International Trade, Growth and Wage Inequality -
The Role of Human Capital Formation

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Abstract
In a “New Trade” model with endogenous supply of skill, how knowledge diffusion through international trade affects growth and skill premium depends on responsiveness of skill supply to technological change. The less responsive is skill supply, the more likely that lower skill-premium accompanies higher growth after opening to trade.

1 Introduction
This project deals with the question of what effects international trade has in determining economic growth and wage inequality. The answer turns out to depend critically on the responsiveness of skill-supply in a country. If skill supply is highly resistant to inflow of new ideas through international trade, trade tends to increase growth and release high-skilled labor to production sector, lowering wage inequality. However, if skill supply is rather responsive to newly available ideas higher growth rate accompanied by increasing inequality is possible. This theoretical flexibility has the potential to explain the uneven growth in supply and demand for skilled labor in the US in the 1970’s and the rising wage inequality widely observed in developing countries in the last few decades.

In section 2 I build a closed economy expanding variety model with high-skilled labor being the input of R&D research. Growth, however, is only possible if R&D benefit from the externality of ideas available in the economy. This version of the model establish skill supply as a source of growth. In section 3 I allow for skill supply to respond to expected wage income and labor’s innate talent in a Blanchard (1985) and Acemoglu (1998) set up. As a consequence, supply of skill responds positive to growth since in an under high-growth regime, the opportunity cost of forgoing low-skilled wage is smaller. Section 4 puts together R&D growth and skill-supply in a trade model between two symmetric countries in a “new-trade” model in which the demand for traded good comes from love for variety and supply of traded good relies on monopolistic power of
producers. Trade increase static welfare but only diffusion of knowledge has an effect on growth and skill supply. Section 5 concludes.

2 An endogenous growth model with two types of labor

In this section, I set up a Romer-style growth model with expanding variety. At each time period, there is a finite set of intermediate goods produced by producers who set prices monopolistically. Free entry to the market creates incentives for new start-ups to extend the variety of good, which lowers prices and reduces the profit margin of each monopolies. Given the labor cost of research and development (R&D), the free entry condition ensures that monopolistic producers make no profit in the long run. The section establishes a relationship between exogenous high-skilled labor supply and economic growth through R&D activities that employ high-skilled labor. The following description follows Grossman and Helpman (1993)'s closely and adds a twist by including two types of labor inputs in production.

2.1 Consumer

I use a well-known specification for intertemporal preference

$$\max_{C(t)} \int_0^\infty e^{-rt} \log C(t) \, dt$$

subject to a budget constraint

$$p_C(t) [C(t) + \dot{k}(t)] = r(t) k(t) + W(t)$$

The budget constraint reads: At each period $t$, a representative worker earns nominal wage income $W(t)$ and capital gain $r(t) k(t)$ for $r(t)$ is the prevailing nominal interest rate and $k(t)$ is the stock of capital. Wage income $W(t)$ is earned from inelastically supply two types of labor that the worker is endowed with: $L$ units of low skilled labor and $H$ units of high-skilled labor. In this section, $H$ and $L$ are assumed to be fixed. The worker maximizes utility by choosing to allocate this total income into either consumption $C(t)$ or saving $\dot{k}(t)$, taking price $p_C$ as given. As a result

$$r(t) - \rho = \frac{\dot{E}}{E}$$

for nominal expenditure on consumption $E \equiv p_C(t) C(t)$.

I assume that the representative consumer of the economy chooses a finite bundle of intermediate goods indexed $j \in [0, n]$ by the following preference:

$$C = \left[ \int_0^n x(j)^\alpha \, dj \right]^{1/\alpha}$$
where \( x(j) \) is the quantity of good \( j \) consumed and \( \alpha > 0 \) and \( n_t \) is possibly time-variant. Suppose that the nominal expenditure on all goods \( E = \int_0^n x(j) p(j) \, dj \) is normalized to be \( E = 1 \). The derived demand for each good is

\[
x(j) = \frac{p(j)^{-\epsilon}}{\int_0^n p(i)^{1-\epsilon} \, di}
\]

(5)

where \( p(j) \) is the nominal price of good \( j \) and \( \epsilon \equiv \frac{1}{1-\alpha} \) is the elasticity of substitution between intermediate goods \( j \in [0, n] \).

### 2.2 Prices and costs

Given this derived demand, the unit cost of production for the final good is

\[
p_C = \left[ \int_0^n p(i)^{1-\epsilon} \, di \right]^{\frac{1}{\epsilon}}
\]

Since in equilibrium, all intermediate good firms are identical and face identical demand, \( x(j) = x, p(j) = p, \forall j \in [0, n] \), we have

\[
C = n^{1/\alpha} x
\]

(6)

\[
x = \frac{1}{np}
\]

(7)

\[
p_C = n^{1/(1-\epsilon)} p
\]

(8)

The number of varieties \( n \) measures technological level of the economy. The higher is \( n \), the lower quantity of resources \( x \) is needed to produced a unit of final consumption good. Increasing varieties decrease price of each intermediate good and lower overall cost of consumption good.

Equation (3) implies that nominal interest rate is constant:

\[
r(t) = \rho
\]

(9)

### 2.3 Monopolistic competition and innovation

#### 2.3.1 Monopolistic pricing and value of ideas

Each good \( j \) is produced by a monopolistic firm that choose price \( p(j) \) to maximize profit

\[
\pi(j) = p(j) x(j) - c(j) x(j)
\]

\[
= \frac{p(j)^{-\epsilon}}{p_C^{1-\epsilon}} [p(j) - c(j)]
\]

after having solved for the unit cost function \( c(j) \) and taking aggregate price level \( p_C \) as given. Solution to this problem yields optimal price and profit
Combining equations (7) and (12), we have

\[ \pi(j) = \pi = \frac{(1 - \alpha)}{n} \]

By \( v(j) \) I denote the discounted value of an idea that produce good \( j \), that is, the discounted infinite-sum of all profit gains from selling a specific product \( j \). At each time period, this product pays a “dividend” \( \pi(j) \) and helps the owner of the idea earn an amount equal to net change in its value \( \dot{v} \). At the same time, a non-arbitrage condition requires that the total gain of an idea each period equals the return to riskless loan \( rv \), for \( r \) is the prevailing interest rate in the market. Hence

\[ \pi + \dot{v} = rv \]

which leads to

\[ v(t) = \int_t^\infty e^{-\rho(t-\tau)} \pi(\tau) d\tau \]

\[ = \int_t^\infty e^{-\rho(t-\tau)} \frac{(1 - \alpha)}{n(\tau)} d\tau \]

(15)

where \( R(\tau) \equiv \int_0^\tau r(s) ds \).

In equilibrium, interest rate \( r(t) = \rho \) for all \( t \). We have difference equation

\[ \dot{v} = \rho v - \frac{(1 - \alpha)}{n} \]

(16)

### 2.3.2 Free entry condition

I suppose that when a start-up hires \( h \) units of high-skilled labor, it increases the number of ideas \( dn/dt \) by \( h n/b \) units, for some positive constant \( b \). The current number of varieties \( n \) is exploited to measure the externality of the stock of knowledge on innovation: The more types of goods there are in the market, the more productive is investment in R&D. In general, the stock of knowledge may include contribution from other factors than the number of sectors of the economy. One of such factors is the stock of knowledge from another country, as will be explored in the next section. Firms have an incentive to invest in R&D only if the marginal return from new ideas \( \frac{\partial}{\partial n} hv/b \) equals marginal costs of hiring high-skilled labor \( w_h \). Hence

\[ w_h = vn/b \]

(17)

The labor demand for R&D is therefore \( H_{rd} = bn/n \).
2.4 Labor demand for production of intermediate good

All intermediate good $j \in [0, n]$ is produced using technology

$$x(j) = Ah(j)^\beta l(j)^{1-\beta}$$

for $\beta \in (0, 1)$

The unit cost function associated with this technology is

$$c(j) = \frac{1}{A} \beta^{-\beta} (1 - \beta)^{1-\beta} w_h^{\beta} w_l^{1-\beta}$$

for $w_h, w_l$ are high-skilled and low-skilled wages, respectively. Per-unit demand for high-skilled labor

$$h_{pr} = \frac{\partial c(j)}{\partial w_h} = \frac{1}{A} \beta^{1-\beta} (1 - \beta)^{1-\beta} w_h^{\beta-1} w_l^{1-\beta}$$

$$= \frac{1}{A} \left( \frac{w_h}{\beta} \right)^{\beta-1} \left( \frac{w_l}{1-\beta} \right)$$

(19)

2.5 Summary of equilibrium and solution

In an equilibrium with fixed labor supply $H$ and $L$ and technology specification $\alpha, \beta$ and $b$, the following conditions are satisfied:

1. Consumer maximizes intertemporal utility (1) subject to budget constraint (2) by choosing expenditure $E$ and static utility (3) by choosing quantities $x(j), j \in [0, n]$ taking prices $p_C, p(j), j \in [0, n]$ and interest rate $r(t)$ as given.

2. Producer of intermediate goods choose prices $p(j)$ as in equation (11), taking demand (5) and production cost (18) as given, to maximize profit.

3. Start-ups hire high-skilled labor to do research for new ideas, which allow them to enter the market, up to a point where marginal return from ideas equals marginal cost of R&D (17). This condition determine the net change in the total number of varieties at each period.

4. Labor market clear $L = \int_0^n \partial c(j) / \partial w_l dj; H = H_{rd} + H_{pr} = bn/n + \int_0^n \partial c(j) / \partial w_l dj$

I start solving the equilibrium by clearing the labor market in the productions sector

$$\frac{H_{pr}}{L} = \frac{\beta}{1-\beta} \frac{w_l}{w_h}$$

(20)
where $H_{pr}$ is the quantity of high-skilled labor ends up working in production sector. From this cost function and labor demand, equation (11) yields price for intermediate good:

$$p = \frac{c}{\alpha} = \frac{1}{\alpha A} \beta^{-\beta} (1 - \beta)^{1-\beta} w_h w_l^{1-\beta} = \frac{w_h}{\alpha A} \beta^{1-\beta} (1 - \beta)^{1-\beta} w_h^{\beta-1} w_l^{1-\beta} = \frac{w_h}{\alpha A} \left( \frac{L}{H_{pr}} \right)^{\beta-1} = \frac{v_n}{\alpha A b} \left( \frac{L}{H_{pr}} \right)^{\beta-1}$$

Market clearing condition for high-skilled labor

$$H = H_{rd} + H_{pr}$$

$$H = \frac{aH}{n} + \frac{1}{p A} \left( \frac{L}{H_{pr}} \right)^{\beta-1}$$

$$\rightarrow \frac{\dot{H}}{n} = \frac{H}{b} \frac{\alpha \beta}{v_n}$$ (21)

Helpman and Grossman (1993) establish conditions under which a system of differential equations (16) and (21) has a uniquely growing solution in which $\dot{h}/n + \dot{i}/v = 0$. This solution implies

$$\frac{H}{b} + \rho = \frac{\alpha \beta + 1 - \alpha}{v_n}$$

$$\frac{bH}{n} = H - \frac{\alpha \beta (H + \rho b)}{\alpha \beta + 1 - \alpha}$$

$$= H \left[ \frac{1 - \alpha}{\alpha \beta + 1 - \alpha} - \frac{\alpha \beta \rho b}{\alpha \beta + 1 - \alpha} \right]$$ (22)

Under exogenous labor supply, an exogenous increase in relative supply of skill increases technological growth and increases the ratio between high-skilled and low-skilled labor in production sector, leading to lower skill premium. In the end, there is a negative relationship between growth rate and wage inequality.

**Proposition 1** When the skill supply increases exogenously, growth rate increases $\partial g/\partial H > 0$, accompanied by lower wage inequality $\partial (w_h/w_l)/\partial H < 0$.

**Proof.** Equation (22) implies that

$$\frac{\partial g}{\partial H} = \frac{1 - \alpha}{b(\alpha \beta + 1 - \alpha)} > 0$$
Equation (20) implies skill premium as a function of skill supply

\[
\frac{w_h}{w_l} = \frac{\beta}{1 - \beta} \frac{L}{H_{pr}} = \frac{(\alpha \beta + 1 - \alpha) L}{\alpha (1 - \beta) (H + \rho b)}
\]

Hence

\[
\frac{\partial \left( \frac{w_h}{w_l} \right)}{\partial H} = -\frac{(\alpha \beta + 1 - \alpha) L}{\alpha (1 - \beta) (H + \rho b)^2} < 0
\]

3 Endogenous skill formation

In this section, I allow skill supply of the economy to be responsive to wage inequality between low-skilled and high-skilled labor in the economy. The idea is that the labor force is populated by individuals differentiated by ability. Individuals choose to acquire education to become high-skilled labor or otherwise accept wage of a low-skilled labor. The opportunity cost of education decreases with ability because a more talented individual takes less time to become high-skilled labor than a less talented individual.

Suppose that the population of the economy is constant at \( H + L = 1 \). At any instant of time a new generation is born, replacing dying population, both of which have a mass of \( v \in (0,1) \). Typically \( v \) is assumed to be small for mathematical simplicity.

From the individual point of view, the probability of dying \( \tau \) period from now is a function of \( Pr_D(\tau) = ve^{-\nu \tau} \) and the expected life span is \( \int_0^\infty \tau ve^{-\nu \tau} d\tau = v^{-1} \). The larger is \( v \) the shorter is the life span of individuals and the larger is the rate with which young generation replace old generation in the population.

At birth, each individual chooses to be high- or low-skilled labor. If individual \( z \in [0,1] \) chooses to be high-skilled, it will take him \( d(z) \) period acquiring skills, the amount of time he could have spent working as a low skilled labor. Individuals are different by \( d(z) \), the only source of heterogeneity. The larger is \( d(z) \), the less talented is the individual, the larger is the opportunity costs of acquiring skills.

Realistically, the cost of attaining skills depend on educational technology, which take as input more than individual talents. Other factors generally assumed to contribute to education quality are human capital of educator, teaching resources and public provisions of education. All of these factors have direct or indirect links to the stock of knowledge in the economy. At this stage of the project, however, I have not figured out a way to integrate one of such factors. I assume an exogenous distribution of “abilities” that stays unchanged through time.

Suppose that the distribution of individual talent in the population is static through time with the cumulative distribution \( F(d) \).
Discounted for the probability of death, any individual choosing to low-skilled worker has an expected wage income of

\[
R_l (t) = \int_{t}^{\infty} e^{-(\rho + v)(s-t)} w_l (s) \, ds \\
= w_l (t) \int_{t}^{\infty} e^{-(\rho + v)(s-t)} e^{-\frac{1}{\alpha} g(s-t)} \, ds \\
= \frac{w_l (t)}{\rho + v - \frac{1-\alpha}{\alpha} g}
\]  

(23)

where \( g = \frac{n}{n} \).

Similarly, expected discounted value of wage income for an individual with talent \( d \) choosing to be high-skilled worker is

\[
R_h (d; t) = \int_{t+d}^{\infty} e^{-(\rho + v)(s-t)} w_h (s) \, ds \\
= \frac{w_h (t + d)}{\rho + v - \frac{1-\alpha}{\alpha} g} \\
= \frac{w_h (t)}{\rho + v - \frac{1-\alpha}{\alpha} g} \exp \left[ - \left( \rho + v - \frac{1-\alpha}{\alpha} g \right) d \right]
\]  

(24)

Comparing equation (23) and (24) and taking \( w_h / w_l \) as given, an individual with talent \( d \) chooses to be high(low)-skilled if and only if

\[
d < (>) \frac{\log \left( \frac{w_h}{w_l} \right)}{\rho + v - \frac{1-\alpha}{\alpha} g}
\]  

(25)

This condition then determines the skill supply endogenously through distribution \( F (d) \)

\[
\frac{H}{L} = \frac{F (d^*)}{1 - F (d^*)}
\]  

(26)

The labor market clearing condition (20) implies that

\[
\frac{w_h}{w_l} = \frac{\beta}{1 - \beta} \frac{L}{H_{pr}} \\
= \frac{\alpha \beta + 1 - \alpha}{\alpha (1 - \beta)} \frac{L}{H + \rho b}
\]  

(27)

Equations (25) and (27) imply that

\[
\frac{\partial d^*}{\partial (H/L)} < 0; \quad \frac{\partial d^*}{\partial g} > 0
\]

Total differentiate equation (26) yields

\[
\frac{\partial (H/L)}{dg} > 0
\]  

(28)
In equilibrium, higher growth rate decreases the relative opportunity cost of “waiting” for skill-attainment, leading to overall higher return to skill and higher skill supply. This relationship is derived without incorporating any direct benefit of economic resources in education technology.

The balance growth path with endogenous skill supply is specified by the system of equation

\[
g = H \left( \frac{1 - \alpha}{(\alpha \beta + 1 - \alpha) b} - \frac{\alpha \beta \rho}{\alpha \beta + 1 - \alpha} \right) \tag{29}
\]

\[
H = F \left( \frac{(\alpha \beta + 1 - \alpha)(1 - H)}{\alpha(1 - \beta)(\rho + v - \frac{1 - \alpha}{\alpha} g)(H + \rho b)} \right) \tag{30}
\]

I make the following assumptions:

**Assumption 1:** \( F(d) \) is a strictly increasing, continuous function with respect to \( d \).

Under assumption 1, equation (30) establishes \( g \) as a well-defined function of \( H \). Hence, I can rewrite equations (29) and (30), respectively, as

\[
g = g_1(H) \tag{31}
\]

\[
g = g_2(H) \tag{32}
\]

**Assumption 2:** For \( H_1^* \) and \( H_2^* \) solving \( 0 = g_1(H_1^*) \) and \( 0 = g_2(H_2^*) \), respectively

\[
\frac{\alpha \beta \rho b}{1 - \alpha} = H_1^* < H_2^* = F \left( \frac{(\alpha \beta + 1 - \alpha)(1 - H_2^*)}{\alpha(1 - \beta)(\rho + v - \frac{1 - \alpha}{\alpha} g)(H_2^* + \rho b)} \right) \tag{33}
\]

**Assumption 3:** For \( \bar{g} \equiv g_1(1) = \frac{1 - \alpha - \alpha \beta \rho}{(\alpha \beta + 1 - \alpha) b} \), there exists a value \( \bar{H} \in [0, 1] \) that satisfies

\[
\bar{g} = g_2(\bar{H}) \tag{34}
\]

**Assumption 4:** For all \( H \in [0, 1] \)

\[
\frac{dg_2(H)}{dH} > \frac{dg_1(H)}{dH} = \frac{1 - \alpha}{(\alpha \beta + 1 - \alpha) b} \tag{35}
\]

**Proposition 2** Under assumptions 1-3, there exists a stable balance growth path.

**Proof.** Define function \( \gamma(H) \equiv g_1(H) - g_2(H) \). Since both \( g_1 \) and \( g_2 \) are continuous, so is \( \gamma \).

Since both \( g_1 \) and \( g_2 \) are increasing functions, assumption 2 implies that \( \gamma(H_2^*) > 0 \) and assumption 3 implies that \( \gamma(\bar{H}) < 0 \). The intermediate value theorem postulates that there exists a value \( H_e \in [H_2^*, \bar{H}] \) such that \( \gamma(H_e) = 0 \).

**Proposition 3** Under assumptions 1-4, the balance growth path is unique.
Proof. Assumption 4 implies that \( \gamma'(H) = g'_1(H) - g'_2(H) < 0, \forall H \in [0, 1] \). But \( \gamma(H_e) = 0 \) by proposition 1. Hence \( \gamma(H) > 0, \forall H < H_e \) and \( \gamma(H) < 0, \forall H > H_e \).

Assumptions (29) and (30) ensure that there exist a unique solution \((H, g)\) satisfying \( g \geq 0; H \in [0, 1] \). They also make sure that, on a \((H, g)\) diagram, equation (30) intersects equation (29) from below. This latter condition needs to be satisfied so that a more advanced economy where equation (29) yield higher growth rate \( g \) for any value of \( H \) ends up with higher growth rate than a less advanced economy with equation (29) specified by lower growth rate \( g \).

In equilibrium, the higher is the turn-over rate \( v \) of population, the lower is the return to skill. For higher value of \( v \), the economy grows slower and supplies less high-skilled labor.

I demonstrate equilibrium characteristic by an example in which the cost of innovation \( b \) increases. In Figure 1. The curves \( E_1, E'_1 \) are associated with equation (29) before and after an increase in \( b \). \( E'_1 \) lies below \( E_1 \) because for higher cost of innovation, the same amount of skill induces slower growth rate. The curves \( E_2, E'_2 \) depicts equation (30) before and after an increase in \( b \). Higher cost of innovation reduces labor supply available for production of consumption good, raising skill premium, leading to higher supply of skill for each given growth rate. As a consequence, equilibrium shifts from points \((H_0, g_0)\) to point \((H_1, g_1)\). An increase in innovation cost reduces growth and increases inequality.
4 Trade and diffusion of knowledge from abroad

Diffusion of knowledge and flow of resources through national borders by the mean of trades are different conceptually. Realistically, however, diffusion of knowledge are generally not possible without active trade in physical goods. Diffusion of knowledge are embodied in the physical flow of traded good. In this section I extend the growth model above to a traded model between two symmetric but distinct countries. There is a demand for imported good due to love for varieties. This is the source of welfare-improvement of trade. However, trade does not increase growth rate of an economy unless it carries along new ideas beneficial to R&D activities. I then consider the effect of this diffusion of knowledge on growth and wage inequality through the responsiveness of skilled labor supply. To my surprise, unlike the case when skill supply is assumed to be fixed exogenously, the ability of the labor force to adjust supply of skill complicates the effect of new ideas to a trading economy. On the one hand, they make R&D more productive. On the other hand, they act as a “multiplier” freeing high-skilled labor to production activities, leading to lower skill premium and lower supply to skill. Qualitative results hence depend crucially on the responsiveness of skill supply.

4.1 Trade with no diffusion of ideas

In a trade model with two similar countries, countries $A$ and $B$ are identical in autarky and are specified as in section 1. When they are allowed to trade with each other, consumers in each country find imported good as attractive as a newly created domestic good. There is now overlapping between the two sets of goods of the two. Hence the immediate impact of trade on consumers is to expand variety.

Consumers of two countries have the same taste. Suppose total their nominal expenditure is

$$E = E^A + E^B$$

As in section 1 I normalize $E = 1$. Suppose that in equilibrium, each monopolistic producer in country $i = A, B$ charge price $p^i$ to any customer. Since firms of two countries are identical and there are no trade costs, $p^A = p^B$ in equilibrium. Given these prices, the budget share of any household of the world to goods produced in country $i$ is

$$s^i = \frac{n^i (p^i)^{1-\epsilon}}{n^B (p^B)^{1-\epsilon} + n^B (p^B)^{1-\epsilon}}, i = A, B$$

Hence sales per firm and profit

$$x^i = \frac{s^i}{n^i p^i}$$

$$\pi^i = \frac{(1-\alpha) s^i}{n^i}$$
The no-arbitrage condition becomes

\[ \dot{v}^i = \rho v^i + \frac{(1 - \alpha) s^i}{n^i} \quad (36) \]

I suppose that there is no knowledge spill-over so that R&D in each country depends entirely on its current stock of idea. The labor market clearing condition becomes

\[ \frac{\dot{n}^i}{n^i} = \frac{H^i}{b} - \frac{\alpha \beta s^i}{v^i n^i} \quad (37) \]

Equations (36) and (37) yield solution to growth rate

\[ \frac{b \dot{n}^i}{n^i} = H^i - \frac{\alpha \beta (H + \rho b)}{(\alpha \beta + 1 - \alpha)} \]

which is identical to growth rate if the two countries have been otherwise in autarky.

The benefit of trade to a country can only be realized in the increase in level of production and consumption in each period due to increases in variety. Without any direct effect on the R&D process, trade does not affect overall economic growth. The next subsection demonstrate how diffusion of knowledge induces change in economic growth and wage inequality.

### 4.2 Diffusion of ideas, growth and wage inequality.

The effect of opening domestic innovation to ideas from abroad has an effect analogous to a cut in innovation cost \( b \). As a consequence, diffusion of knowledge will increase economic growth and increases high-skilled labor supply, which in turn lower wage inequality.

In this section, I suppose that the two countries \( A \) and \( B \) are as described in the last subsection. Now, not only they trade freely with each other, ideas also follow trade to enhance R&D in each country. World’s stock of idea is

\[ n^W = n^i + \delta^j n^j, i, j = A, B \quad (38) \]

Where \( \delta^i < 1 \) is a factor discounting for the fact that ideas flowing from country \( j \) to country \( i \) does not have the same positive effect on innovation as domestically developed ideas. In a symmetric world trade equilibrium \( n^A = n^B \), hence \( \delta \) is the same for both countries.

This increases in the number of ideas available lowers the costs of producing idea of idea in each country. Equation (17) becomes

\[ v^i = \frac{w^i \delta}{n^i + \delta n^j} \quad (39) \]

\[ v^i n^i = \frac{n^i}{n^i + \delta n^j} w^i \delta \quad (40) \]

Price of each good in the world market is:
\[ p^i = \frac{v^i (n^i + \delta n^j)}{\alpha \beta Ab} \left( \frac{L^i}{H_{pr}} \right)^{\beta - 1} \]

\[ = \frac{v^i n^i (n^i + \delta n^j)}{\alpha \beta Ab} \left( \frac{L^i}{H_{pr}} \right)^{\beta - 1} \quad (41) \]

Market clearing condition for high-skilled labor in each country

\[ \frac{\dot{n}^i}{(n^i + \delta n^j)} = \frac{H^i}{b} - \frac{\alpha \beta}{v^i (n^i + \delta n^j)} \]

\[ H^i = \frac{n^i}{(n^i + \delta n^j)} b \left[ g + \frac{\alpha \beta}{v^i n^i} \right] \quad (42) \]

The non-arbitrage equation

\[ -g^i = \rho - \frac{(1 - \alpha)}{n^i v^i} \]

I now consider the consequences of trade opening to growth and wage inequality.

In equilibrium, \( g^A = g^B \) hence

\[ n^i v^i = \frac{1 - \alpha}{\rho + g} \]

\[ H^i + \delta H^j = b \left[ g + \frac{\alpha \beta (\rho + g)}{1 - \alpha} \right] \]

\[ g = \frac{(1 + \delta) (1 - \alpha) H - \alpha \beta \rho b}{b (\alpha \beta + 1 - \alpha)} \quad (43) \]

Equation (30) becomes

\[ H = F \left( \frac{(\alpha \beta + 1 - \alpha) (1 - H)}{\alpha (1 - \beta) (\rho + v - \frac{1 - \alpha}{\alpha} g) ((1 + \delta) H + \rho b)} \right) \quad (44) \]

The autarky specified in the last section is a special case of the open economy model specified by equations (43) and (44) when \( \delta = 0 \).

I now consider a comparative static in which \( \delta \) declines by a small amount as depicted in figure 2. Equation (29) shifts downward: thanks to less exposure to new ideas from a broad, R&D becomes less productive given any fixed endowment of domestic skill. Equation (44) shifts to the right: lower demand for consumption good shifts high-skilled labor out of the production sector, leading higher skill-premium and higher supply of skill.

Graphically, the two curves specifying the balance growth path moves in the same direction as a consequence of less trade. Since exact functional form of
\( F(d) \) is not specified, unambiguous results with respect to qualitative changes in growth rate \( g \) and skill supply \( H \) cannot be drawn.

Figure 2 is only one possible case. In this case, responsiveness of skill formation in relative increase in productivity of R&D is strong, pulling down growth rate with it. Hence, both growth rate and equality decreases as a result of reducing an economy’s exposure to international trade. But this is only one extreme case. If skill-supply does not react strongly as likely be the case in develop countries where the R&D sector is small, skill responsiveness is relative weaker. Then lower supply of skill can be accompanied by lower growth rate.

5 Conclusion

To understand the effects of international trade on the dynamic of the labor market it is crucial to understand how skill-formation take place. Even in the absence of direct spillover of international knowledge on the skill supply of a country, trade still has an indirect impact on the supply of skill through allocation of skilled labor resources between R&D and production sector. From the policy point of view, human-capital policies will have implication on whether international trade enhances growth and reduces wage inequality.

REFERENCE:
