructure investment, thus

\[ \dot{R} = -P_z (I_z - \delta Z) \]  

(28)

The consolidated public sector budget constraint is

\[ \dot{m} = P_z I_z + \dot{T} - r \dot{R} - \chi m - P_z (I_z - \delta Z) \]  

(29)

2.4 Market-Clearing Conditions

The temporary equilibrium for the economy is defined by equality of demand and supply in the labor market and the nontraded good market. Both markets clear when

\[ L = L_n + L_T \]  

(30)

\[ Q_n = D_n (P_n, E) + b_n \left[ I_n + \frac{v(\frac{I_n}{K_n} - \delta)^2 K_n}{2} + I_T + \frac{v(\frac{I_T}{K_T} - \delta)^2 K_T}{2} \right] + b_z I_z \]  

(31)
where \( L \) is the fixed supply of labor; \( D_n(P_n, E) \) is the Marshallian demand function for consumer goods. Equation (31) states that the total output in nontradable sector is equal to the demand for nontraded consumption goods plus the demand for nontraded inputs used in producing new capital goods and new infrastructure investment.

### 3 The Dynamic System

The perfect foresight solution to the model generates to a dynamic system with five state variables, \( K_n, K_T, Z, R, \) and \( m, \) and three jump variables, \( E, I_n, \) and \( I_T. \) In order to form a system of differential equations, we need to use the first order conditions obtained from the private agent’s optimization problem. Equation (19) to (24) yield

\[
V_{EE}D_n \left( \frac{V_{En}}{V_{EE}D_n} \hat{P}_n + \frac{1}{D_n} \hat{E} \right) = (\rho + \chi)V_E - h\phi' \left( \frac{m}{c(P_n)} \right) \frac{1}{c(P_n)} \quad (32)
\]

where \( V_{EE} = \partial^2 V/\partial E^2, \) and \( V_{En} = \partial^2 V/\partial E\partial P_n. \) The first order conditions from private agent’s optimization problem yield

\[
\dot{\lambda}_2 = \lambda_1 \left[ P_k(\rho + \delta) \left( 1 + v \left( \frac{I_n}{K_n} - \delta \right) \right) - r_n + P_k v \left( \frac{I_n}{K_n} - \delta \right)^2 - P_k v \left( \frac{I_n}{K_n} - \delta \right) \frac{I_n}{K_n} \right] \quad (33)
\]

\[
\dot{\lambda}_3 = \lambda_1 \left[ P_k(\rho + \delta) \left( 1 + v \left( \frac{I_T}{K_T} - \delta \right) \right) - r_T + P_k v \left( \frac{I_T}{K_T} - \delta \right)^2 - P_k v \left( \frac{I_T}{K_T} - \delta \right) \frac{I_T}{K_T} \right] \quad (34)
\]

In order to solve the time paths for the three jump variables, it is necessary to take into account the induced variations in \( L_n, L_t, \) \( r_n, \) \( r_t, \) \( P_n, \) and \( P_k. \) This involves solving the pseudo-static variant of the model in which the eight endogenous variables are treated as exogenous variables. The solutions are

\[
\frac{\hat{P}_n}{P_n} = \frac{1}{JP_nQ_n} \left[ \gamma \hat{E} + \beta P_k \left( 1 + v \left( \frac{I_n}{K_n} - \delta \right) \right) \hat{l}_n + \beta P_k \left( 1 + v \left( \frac{I_T}{K_T} - \delta \right) \right) \hat{l}_T \right]
\]

\[
+ \left[ \beta P_k v \left( \frac{I_n}{K_n} - \delta \right)^2 - \beta P_k v \left( \frac{I_n}{K_n} - \delta \right) \frac{I_n}{K_n} - r_n - \frac{\theta^1_{1} r_n}{\theta^2_{k} a L_T + a L_n} \right] \hat{K}_n
\]

\[
+ \left[ \beta P_k v \left( \frac{I_T}{K_T} - \delta \right)^2 - \beta P_k v \left( \frac{I_T}{K_T} - \delta \right) \frac{I_T}{K_T} + \frac{\theta^2_{T} r_T a L_T + a L_n}{\theta^1_{k} a L_T + a L_n} \right] \hat{K}_T - \frac{\eta}{Z} P_n Q_n \hat{Z}
\]

\[
\hat{L}_n = \frac{a_3 L_T}{a_3 L_T + a L_n} \hat{K}_n - \frac{a_1 L_T}{a_3 L_T + a L_n} \hat{K}_T + \frac{a_2 a_3 L_T}{a_3 L_T + a L_n} \hat{P}_n \quad (35)
\]

\[
\hat{L}_T = \frac{a_1 L_n}{a_3 L_T + a L_n} \hat{K}_T - \frac{a_3 L_n}{a_3 L_T + a L_n} \hat{K}_n - \frac{a_2 a_3 L_n}{a_3 L_T + a L_n} \hat{P}_n \quad (36)
\]
\[
\hat{r}_n = \frac{1}{\theta_k} \left( 1 + \frac{a_2 L_n \theta_l^n}{a_3 L_T + a_1 L_n} \right) \hat{P}_n + \frac{\theta_l^n}{\theta_k} \frac{L_T}{a_3 L_T + a_1 L_n} \hat{K}_T + \frac{\theta_l^n}{\theta_k} \frac{L_n}{a_3 L_T + a_1 L_n} \hat{K}_n + (1 - \theta_l^n) \frac{\eta}{\theta_k^0} \hat{Z}
\] (38)

\[
\hat{r}_T = \frac{\theta_l^T}{\theta_k} \frac{a_2 L_n}{a_3 L_T + a_1 L_n} \hat{P}_n + \frac{\theta_l^T}{\theta_k} \frac{L_T}{a_3 L_T + a_1 L_n} \hat{K}_T + \frac{\theta_l^T}{\theta_k} \frac{L_n}{a_3 L_T + a_1 L_n} \hat{K}_n + (1 - \theta_l^T) \frac{\eta}{\theta_k^0} \hat{Z}
\] (39)

where \(a_1 = \theta_l^n (\sigma_u^n + \sigma_k^n - 2 \sigma_l^n) < 0, a_2 = \sigma_l^n - \sigma_k^n > 0, a_3 = \theta_l^T (\sigma_u^T + \sigma_k^T - 2 \sigma_l^T) < 0,\) and \(a_4 = \sigma_l^T - \sigma_k^T > 0.\) \(J = \theta_l^n \frac{a_2 a_3}{a_3 + a_2 L_n / L_T} + (\epsilon + \gamma) \frac{P_n}{Q_n}, \beta = \frac{b_a P_n}{P_k}\) is the cost share of the nontradable component in the production of capital goods, \(\epsilon\) is the compensated own-price elasticity of demand for nontradable goods. \(\theta_l^k\) is the cost share of factor \(i\) in sector \(k; \sigma_{ij}^k\) is the Allen partial elasticity of substitution between factors \(i\) and \(j\) in sector \(k.\) A circumflex denotes the percentage change in a variable \((\hat{x} = dx / x).\)

The above results are very intuitive. Equation (35) states that \(P_n\) rises when higher consumption and investment spending strengthen demand, and falls when infrastructure accumulation increases supply. The capital accumulation in both sectors has ambiguous effects on \(P_n,\) but more probably, the capital accumulation in the nontradable sector increases supply and results in a decrease in \(P_n,\) while the capital accumulation in the tradable sector may have negative impact on \(P_n.\) Equation (36) shows that both \(K_n,\) and \(P_T\) have favorable effects on labor demand, as the complementarity between labor and capital is high, and high price level decreases the real cost of labor in the nontradable sector, so the demand for labor increases; while \(K_T\) has the opposite effect on labor demand in the nontradable sector. Equation (37) indicates that similar to the nontradable sector, \(K_T\) has favorable effect on labor demand in the tradable sector; while \(K_n\) and \(P_T\) have the opposite effects. Equation (38) says that the accumulation of \(K_n\) and \(K_T\) will drive down the capital rental in the nontradable sector, but the effect of \(P_n\) on \(r_n\) is ambiguous. However, \(r_T\) decreases with \(P_n, K_n,\) and \(K_T.\) The accumulation of infrastructure stock will result in a higher capital rentals in both sectors.

Use equation (35) to (39), equation (32), (33), and (34) can be solved for \(\hat{E}, \hat{I}_n,\) and \(\hat{I}_T\) as a function of the endogenous variables only.

\[
\hat{E} = f_1(E, I_n, I_T, K_n, K_T, Z, m)\] (40)

\[
\hat{I}_n = f_2(E, I_n, I_T, K_n, K_T, Z, m)\] (41)

\[
\hat{I}_T = f_3(E, I_n, I_T, K_n, K_T, Z, m)\] (42)

The above three equations together with the consolidated public sector budget constraint (equation (29)), and two capital accumulations in both sectors (equation (16) and (17)), and infrastructure accumulation (equation (27)), infrastructure financing scheme (equation (28)) form a system of eight differential equations. It is impossible to derive a closed-form solution analytically, so we will solve the above system of equations numerically.

\[^{4}\text{Uzawa, (1962)}\]
4 Model Calibration and Numerical Solutions

4.1 Calibration

Table 1 lists the parameter values used to calibrate the model.
Table 1: Calibration of the Model

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>( \beta )</td>
<td>0.35</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.5</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.6</td>
</tr>
<tr>
<td>( \beta_c )</td>
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</tr>
<tr>
<td>( \Omega )</td>
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</tr>
<tr>
<td>( \delta )</td>
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<tr>
<td>( \eta )</td>
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<tr>
<td>( \mu )</td>
<td>0.1</td>
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<tr>
<td>( \sigma^m )</td>
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<tr>
<td>( \sigma^T )</td>
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<tr>
<td>( \chi )</td>
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</tr>
<tr>
<td>( \tau )</td>
<td>0.5</td>
</tr>
<tr>
<td>( R_Y )</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Notation

- \( \beta \): cost share of nontraded inputs in the production of capital
- \( \gamma \): consumption share of the nontraded good
- \( \alpha \): cost share of nontraded inputs in the production of infrastructure
- \( \beta_c \): elasticity of substitution between traded and nontraded consumption goods
- \( \Omega \): q-elasticity of investment spending
- \( \delta \): depreciation rate
- \( \rho \): time preference rate
- \( r \): interest rate
- \( \eta \): marginal utility of infrastructure stock
- \( \theta^1_k \): cost share of capital in sector i
- \( \theta^1_l \): cost share of labor in sector i
- \( \mu \): ratio of money balance to expenditure
- \( \sigma^1 \): elasticity of substitution between labor and capital in sector i
- \( \chi \): rate of currency depreciation
- \( \tau \): intertemporal elasticity of substitution
- \( R_Y \): initial ratio of foreign exchange reserves to real GDP

From empirical evidence, the value of \( \beta \) lies between 0.3 to 0.65, but LDCs tend to have lower values of \( \beta \) as the capital market is not quite mature, thus 0.35 is chosen. On the contrary, the value of \( \alpha \) tends to be big in LDCs, and the Chinese government will prefer to use more nontraded inputs in the production of infrastructure. The empirical estimate for the q-elasticity of investment spending is quite low, typically between 0.2 to 2. More recent studies suggest that these numbers underestimate the true elasticity and the real value should be somewhere between 2 to 5, so a slightly larger value is chosen here.\(^5\) The time preference rate is set quite large due to the fact that it determines the net rate of return on private capital. The real interest rate is consistent with a value between the long-term real returns paid by U.S. stocks and treasury bonds. Those returns have been quite low in recent years. The Chinese economy is labor intensive in both sectors, especially so in the tradable sector, thus \( \theta^T_l = 0.65 \) and \( \theta^n_l = 0.55 \) are chosen to reflect this characteristic.

Since the Chinese government stimulus package, the domestic rate of inflation has gone up continuously, so a slightly high rate of currency depreciation is chosen to match this fact. Intertemporal elasticity of substitution tends to be small in developing countries, with an extremely high savings rate in China, a relatively small value of \( \tau \) is assumed. For the rest of the parameter values, the neutral values are chosen. One particular feature of the calibration here is the initial ratio of foreign exchange reserves to real GDP, which is extremely large. It is used to match the Chinese economy, as China held more than $2 trillion foreign exchange reserves

\(^5\)Jovanovic and Rousseau, 2007
by the end of 2009. The value of 0.4 is the average ratio of $\frac{R}{Y}$ from 2000 to 2009. Data used to estimate this value comes from IFS and the People’s Bank of China.

### 4.2 Numerical Solution

At $t = 0$, the Chinese government decides to increase the infrastructure investment by 50% using the foreign exchange reserves, therefore, infrastructure investment jumps up to a higher level. This is an unanticipated, permanent shock. Figure 1 shows the perfect foresight solutions for the base case in the benchmark model.

![Figure 1: Solution Paths for the Endogenous Variables](image)

The solution paths were generated by log-linearizing the model. Mathematica supplies the results for the log linearization and numerical solutions for the transition path.

7 The plots show the percentage difference between all the variables at time $t$ and their initial values at $t = 0$. The dashed line is the percentage change between the new steady state and the old steady state value.
The above solution paths are the core variables directly from the dynamic system. Total expenditure and money holdings are measured in units of traded goods. In order to have a better picture on the effect of the increase in government infrastructure investment, we need to solve for the transition paths for the real variables in the economy.
The Chinese government increases the infrastructure investment at $t = 0$ by surprise, so the infrastructure investment jumps to a higher level at $t = 0$, and stays constant thereafter. Foreign exchange reserves are used to finance the higher investment spending. As expected, foreign exchange reserves fall over time, and the stock of infrastructure increases continuously. However, both $Z$ and $R$ will not change immediately.

By the market clearing condition in the nontradable sector, when government infrastructure increases, a large proportion of output from nontradable sector is taken away from the private sector, therefore, the consumption of nontraded goods decreases, and the investment in the nontradable sector falls, creating a downward pressure on $P_n$ right after the shock. The lower $P_n$ has positive impact on the rental return on the tradable sector as equation (39) shows, so $I_T$ jumps up immediately, while the total investment increases. Inflation goes down immediately, so the real cost of holding money decreases, real money balances increases immediately after the shock.

The plots also show the percentage differences, not in absolute term, except the rate of inflation, which is presented in level term.
On the supply side, the lower level of $P_n$ increases the real cost of capital in the nontradable sector, so the capital inputs decrease, which results in the decrease of output. Different from the nontradable sector, the price for the traded good goes up at a constant rate, so it is more profitable to invest in the tradable sector, $K_T$ increases continuously, output rises in the tradable sector. Even though there exists a contractionary effect on the nontradable sector due to the higher infrastructure investment, the boom in the tradable sector is greater than the bust, real GDP increases over time.

After the implementation of this new policy, perfect foresight private agents know that the government will draw down the reserves to pay for the infrastructure investment, so the government will get rid of the "unwanted" foreign reserves, which will result in the decrease in the nominal money supply. Notice that $\tau$ is small here, so the private agents not only want to smooth their consumption paths, but also would like to hold more real money balances when the nominal interest rate is low. The exchange rate system is predetermined, so the rate of currency depreciation is controlled by the government, which means $e$ will increase at a constant rate, that is 14%. Therefore, the excess demand for money creates an upward pressure on the price level, real money balance goes down right after the initial upward jump.

Productivity starts to rise as $Z$ increases continuously. Expecting higher future income, people want to consume more, total expenditure goes up. Demand for the nontraded goods rises sharply, $P_n$ is temporarily lower. The excess demand for the nontraded goods drives up the price, so $P_n$ starts to rise, inflation goes up. Higher price level in the nontradable sector makes the investment more profitable, so the investment in the nontradable sector rises. On the contrary, investment in the tradable sector decreases partially because the higher $P_n$ reduces the rental return in the tradable sector, and also due to the "competing asset effect". The increase in the price level reduces the real money balances, creating a liquidity shortage, so the marginal utility of money balances rises sharply, which draws savings away from capital accumulation. Facing a severe liquidity shortage and an increasing rental return in the nontradable sector, the private agent may find it optimal to reduce the investment in the tradable sector in order to rebuild the real money stocks. Total investment decreases.

In the later phase of the reform, the continuous increase in $P_n$ reduces the real cost of capital and labor in the nontradable sector, so the capital and labor inputs increase, higher infrastructure stock enhances the productivity, output in the nontradable sector rises. For the tradable sector, after the initial boom, investment falls due to the strong motivation of rebuilding the real money balances. Capital and labor inputs in the tradable sector keep falling, output decreases but still much higher than the original level. Real GDP starts to decrease when capital stock in the tradable sector falls, but still higher than its initial steady state value.

In the long run, higher infrastructure investment increases the productivity and efficiency in the private sector, real expenditure, real money holdings, real GDP, private investment, and capital stocks all rise to a higher level. The rate of inflation returns back to its original level. Because the government finances the higher infrastructure investment by using the foreign exchange reserves, so there is no fiscal deficit along the transition path, the public sector’s budget is balanced. Foreign exchange reserves settle down at a lower level, but there still exists substantial amount of international reserves.

The policy of increasing government infrastructure investment by drawing down the foreign exchange reserves is quite successful in the long run. In a country like China with a thin stock of infrastructure but a huge amount of foreign exchange reserves, this policy increases the
general productivity and efficiency in the private sector. The depreciation of dollars, the lower return from US dollar assets, and the higher return on infrastructure make it profitable for the Chinese government to use those reserves to increase the infrastructure investment without any negative impact on the public sector’s budget. However, this policy is not perfect. As can be seen from the transition paths, there exists an initial long period of lower level of investment, capital stock, and output in the nontradable sector, lower level of real expenditure and money balances, even though there is an initial boom in the tradable sector, and higher real GDP with lower level of inflation initially. The adjustment process is very slow. It takes nearly 40 periods in order for the most variables converge to their higher new steady state level. In order to enjoy the higher output, income, and expenditure, the economy has to go through an initial long period of contraction in the nontradable sector. This is the price that needs to be paid for the implementation of the policy.

5 Sensitivity Analysis

In Figure 3, we carried out runs for an alternative value for $\tau$. Instead of $\tau = 0.5$, we set $\tau = 1.5$, which means the intertemporal substitution is very large and marginal utility of money is small, so there will be less desire to smooth the consumption and accumulate more money.

Figure 3: Solution Paths for $\tau = 1.5$

Only the real variables are reported, not the nominal ones.
With the big value of intertemporal rate of substitution, the private agents do not have strong desire to smooth their consumption path, so the magnitude of the initial prolonged period of contraction in the nontradable sector is much smaller than the benchmark case, even though it still takes a long time for the nontradable sector to recover from the contraction. The transitional paths are quite similar to the case when $\tau = 0.5$, the only difference here is the expansionary
effect is greater, the after shock steady state values are higher. The lowest values of the variables during the bust phase are much smaller. It still takes quite a long period of time for all the variables to converge to their new steady state values. (This part is incomplete, need further revision.)

6 Conclusion

In this paper we have analyzed the effects of increasing the government infrastructure investment by using foreign exchange reserves in a model matching the Chinese economy. The decreasing interest returns of foreign assets and the higher returns from infrastructure investment make it more profitable to draw down the foreign exchange reserves to finance the infrastructure investment. The solution path shows that the higher level of infrastructure increases the general productivity and efficiency of the private sector, making it more productive to invest in both sectors. In the long term, investment, capital stock and output in both sectors, real GDP, real expenditure and money holdings all rise to a higher level, even though there exists a transitory long period of contraction in the nontradable sector. This may be the cost of this policy reform. Inflation goes down on the transition path due to the decrease of price level in the nontradable sector, however, the equilibrium rate of inflation is unchanged. The government effectively pays for the cost of higher infrastructure investment using the foreign exchange reserves without any adverse impact on government budget.

However, the prolonged period of contraction in the nontradable sector suggests that government should use this policy with caution. A stimulus plan for the nontradable sector should be combined with this policy reform to shorten the periods of lower output in the nontradable sector, and reduce the magnitude of contraction.

Labor supply is inelastic in our model, this is not a realistic assumption for the real economy. The main task for future research is to analyze the effects of the same policy under the assumption of elastic labor supply. We hope the changes of labor supply will introduce the wealth effect into the model, so the labor supply and consumption will absorb the effect of the shock together, and the adjustment process will be faster with endogenous labor supply.
References


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