Inflation Persistence and Incredible Disinflation in a Small Open Economy

[Incomplete Version]

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Abstract

In this paper, we investigate the effects of incredible disinflation policy under the circumstances that disinflation policy has a short-run inflation-output tradeoff due to inflation persistence. Under the ERBS system, incredible policies with and without inflation inertia produce similar transition paths (small boom-deep bust-recovery cycles) and magnitudes, but the case of inertial inflation has the shortened cycle. Under the inflation targeting, incredible policies produce immediate recession-recovery during the stabilization phase and the case of persistent inflation has relatively larger effects on real variables. Incredible IT can show the opposite cycle of consumption paths compared to ERBS regardless of inflation persistence, but this result relies on the existence of durables.

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Keywords: inflation inertia, exchange-rate-based stabilization, inflation targeting, incredible monetary policy

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1 Introduction

While there is huge volume of literature related to exchange rate based stabilization (ERBS)
in both theoretical and empirical aspects, it seems that there is relatively little literature
on the policy effects of inflation targeting in the context of developing and less developed
countries which are most small open economies. Many of those countries have adopted or
considered adopting inflation targeting as their monetary policy regime and used the interest
rate policy as an operating tool. Considering that those countries’ policy actions easily tend
to lose the public’s confidence, this paper investigates the macroeconomic effects of incredible
disinflation under both ERBS and inflation targeting in a small open economy in the context
of developing and less developed countries.

As is well recognized in incredible ERBS systems, the credibility of the central bank’s com-
mitment plays an important role in influencing transitional dynamics of important macroe-
conomic variables. Incredible ERBS systems can produce empirically observable boom-bust
cycles mainly based on the intertemporal substitution mechanism. On the other hand, Calvo
(2007) shows that incredible inflation targeting based on interest rate rules can lead to the
opposite results (high real interest rates and output loss over the first stages of the program).

This paper mainly compare the effects of weak credibility under different monetary policy
regimes in a quantitative manner using various set-ups. Since Calvo & Vegh (1993) origi-
nally investigated the role of weak credibility under the ERBS system, the literature trying to
quantitatively explain boom-bust cycles has focused on flexible price systems.\footnote{Calvo & Vegh (1993) and Calvo & Vegh (1994) used the original sticky price model specified in Calvo (1983). Recently, Celasun (2006) introduced the sticky inflation model to the temporary ERBS model. In his model, the inflation stickiness or persistence is caused by some specification of backward-looking pricing. For the comprehensive review about inflation persistence, see Gorodnichenko (2008).} Considering
many LDC’s and developing countries suffer from chronically high inflation, we pay special
attention to the role played by inflation persistence. Specifically, we introduce inertial infla-
tion dynamics to the nontradable sector by Calvo, Celasun & Kumhof (2007). In their model
in which firms optimize under forward-looking staggered price setting, even credible disin-
flation displayed a delayed and gradual inflation response and strong output losses. While
this model produces short-term inflation-output tradeoff, the economy enjoys long-term gain
- i.e. output expansion. [More detailed literature review to-be-described]

In this paper, we try to answer whether the modeling strategy with inflation persistence
can improve the explanatory power of traditional weak credibility hypothesis surrounding
ERBS episodes.\footnote{boom-bust in real output, huge increase in consumption spending, slow adjustment in inflation, real
exchange appreciation, high real interest rate etc.} We also try to figure out how different results weakly credible disinflation
policy can produce?

We briefly present the main results concerning incredible disinflation policy under in-
flation persistence on the nontradable sector. The central bank holds a bit strong policy
stance in terms of the magnitude of policy rule changes (ie. 40% points decreases in the
depreciation rate or 10 % points decrease in the domestic inflation target). Since the pricing
scheme stresses out the short-term inflation-output tradeoff by construction, even ERBS sys-
tems with weak credibility shows fairly weak boom period in the first phase of the program. ERBS programs with or without inflation inertia produce similar transition paths - ie. small boom-deep bust-recovery cycles. Inflation targeting with or without inflation inertia enter into recession immediately after the policy announcement and recover during the stabilization period. In ERBS case, inflation inertia matters for the length of cycles. The case of inertial inflation has the shortened cycle. In inflation targeting case, inflation inertia seems to have relatively larger effects on (nondurable) tradable and nontradable consumption. In the contrary to Calvo (2007)’s analysis, the model economy can experience sharp recovery even over the initial steady state level at the time of policy reversal. In other words, incredible IT can show the opposite cycle of consumption paths compared to ERBS regardless of inflation persistence, but this result heavily relies on the existence of durables in this model.

The rest of the paper is organized as follows. Section 2 lays out the model and Section 3 discusses calibration assumptions and numerical methods. Section 4 present numerical solutions. Section 5 concludes.

2 Model

2.1 Economy

In a small open economy, there are an infinitely-lived representative household, a tradable good manufacturing firm, a continuum of monopolistically competitive nontradable goods manufacturing firms, and a government. Both the household and the tradable producing good firm are price-takers, while nontradables producing firms set prices following Calvo-pricing.

The economy is perfectly integrated with the rest of world in a tradable good market whose price is given by the law of one price. The world price is assumed to be one. Therefore, the domestic price of a tradable good is simply nominal exchange rate ($E_t$). The uncovered interest parity is also assumed to be hold, so that the economy can freely buy or sell international risk-free real bonds ($b_t$) from or to the rest of the world:

$$i_t = r + \varepsilon_t,$$

where $i_t$ is the domestic nominal interest rate, the interest rate on the traded bond is assumed to be fixed at $r$ and $\varepsilon_t (\equiv \dot{E_t}/E_t)$ is the domestic currency depreciation rate.

2.2 Firm

There are two types of firms in this economy. Both the representative tradable producer and the heterogeneous nontradables producers employ labor as production inputs. The labor market is competitive and labor is mobile between two sectors.
2.2.1 Tradable Good Firm

A competitive tradable good producer maximizes its profit \((Z^T_t)\) at any point in time\(^3\):

\[
Z^T_t \equiv E_t Q^T(l^T_t) - W_t l^T_t,
\]

where \(Q^T(\cdot)\) is the firm’s production technology, \(l^T_t\) is employment in the tradable good sector and \(W_t\) is a nominal wage. The tradable good firms is owned by the representative household.\(^4\)

2.2.2 Nontradables Firms

Nontradable good manufacturing firms are distributed uniformly along the unit interval \((j \in [0, 1])\) and produce their outputs by the production technology:

\[
y^n_t(j) = F(l^n_t(j)),
\]

where \(l^n_t(j)\) is employment in the nontradables sector.

The period nominal profit of nontradable good firm \(j\) is given by

\[
Z^n_t(j) \equiv P^n_t(j) y^n_t(j) - W_t l^n_t(j)
\]

where \(P^n_t(j)\) is a price of good \(j\).

The nontradable goods market is monopolistically competitive. Following Dixit and Stigliz (1977), the aggregate nontradable goods consumption \(c^n_t\) is given by a CES aggregator:

\[
c^n_t = \left( \int_0^1 c^n_t(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
\]

with \(\sigma > 1\), and the aggregate nontradables price index \(P^n_t\) is given by

\[
P^n_t = \left( \int_0^1 P^n_t(j)^{1-\sigma} \right)^{\frac{1}{\sigma-1}}.
\]

The demand of each nontradable good \(j\) satisfy

\[
c^n_t(j) = c^n_t \left( \frac{P^n_t(j)}{P^n_t} \right)^{-\sigma}.
\]

Following Calvo, Celasun and Kumhof (2007)\(^5\), we assume that each nontradable good manufacturing firm can change its price and the updating rate at the time when a price-change signal\(^6\) is received. When a firm receive a price-change signal, it determines today’s

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\(^3\)From now on, the upper \(T\) stands for the tradable sector and the upper \(n\) stands for the nontradable sector.

\(^4\)The firm’s maximization problem will be combined directly into the household’s optimization problem later.

\(^5\)We closely follow Calvo, Celasun and Kumhof (2007) here. Unlike them in which a logarithm utility function and a linear production function are used, we introduce more general functional forms. For the quantitative analysis, we use CES-CRRA utility function and CES production function. However, the basic derivation is almost the same.

\(^6\)As usual in Calvo contracts, the probability of receiving a price-change signal \(s - t\) periods from time \(t\) \((s > t)\) is given by \(\delta e^{-\delta(s-t)}\), where \(\delta\) is the number of signals received per unit of time.
price level \( V_t^j \) and update the firm specific inflation rate according to its optimal choice as of time \( t \) (\( v_t^j \)).\(^7\) Therefore, until another price is received, the firm specific price \( (P^n_s(j)) \) at time \( s > t \) is set according the following pricing policy

\[
P^n_s(j) = V_t^j e^{v_t^j(s-t)},
\]

(8)

Note that since the nontradable good inflation converges to the domestic inflation target (depreciation rate) that the central bank sets under the inflation targeting (ERBS), the final steady state is equivalent to the inflation target (depreciation rate). While the nontradable goods producers with Calvo-Yun pricing schemes set their updating rule optimally. firms choose their updating rule by monitoring the central bank’s explicit target, producers with Calvo-Celasun-Kumhof pricing schemes set their updating rule optimally.

Specifically, nontradables manufacturing firms maximize the present discounted value of real future profits in the following manner:

\[
\max_{V_t^j, v_t^j} \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} \left[ \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right] y_s(j) - w^n_s l^n_s(j) \right] ds,
\]

(9)

subject to each nontradable good producer’s production function (3) and nontradable good demands (7), where \( w^n_s(\equiv W_t/P^n_t) \) is the real wage in terms of nontradable goods and \( U_n(t) \equiv U^n(c^n_t, c^n_t) \). Note that \( e^{-\rho(s-t)} U_n(s)/U_n(t) \) is the intertemporal marginal rate of substitution between two periods \( (s > t) \). Using nontradable goods market clearing condition \((y^n_t(j) = c^n_t(j))\) and the inverse form of the production function \((l^n_t(j) = F^{-1}(y^n_t(j)))\), firms’ maximization problem can be written as:

\[
\max_{V_t^j, v_t^j} \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} c^n_s \left[ \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{1-\sigma} - w^n_s F^{-1} \left( \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{-\sigma} c^n_s \right) \right] ds.
\]

(10)

Then we have the following first-order conditions for \( V_t \) and \( v_t \):

\[
\begin{align*}
(\sigma - 1) & \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} c^n_s \left[ \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{1-\sigma} ds
\end{align*}
\]

\[
= \sigma \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} c^n_s \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{-\sigma} w^n_s F^{-1} \left( \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{-\sigma} c^n_s \right) ds,
\]

(11)

\[
(\sigma - 1) \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} c^n_s \left[ \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{1-\sigma} (s-t) ds
\end{align*}
\]

\[
= \sigma \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} c^n_s \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{-\sigma} (s-t) w^n_s F^{-1} \left( \left( \frac{V_t^j e^{v_t^j(s-t)}}{P^n_s} \right)^{-\sigma} c^n_s \right) ds,
\]

(12)

where the term \( w^n_s F^{-1} \) can be seen a marginal cost \( (mc^n) \) for a firm \( j \).\(^8\)

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\(^7\)Note that, unlike the commonly used Yun (1996)’s pricing scheme which modified Calvo (1983), firms here do not update the firm specific inflation rate at the steady state. Instead, firms choose their updating rule optimally. For comparison, we show results derived by using Yun (1996)’s pricing scheme.

\(^8\)Since we don’t assume a homogenous production function here, the marginal cost depends on an output as well as prices.
The maximization problem is identical for all nontradable manufacturing firms that receive a price-change signal. Therefore, we can proceed without the firm index \( j \). Define \( \mu \equiv \sigma / (\sigma - 1) \) and \( p^n_t \equiv V_t / P^n_t \) which is the relative price of a nontradable good for a new price setter. Note that, for \( s > t \), \( P^n_t = P^n_t e^{\int_t^s \pi dz} \), where \( \pi_t \equiv P^n_t / P^n_t \). Multiply both sides of the first-order conditions by \( V_t \). Then the equations (11) and (12) can be rewritten as:

\[
\int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} e^n_s (p^n_t e^{-\int_t^s (\pi_s - \psi) dz})^{1-\sigma} ds \\
= \int_t^\infty e^{-(\delta + \rho)(s-t)} \frac{U_n(s)}{U_n(t)} e^n_s (p^n_t e^{-\int_t^s (\pi_s - \psi) dz})^{1-\sigma} \mu w^n s_F^{-1}((p^n_t e^{-\int_t^s (\pi_s - \psi) dz})^{1-\sigma} c^n_t) ds. \tag{13}
\]

Meanwhile, as usual in Calvo pricing, by appealing to the 'law of large numbers', we can take some algebraic manipulations, we obtain the following expressions

\[
\frac{1 + \sigma \chi}{\delta + \rho} \hat{p}^n_t + \frac{1 + \sigma \chi}{(\delta + \rho)^2} \hat{v}_t = \int_t^\infty e^{-(\delta + \rho)(s-t)} \left[ (1 + \sigma \chi) \int_t^s \hat{\pi}_dz + \frac{\hat{\psi}_n}{w^n} + \xi \frac{e^n_t}{c^n_t} \right] ds, \tag{15}
\]

\[
\frac{1 + \sigma \chi}{(\delta + \rho)^2} \hat{p}^n_t + \frac{2(1 + \sigma \chi)}{(\delta + \rho)^3} \hat{v}_t = \int_t^\infty e^{-(\delta + \rho)(s-t)} (s-t) \left[ (1 + \sigma \chi) \int_t^s \hat{\pi}_dz + \frac{\hat{\psi}_n}{w^n} + \xi \frac{e^n_t}{c^n_t} \right] ds. \tag{16}
\]

where \( \chi \equiv \frac{1}{e^{1-n} \mu_y} (\frac{\partial m}{\partial \mu} \frac{\partial y^n}{\partial \mu} \frac{y^n}{m \mu^n}) \) is the elasticity of the marginal cost with respect to the nontradable good output. Differentiating equations (15) and (16) with respect to time \( t \) and taking some algebraic manipulations, we obtain the following expressions

\[
\hat{p}^n_t + \frac{1}{\delta + \rho} \hat{v}_t = \left( \delta + \rho \right) \hat{p}^n_t + \hat{v}_t - \hat{\pi}_t - \frac{\delta + \rho}{\delta + \sigma \chi} \frac{\hat{\psi}_n}{w^n} - \frac{\delta + \rho}{1 + \sigma \chi} \xi \frac{e^n_t}{c^n_t}, \tag{17}
\]

\[
\hat{p}^n_t + \frac{2}{\delta + \rho} \hat{v}_t = \hat{v}_t - \hat{\pi}_t. \tag{18}
\]

Meanwhile, as usual in Calvo pricing, by appealing to the 'law of large numbers', we can express the aggregate price index (6) into the following form:

\[
P^n_t = \left( \delta \int_{-\infty}^t e^{-\delta(t-s)} \left( V_s e^{n_s(t-s)} \right)^{1-\sigma} ds \right)^{1/\sigma}. \tag{19}
\]

When we differentiate equation (19) with respect to time \( t \) and linearize the resulting expression around the steady state, we have

\[
\hat{\pi}_t = \delta \hat{p}_t + \hat{\psi}_t, \tag{20}
\]

where

\[
\hat{\psi}_t \equiv \int_{-\infty}^t \delta e^{-\delta(t-s)} \hat{\psi}_s ds. \tag{21}
\]

Note that \( \psi_t \) is a predetermined variable representing the weighted average of firm-specific inflation rates which are still alive. [comparison with Calvo-Yun pricing and source of inflation inertia \rightarrow here]
After some algebraic manipulations of equations (20) and (21), the resulting expressions with equations (17) and (18) are reduced to the following nontradable good pricing block consisting of $\pi^t_n$, $v_t$ and $\psi_t$

\[
\dot{\pi}_t = 2\delta \dot{v}_t + (\delta + 2\rho)\dot{\pi}_t - (3\delta + 2\rho)\dot{\psi}_t - \frac{2\delta(\delta + \rho)\dot{v}_t^n}{1 + \sigma \chi} \frac{\dot{w}_t^n}{w^n} - \frac{2\delta(\delta + \rho)\chi \dot{c}_t^n}{1 + \sigma \chi} e^n, 
\]

\[
\dot{v}_t = -\frac{(\delta + \rho)^2}{\delta}(\pi_t - \dot{\psi}_t) + \frac{(\delta + \rho)^2 \dot{w}_t^n}{1 + \sigma \chi} \frac{\dot{c}_t^n}{c^n} + (\delta + \rho)^2 \chi \dot{c}_t^n \frac{\dot{c}_t^n}{c^n},
\]

\[
\dot{\psi}_t = \delta \dot{v}_t - \delta \dot{\psi}_t
\]

For the later comparison with CCK pricing scheme, Calov-Yun pricing scheme is reduced to

\[
\dot{\pi}_t = \rho \dot{\pi}_t - \frac{\delta(\delta + \rho) \dot{w}_t^n}{1 + \sigma \chi} \frac{\dot{c}_t^n}{c^n}.
\]

2.3 Household

We assume that the representative household consumes three types of consumption goods: tradable non-durable goods ($c_t^0$), tradable durable goods ($D_t^T$)\(^9\) and nontradable goods ($c_t^n$). The household possesses an instantaneous utility function of the form $U(c_t^0, c_t^n) + H(D_t^T) - \Psi(L_t^s) - R(S_t^T/D_t^T - c)D_t^T$, where $c_t^0$ and $c_t^n$ constitute composite non-durable consumption ($C_t$), $S_t^T$ is a new purchase of durable, and $D_t^T$ is the stock of durables. $R(\cdot)D_t^T$ component is introduced to impose a friction that constrain a volatility of durable purchases.\(^10\) The total labor supply ($L_t^s$) is endogenously determined. By the total labor market equilibrium condition, the household is assumed to allocate the total labor supply to tradable and nontradable sectors ($L_t^s$ and $L_t^n$).

The representative household chooses \(\{c_t^0, c_t^T, S_t^T, D_t^T, m_t, b_t, L_t^s, l_t^n\}_{t=0}^{\infty}\) to maximize a discounted sum of utilities

\[
\int_{0}^{\infty} [U(c_t^0, c_t^n) + H(D_t^T) - \Psi(L_t^s) - R(S_t^T/D_t^T - c)D_t^T]e^{-\rho t} dt,
\]

subject to the wealth constraint

\[
a_t = m_t + b_t,
\]

\(^9\)ERBS literature including papers focusing on weak credibility has been able to explain only a small fraction of actual boom-bust cycles in terms of quantitative magnitudes. Recently, Buffie & Atolia (2007) and Buffie & Atolia (2009) produced paths consistent with empirical stylized facts by introducing durable consumptions to their model. Three main factors behind Atolia & Buffie (2009)’s success are durables goods for both tradable and nontradable sectors, the discrete devaluation at the time of policy reversal, and the imperfect capital market. Although introducing durables to both sectors is more balanced approach, we only deal with tradable durables in this paper for simplicity.

\(^{10}\)This type of cost can be interpreted as a deliberation cost. The adjustment cost is assumed to be increasing, symmetric, and convex in net purchases of durable goods: $R(0) = 0$, $R' \geq 0$ as $D_t^T \geq 0$, and $R'' > 0$. For a detailed explanation, see Buffie & Atolia (2007) and Buffie & Atolia (2009).
the flow budget constraint
\[
\dot{a}_t = ra_t + Q^T(L_t^s - l_t^n) + \frac{w^n l^n}{e_t} + \frac{\int_0^1 Z^n(j)dj}{E_t} + \tilde{g}_t - \left( c_T^t + S_T^t + \frac{c^n_t}{e_t} \right) \left[ 1 + L \left( \frac{m_t}{c_T^t + S_T^t + \frac{c^n_t}{e_t}} \right) \right] - \int_t m_t, \tag{28}\]
and the law of motion for a durable stock
\[
\dot{D}_t^T = S_T^t - cD_t^T, \tag{29}\]
where \(e_t \equiv E_t/P_t^n\) is a real exchange rate and \(Z^n_t(j)\) is the nominal profit of the nontraded good producing firm \(j\). Domestic currency \((m_t \equiv M_t/E_t)\) is held to reduce transaction costs \(((c_T^t + S_T^t + c^n_t)/e_t)L[m_t/(c_T^t + S_T^t + c^n_t/e_t)])\).\(^{11}\) Lump-sum transfers consist of real government transfer and rebates from firms providing liquidity services \((\tilde{g}_t = g_t + (m_t/(c_T^t + S_T^t + c^n_t/e_t)))L[m_t/(c_T^t + S_T^t + c^n_t/e_t)])\). As usual in a small open economy where the uncovered interest parity holds, we assume that the household’s time preference rate \((\rho)\) is the same as the world interest rate \((r)\). \(c\) is a depreciation rate of the durable good.

The present value Hamiltonian is given by
\[
H = e^{-\rho t} \left\{ U(c_t^T, c_t^n) - \Psi(L_t^s) + \lambda_t \left[ ra_t + Q^T(L_t^s - l_t^n) + \frac{w^n l^n}{e_t} + \frac{\int_0^1 Z^n(j)dj}{E_t} + \tilde{g}_t - \left( c_T^t + S_T^t + \frac{c^n_t}{e_t} \right) \left[ 1 + L \left( \frac{m_t}{c_T^t + S_T^t + \frac{c^n_t}{e_t}} \right) \right] - \int_t m_t \right] + \phi_t[S_T^t - cD_t^T] \right\}. \tag{30}\]
where \(\lambda_t\) is a multiplier associated with the flow budget constraint, \(\phi_t\) is a multiplier associated with the law of motion for durable.

The optimization problem yields the standard necessary conditions:
\[
\dot{\lambda}_t = \lambda_t(r - \rho) = 0 \quad (\Rightarrow \lambda_t = \lambda), \tag{31}\]
\[
U_{c_t^T}(c_t^T, c_t^n) = \lambda(1 + L(X_t) - X_tL'(X_t)), \tag{32}\]
\[
\frac{U_{c_t^n}(c_t^T, c_t^n)}{U_{c_t^T}(c_t^T, c_t^n)} = e_t, \tag{33}\]
\[
-L'(X_t) = i_t, \tag{34}\]
\[
\Psi'(L_t^s) = \lambda Q_t^{T'}(l_t^T), \tag{35}\]
\[
Q_t^{T'}(l_t^T) = \frac{w^n_l}{e_t}, \tag{36}\]
\[
\phi_t = \lambda(1 + L(X_t) - X_tL'(X_t)) + R'(S_t^T/D_t^T - c), \tag{37}\]
\[
\dot{\phi}_t = \phi_t(\rho + c) + R(S_t^T/D_t^T - c) - R'(S_t^T/D_t^T - c)S_t^T/D_t^T - H'(D_t^T), \tag{38}\]
\(^{11}\)The liquidity cost \((L)\) is decreasing and strictly convex in the ratio of liquidity services \((m_t)\) to total spending \((L' < 0, L'' > 0)\).
where \(X_t \equiv m_t/(c_t^T + S_t^T + c_t^i/e_t)\) which is the inverse of expenditure based money velocity. In equation (31), \(\lambda_t\) is a constant due to the assumption of non-growing stationary path of consumption. Equation (32) implies that the marginal utility of the numerarie good (tradable good) equals the shadow price of wealth times the effective price of consumption \((1 + L(X_t) - X_tL'(X_t))\). Equations (33) state that the marginal rate of substitution between (non-durable) tradable and nontradable consumptions equals their relative price which is the real exchange rate. Equation (34) says that the opportunity cost of money (marginal liquidity cost) must be the same as the nominal interest rate. Equation (35) with equation (32) requires that the marginal rate of substitution between leisure and consumption equal the marginal product of labor which is a real wage in terms of the tradable good. Equation (36) says that labor costs in both tradable and nontradable sectors are the same. Equations (37) and (38) present Tobin’s \(q\) model of durables purchases.\(^{12}\)

### 2.4 Government

The flow budget constraint of the consolidated public sector is given by

\[
\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - g_t, \tag{39}
\]

where \(h_t\) is the net foreign asset held by the government.

In this paper, we assume that fiscal policy is passive in the sense that the government transfer \(g_t\) adjusts to offset changes in interest income and the revenue from the inflation tax \((rh_t + \varepsilon_t m_t)\).\(^{13}\)

Household’s and firm’s problems are the same for both ERBS and inflation targeting cases which we will deal with. The main difference between the two cases comes from the specifications of the monetary policy. In the ERBS system, the exchange rate is a nominal anchor while an inflation target plays a role of the nominal anchor in the inflation targeting. Following the literature considering temporariness in ERBS, the length of stabilization policies are exogenously given.\(^{14}\)

\(^{12}\)If fiscal policy does not accommodate the net revenue change using the transfer payment (i.e. fixed transfer), the regime with this kind of unsustainable fiscal policy is easily susceptible to the balance of payments (BOP) crises. In this paper, we do not deal with this crisis. There is large literature related to the BOP crises. In the context of the ERBS system, see Calvo & Vegh (1999). For the BOP crises in the context of inflation targeting, see Kumhof, Li, & Yanal (2007).

\(^{13}\) Unlike models with exogenously given duration of policies, there is a literature focusing on uncertain duration of policies. For example, Calvo & Drazen (1998) showed that policies characterized by uncertain duration can generate consumption booms. Mendoza & Uribe (2000, 2001) introduced uncertain duration to their model by using a time-varying devaluation risk and endogenously determined probability of devaluation.
2.4.1 ERBS

The economy is initially in a steady state consistent with a higher level of a constant depreciation rate $\varepsilon_h$. At time 0, the central bank announces that it maintain a lower rate of crawl permanently. However, we assume that the public knows that the government reverse this policy back to the original one at time $t_1$.

\[
\varepsilon_t = \begin{cases} 
\varepsilon^l & 0 \leq t < t_1 \\
\varepsilon^h & t \geq t_1 
\end{cases}
\]  

(40)

2.4.2 Domestic Inflation Targeting

The central bank sticks with the strict inflation targeting in the sense that it implements its monetary policy by Taylor-rule-type interest rate feedback rule which is a function of inflation. It sets the nominal interest rate according to the following rule:

\[
i_t = R(\pi_t),
\]

(41)

where $R(\cdot)$ is continuous, non-decreasing and strictly positive, and there exists at least one $\pi^* > -\rho$ such that $R(\pi^*) = \rho + \pi^*$. For simplicity, the inflation target is set against the domestic inflation rate ($\pi$) and the output gap is not considered for now.

["CPI inflation targeting" and "flexible target considering output" to-be-described]

The economy is initially in a steady state consistent with a higher level of an inflation target rate $\pi^h$. At time 0, the central bank announces that it decrease an inflation target rate to a lower rate $\pi^l$ permanently. However, we assume that the public knows that the government reverse this policy back to the original one at time $t_1$.

\[
\pi^* = \begin{cases} 
\pi^l & 0 \leq t < t_1 \\
\pi^h & t \geq t_1
\end{cases}
\]  

(42)

In order to understand the incredibility of the inflation targeting regime, we can interpret the credibility as the proximity of private sector inflation expectations.\textsuperscript{16} Since private sector expects that the target set by the central bank will not last forever, private agents’ behavior will be different from the perfect credibility case. Therefore, regardless of whether the policy is reversed at time $T$, actual inflation dynamics will be expected to be different from that of the policy which is gaining credibility from the private sector.

\textsuperscript{15}For simplicity, we consider only the change of the rate of crawl at the time of policy reversal ($t_1$). However, as Buffie & Atolia (2009) reports, many ERBS systems collapse with a large discrete devaluation of their currencies which will strengthen the intertemporal substitution effects. See the table 1 in Buffie & Atolia (2009) for devaluation examples in ERBS programs.

\textsuperscript{16}For example, Svensson (2008) points out that "private-sector expectations of inflation affect current pricing decisions and inflation for the next few quarters. Therefore, the anchoring of private sector inflation expectations on the inflation target is a crucial precondition for the stability of actual inflation."
2.5 Equilibrium

The nontradables market for all goods and the labor market clear at all times:

\[ g_t^n(j) = c_t^n(j) \quad \forall t, \forall j \in [0, 1], \quad (43) \]
\[ l_t^n = \int_0^1 l_t^n(j) \, dj \quad \forall t. \quad (44) \]

For the inflation targeting case, the domestic market nominal interest rate is the same as the interest rate the central bank sets:

\[ i_t = i_t^p. \quad (45) \]

The flow budget constraints of the household and the government with equilibrium conditions imply that the following resource constraint must hold:

\[ \dot{k}_t = r k_t + Q^T(T_t^l) - c_t^T - S_t^T, \quad (46) \]

where \( k_t \equiv b_t + h_t \) is the consolidated foreign asset holdings of the government and the household.

3 Calibration

3.1 Assumption

We assume the following functional forms for technology, preference, labor supply, durable adjustment cost, and liquidity cost:

\[ Q^T(T_t^l) = (k_1 + k_2 l_t^n)^{\frac{n-1}{n}} \eta^{-1}, \quad (47) \]
\[ F(l_t^n(j)) = (k_3 + k_4 l_t^n)^{\frac{n-1}{n}} \eta^{-1}, \quad (48) \]
\[ U(c_t^T, c_t^n) = \left[ \omega \frac{1}{\gamma} (c_t^T)^{\frac{1}{\gamma}} + (1 - \omega) \frac{1}{\gamma} (c_t^n)^{\frac{1}{\gamma}} \right]^{\frac{\gamma-1}{\gamma}} (1 - \frac{1}{\tau})^{1 - \frac{1}{\tau}}, \quad (49) \]
\[ H(D_t) = a_2 \frac{D_t^{1 - \frac{1}{\nu}}}{1 - \frac{1}{\nu}}, \quad (50) \]
\[ \Psi(L_t^d) = a_1 (L_t^d)^{\nu}, \quad \nu > 1 \quad (51) \]
\[ R \left( \frac{S_t^T}{D_t^T} - c \right) = \frac{x}{2} \left( \frac{S_t^T}{D_t^T} - c \right)^2, \quad x > 0, \quad (52) \]
\[ L(X_t) = h X_t^{1 - \frac{1}{\beta}}, \quad h > 0, \quad 0 < \beta < 1 \quad (53) \]
\[ i^p(\pi) = r + \pi^* + \alpha_\pi (\pi_t - \pi^*), \quad (54) \]

where \( X_t \equiv m_t / (c_t^T + S_t^T + c_t^n / e_t) \), \( k_1 \sim k_4 \) are distribution parameters and \( \eta, \tau, \) and \( \gamma \) denote, respectively, the elasticity of substitution between labor and a fixed input, the intertemporal
elasticity of substitution, the elasticity of substitution between trade and nontraded (non-
durable) goods. Firms operate with CES production functions. Utility from tradable and
nontradable (non-durable) goods has a form of CES-CRRA function and utility from (trad-
able) durable good services has an isoelastic functional form. Disutility from total labor is
increasing and convex. The adjustment cost function for durable goods is quadratic. Total
liquidity costs \((c^T_t + S^T_t + c^n_t / e_t)L(X_t)\) are a convex Liquidity cost function of total expend-
diture and real balances. The specification of the interest rate rule is a linear function of the
deivation of the domestic inflation rate from the target.

Our baseline parameter values are described below. All parameter value are on a yearly
basis and most values are selected relying on the literature focusing on ERBS episodes (eg. see Buffie and Buffie (2007), Buffie and Atolia (2009), Reinhart and Vegh (1995), Rebelo and
Vegh (1995)).

- Length of stabilization \(t_1\). The low depreciation rate lasts three years following the
literature. For the inflation targeting case, there is no available episode in the literature.
For the comparison purpose, the low inflation target is assumed to last for three years
equivalent to ERBS cases.

- Time Preference rate \(\rho\): 0.1. Since we have developing or less developed countries
in mind, this value may be well above figures for developed countries. This value is
assumed to be the same as the fixed world market interest rate \(r\).

- Elasticity of substitution between tradable and nontradable consumption goods \(\gamma\):
0.5

- Elasticity of substitution between nontradable goods varieties \(\sigma\): The value of 6 for
\(\sigma\) implies that the markup rate is 20% \(\mu \equiv \sigma / (\sigma - 1) = 1.2\)

- Elasticity of intertemporal substitution \(\tau\): 0.25

- Elasticity of substituteion between capital and labor \(\eta\): 0.5

- Number of price change signals per unit time \(\delta\): We set the average duration of pricing
schemes \(1/\delta\) at 1.

- Convexity of the transactions cost function \(\beta\): 0.4

- Share of tradable (nondurable) good consumption in total (nondurable) consumption
expenditure \(\omega\): 0.5. With the normalization of \(c^T_t = c^n_t = 1\) evaluated at the initial
steady state, this implies the initial steady state level of the real exchange rate \(e_t\) is
one.

- Sectoral cost shares for labor \(\theta_{lT}, \theta_{ln}\): We assume that nontradable manufacturing
sector shows higher labor intensity. \(\theta_{lT}\) is set as 0.4 and \(\theta_{ln}\) at 0.6. With the normalization
of consumptions and labor market conditions, this will fix the values of distribution
parameters of production functions \(k1 \sim k4\).
• Ratio of real money balances to total consumption \( \frac{m_t}{(c_t^T + S_t^T + c_t^n/e_t)} \): 0.2 evaluated at the initial steady state.

• Elasticity of labor supply \( 1/(\nu - 1) \): For \( \nu = 2 \), the elasticity of labor supply with respect to the real wage in term of tradable good is unity.

• Depreciation rate of the durable good \( c \): This value is assumed to be 0.1.

• Elasticity of durables spending with respect to Tobin’s q \( \Omega \): 10 evaluated at the initial steady state. This value with the depreciation rate of the durable good determines the degree of convexity of the durable good adjustment cost function \( R(\cdot) \).

• Tradable consumption share of durables \( \theta_S \): The share of tradable durables in aggregate tradable consumption spending \( \frac{S_t^T}{c_t^T + S_t^T} \) is assumed to be 0.2 evaluated at the initial steady state.

• Inflation targets for inflation targeting \( \pi^* \): the value during stabilization period \( \pi^l, t \in [0, t_1] \) is 0.1. The initial value of the inflation target and the value after the policy reversal \( \pi^h, t \in [t_1, \infty) \) is 0.2.

• Depreciation rates for ERBS \( \bar{\varepsilon} \): the value during stabilization period \( \varepsilon^l, t \in [0, t_1] \) is 0.1. The initial value of the depreciation rate and the value after the policy reversal \( \varepsilon^h, t \in [t_1, \infty) \) is 0.5.

• Initial level of consolidated foreign asses holdings \( k_0 \): This value is assumed to be zero.

• Monetary policy stance \( \alpha_\pi \): The simple Taylor rule parameter value is 1.5.

### 3.2 Summarized dynamic system

For further reference, we collects the conditions which characterize the dynamic equilibrium. The dynamic system contains variables \{\pi_t, v_t, \psi_t, c_t^T, c_t^n, Z_t, S_t^T, D_t^T, e_t, m_t, i_t, l_t^T, l_t^n, w_t, f_t, \phi_t, \lambda\} and consists of the following equations:

**Pricing blocks:**

\[
\dot{\pi}_t = 2\delta \hat{v}_t + (\delta + 2\rho)\pi_t - (3\delta + 2\rho)\psi_t - \frac{2\delta(\delta + \rho)\hat{w}_t^n}{1 + \sigma\chi} - \frac{2\delta(\delta + \rho)\chi \hat{c}_t^n}{1 + \sigma\chi}, \quad (55)
\]

\[
\dot{v}_t = -\frac{(\delta + \rho)^2}{\delta}(\pi_t - \psi_t) + \frac{(\delta + \rho)^2\hat{w}_t^n}{1 + \sigma\chi} + \frac{(\delta + \rho)^2\chi \hat{c}_t^n}{1 + \sigma\chi}, \quad (56)
\]

\[
\dot{\psi}_t = \delta \hat{v}_t - \delta \hat{\psi}_t, \quad (57)
\]
Nontradable Market Clearing:

Household:

\[ c_t^{D} = \frac{c_0}{1 - \omega} - \gamma \frac{\omega}{1 - \omega}, \quad (58) \]

\[ c_t^{T} = \left( \frac{\omega^{\frac{1}{\gamma}} Z_t^{\frac{1}{1-\gamma}}} {\lambda(1 + k_i t_1^{1-\beta})} \right)^{\gamma}, \quad (59) \]

\[ Z_t = \omega^{\frac{1}{\gamma}} (c_t^{T})^{\frac{1}{\gamma}} + (1 - \omega)^{\frac{1}{\gamma}} (c_t^{n})^{\frac{1}{\gamma}}, \quad (60) \]

\[ \lambda Q^T(t^T_t) = a_1 \nu (l_t^T + l_t^n)^{-1}, \quad (61) \]

\[ w_t^n = \frac{Q^T(l_t^T) e_t}{\nu}, \quad (62) \]

\[ \phi_t = \frac{1}{\gamma} c_t^{T} - \gamma Z_t^{\frac{1}{\gamma}} + x \left( \frac{S_t^T}{D_t^T} - c \right), \quad (63) \]

\[ \dot{\phi}_t = \phi_t (\rho + e_t) + \frac{x}{2} \left( \frac{S_t^T}{D_t^T} - c \right) - x \left( \frac{S_t^n}{D_t^n} - c \right) \frac{S_t^T}{D_t^T} - a_2 \frac{D_t^T}{D_t^n} - \gamma, \quad (64) \]

Nontradable Market Clearing:

\[ c_t^n = \left[ \int_0^1 F(l_t^n(j)) \frac{8}{\sigma - 1} \right]^{\frac{\sigma - 1}{\sigma}}, \quad l_t^n = \int_0^1 l_t^n(j) dj, \quad (65) \]

Current account:

\[ \dot{k}_t = r k_t + Q^T(l_t^T) - c_t^T - S_t^T, \quad \lim_{t \to \infty} \dot{k}_t = 0, \quad (66) \]

Monetary policy rule:

\[ i_t = r + \pi^* + \alpha_\pi (\pi_t - \pi^*), \quad (67) \]

Real exchange rate:

\[ \dot{e}_t = (\alpha_\pi - 1)(\pi_t - \pi^*) e_t, \quad (68) \]

where \( k = \frac{h^p}{\beta^p (1 - \beta)^p} \) and \( \chi \equiv \frac{F^{-1} y^*}{F - y^*} (\partial m c^\alpha / \partial y^*) y^* / m c^\alpha \) is the elasticity of the marginal cost with respect to the nontradable good output. The equation denoting real exchange rate evolution is derived from the definition of the real exchange rate, the uncovered interest rate parity, and the specification of the interest rate rule. Note that tradable consumption equation (59) is obtained by combining the marginal utility of tradable consumption (32) and the liquidity opportunity cost (34) with functional assumptions.\(^{17}\) Note that equation (63) and (64) governing durable goods dynamics are connected with other equations with the marginal utility of tradable consumption.\(^{18}\)

\(^{17}\)Since we can express the effective price of consumption \((1 + L(X_t) - X_t L'(X_t)), \) where \( X_t \equiv m_t / (c_t^T + S_t^T + c_t^n / e_t) \) as a term with nominal interest \((1 + k_i t_1^{1-\beta}) \), where \( k = h^p / (\beta^p (1 - \beta)^{p-1}) \), we proceed without terms related to real money balances \((m_t) \) when we solve for the model.

\(^{18}\)Since we assume that there are durable goods only in the tradable sector and that utility functions for nondurable and durable goods are separable, the effects of durable goods dynamics on other variables arise through those on the shadow price of wealth which are relatively limited.
For the ERBS stabilization case, the nominal interest equation and the real exchange rate equation are replaced by

\[ i_t = r + \bar{\varepsilon}, \]  
(69)

\[ \dot{e}_t = (\bar{\varepsilon} - \pi_t)e_t. \]  
(70)

3.3 Solution Method

Since it is not affordable to present a closed form solution, we heavily rely on computational methods. We provide detailed solution procedures in the Appendix. The basic solution method is related to the nested reverse shooting.\(^{19}\) Since we assume a perfect capital market in which an exogenous world market interest rate is fixed and equivalent to the household’s time preference rate, there is a unit root problem.\(^{20}\) Therefore, in order to compute the transition paths of model variables, we must solve for steady states and transition paths simultaneously.\(^{21}\)

Meanwhile, since the main models possesses the characteristic of temporary shocks, we also use two different systems of differential equations which governs dynamics before and after the policy reversal. Therefore, using optimality conditions and some continuity conditions, we derive discrete jumps of jump variables at the time of policy reversal ($t = t_1$).

4 Numerical Solutions

In this section, we present numerical solutions based on models and calibration assumptions described in section 2 and 3. Before we provide the main results related to incredible policies, we first present credible disinflation policies for a comparison reason.

\(^{19}\)Although we intend to solve for non-linear saddle path problems using numerical methods, it may be not affordable to maintain the nonlinearity of the pricing block in which equations are involved with two individual positive and negative infinite integral terms. In this paper, while we use linearized equations for the pricing block, we maintain the nonlinearity for other equations. *The use of this kind of mixed linear and nonlinear system need to be justified. Some papers using this strategy will be cited later.*

\(^{20}\)For detailed explanation about the unit root problem, see Atolia & Buffie (2009c) and Schmitt-Grohe & Uribe (2003).

\(^{21}\)Using an inside (nested) loop over the lagrange multiplier associated with the household budget constraint ($\lambda$), we pin down the value of $\lambda$ for each guess for a small perturbation around the terminal steady state which also varies according to each round of the inside loop. When the convergence of $\lambda$ is ensured, the terminal steady state can be pinned down. For the outside loop over a perturbation around the terminal steady state, we use a manual guess and verify strategy instead of automated algorithm which ensures a high degree of accuracy and reliability. To develop an automated reverse shooting algorithm which easily accommodate a unit root case will be fruitful for this research area. For recent developments of automated algorithm using shooting methods, see Atolia & Buffie (2009b) for the forward shooting and Atolia & Buffie (2009a) for the reverse shooting.
4.1 Credible Disinflation

Figure 1∼4 present the solution paths of model variables for credible disinflation policies for both ERBS system and inflation targeting with/without inflation persistence. Figure 1 shows credible ERBS system with Calvo-Yun pricing scheme which has no inflation persistence, figure 2 credible ERBS system with Calvo-Celasun-Kumhof pricing scheme which has inflation persistence, figure 3 credible inflation targeting with Calvo-Yun pricing scheme, and figure 4 shows credible inflation targeting with Calvo-Celasun-Kumhof scheme.

Under the assumptions of Calvo-Yun pricing, both the ERBS system and inflation targeting show a quite similar pattern. In the inflation targeting, there is no state variable in a core dynamic system (which does not include durable variables). Therefore, all variables except durable variables immediately jumps to the new steady states. In the ERBS system, initial jumps are very close to the new steady state and the transitions show almost flat paths.

However, when we introduce inflation persistence, transitions for both the ERBS and IT show quite huge variations and domestic inflation rate itself shows staggering movements. In both cases, there is a quite strong short-run inflation-output trade-off due to a huge fall in nontradable consumption despite an increase in tradable consumption. Initial responses from the ERBS system and inflation targeting are different from each other. In the ERBS system, those recession-recovery process follows almost continuous path from the initial impact because the real exchange rate is a state variable. However, in the inflation targeting, the real exchange rate appreciates quite strongly at impact and the nontradable sector goes into deeper recession immediately. Then, the economy starts to recover gradually.

4.2 Incredible Disinflation

Figure 5∼8 present the solution paths of model variables for incredible disinflation policies with weak credibility. Figure 5 shows incredible ERBS system without inflation persistence, figure 6 incredible ERBS system with inflation persistence, figure 7 credible inflation targeting with Calvo-Yun pricing scheme, and figure 8 shows credible inflation targeting with Calvo-Celasun-Kumhof pricing scheme.

While credible disinflation policies produce long-run positive output effects, temporary (failed) disinflation policy under both exchange rate based stabilization program and inflation targeting do not benefit this kind of long-run disinflation gain.

Since the pricing scheme stresses out the short-term inflation-output tradeoff by construction, even ERBS systems with weak credibility shows fairly weak boom period in the first phase of the program. ERBS programs with or without inflation inertia produce similar transition paths - ie. small boom-deep bust-recovery cycles. Inflation persistence seems to matter for the length of cycles. The case of inertial inflation has the shortened cycle especially for non-durables. (ie. There is a very short period of boom in CCK pricing compared to Calvo-Yun pricing scheme.) This effect is reduced when we introduce durables.

In the inflation targeting, the economy enters into recession immediately after the policy announcement and recover during the stabilization period. In inflation targeting case, inflation inertia seems to have relatively larger effects on (nondurable) tradable and non-
tradable consumption. Since the nominal exchange rate can discretely change on impact, the real exchange rate need not be constrained to be predetermined as in the ERBS case and appreciates immediately. Furthermore, disinflation policy under the circumstances with inflation persistence can easily make both the nominal and real interest rate increase if the central bank sets the nominal interest rate according to 'active' Taylor-rule type interest rate feedback rule. In this case, the real economy centering on the nontradable sector can go into a deep recession phase. In case of incredible inflation targeting, the calibrated model shows that the total consumption level goes over the initial steady state when the policy is reversed. This is mainly because interest rate levels decrease at the time of policy reversal which reduces effective price of consumption. However this kind of overshooting effect diminishes soon.

5 Conclusion

Incredible disinflation policy under inflation persistence shows only weak boom-bust cycle and even shortened the boom phase. For these reasons, the pricing scheme with inflation persistence can hardly be used to improve the explanatory power of traditional weak credibility hypothesis surrounding ERBS experiences except for the slow adjustment of inflation rate. Meanwhile, in case of incredible disinflation under inflation targeting, the effects on real variable can be larger during the transition path due to higher interest rates driven by the combination of inertial inflation and active interest rate policy. However, the magnitude of this effect is only limited. As mentioned before, one last interesting point is that inflation targeting can produce the opposite cycle of consumption variables. In this model, this is only possible when we introduce durable goods which is highly sensitive to the relative price - the effective price of consumption.

[Concluding remarks, limitations and future research direction to-be-described; currently, distribution cost is being combined]
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Credible ERBS: Calvo-Yun Pricing\textsuperscript{22}

\begin{equation*}
\varepsilon
\end{equation*}

\begin{equation*}
\pi
\end{equation*}

\begin{equation*}
\rho^n
\end{equation*}

\begin{equation*}
i
\end{equation*}

\begin{equation*}
i - \pi
\end{equation*}

\begin{equation*}
\rho^n
\end{equation*}

\begin{equation*}
C'
\end{equation*}

\begin{equation*}
C''
\end{equation*}

\begin{equation*}
C
\end{equation*}

\begin{equation*}
\rho^n
\end{equation*}

\begin{equation*}
\rho
\end{equation*}

\begin{equation*}
L'
\end{equation*}

\begin{equation*}
S
\end{equation*}

\begin{equation*}
D
\end{equation*}

\begin{equation*}
\text{RealCon}
\end{equation*}

\begin{equation*}
\text{RealGDP}
\end{equation*}

\text{Figure 1: Credible disinflation under ERBS: Calvo-Yun Pricing}

\text{\textsuperscript{22}} \rho^n_t \text{ is the relative price of nontradable goods in dollar terms } (1/\varepsilon_t). \text{ Real consumption is calculated by } c^n_t + S_t^n + \rho_t^n c^n_t. \text{ Real GDP is calculated by } Q(l^n_t) + \rho^n_t c^n_t.
Credible ERBS: Calvo-Celasun-Kumhof Pricing

Figure 2: Credible disinflation under ERBS: CCK Pricing
Credible IT: Calvo-Yun Pricing

Figure 3: Credible disinflation under inflation targeting: Calvo-Yun Pricing
Figure 4: Credible disinflation under inflation targeting: CCK Pricing
Figure 5: Incredible disinflation under ERBS: Calvo-Yun Pricing
Incredible ERBS: Calvo-Celasun-Kumhof Pricing

Figure 6: Incredible disinflation under ERBS: CCK Pricing
Incredible IT: Calvo-Yun Pricing

Figure 7: Incredible disinflation under inflation targeting: Calvo-Yun Pricing
Figure 8: Incredible disinflation under inflation targeting: CCK Pricing
Appendix

Appendix A. Solution Procedure

In this appendix, we outline the algorithm we use to compute the solution paths of the model with temporary shocks. As mentioned in the main text, we use a nested reverse shooting. To handle a unit root problem, we nest a inside loop for pinning down the lagrange multiplier λ into an outside loop which ensures that the initial values of state variables derived from solving the model should be the same as true initial values of state variables within some tolerance level. For the outside loop over a perturbation around the terminal steady state, we use a manual guess and verify strategy.

Due to the temporary shocks, we derive transition paths using two different systems of differential equations which governs dynamics before and after the policy reversal. To connect two different systems, we must calculate discrete jumps of jump variables at the policy reversal (t = t1). In doing so, we use optimality conditions and continuity conditions which will be explained below. I describe only cases with Calvo-Celasun-Kumhof pricing because Calvo-Yun pricing cases are relatively simpler.

When we apply numerical methods, we use the following working version of the equations system for ERBS system

\[ \dot{x} = 2\delta \dot{v}_t + (3\delta + 2\rho) \dot{\pi}_t - (3\delta + 2\rho) \dot{\psi}_t - \frac{2\delta(\delta + \rho) \dot{w}_t}{1 + \sigma \chi} \frac{w_t^\rho}{w_t} - \frac{2\delta(\delta + \rho) \chi \dot{c}_t}{1 + \sigma \chi} \frac{c_t^\rho}{c_t}, \]

\[ \dot{v}_t = -\frac{(\delta + \rho)^2}{\delta} (\dot{\pi}_t - \dot{\psi}_t) + \frac{(\delta + \rho)^2 \dot{w}_t}{1 + \sigma \chi} \frac{w_t^\rho}{w_t} + \frac{(\delta + \rho)^2 \chi \dot{c}_t}{1 + \sigma \chi} \frac{c_t^\rho}{c_t}, \]

\[ \dot{\psi}_t = \delta \dot{v}_t - \dot{\psi}_t, \]

\[ \frac{\dot{c}_t^r}{c_t^r} = \frac{\dot{c}_t^r}{c_t^r} - \gamma \frac{\dot{c}_t^r}{c_t^r}, \]

\[ \frac{\dot{c}_t^r}{c_t^r} = \gamma(\gamma - \tau) \dot{Z}_t - \gamma(\gamma - \tau) \dot{Z}_t, \]

\[ \dot{Z}_t = \omega^\frac{1}{n} \left( \frac{\gamma - 1}{\gamma} \right) (c_t^r)^{-\frac{\gamma - 1}{\gamma}} \frac{\dot{c}_t^r}{c_t^r} + \omega^\frac{1}{n} \left( \frac{\gamma - 1}{\gamma} \right) (c_t^r)^{-\frac{\gamma - 1}{\gamma}} \frac{\dot{c}_t^r}{c_t^r}, \]

\[ \dot{c}_t = (\bar{x} - \pi_t) \epsilon_t, \]

\[ \lambda Q''(l_t^\nu) \dot{l}_t^\nu = a_1 \nu(\nu - 1)(l_t^\nu + l_t^\nu)^{\nu-2}(l_t^\nu + l_t^\nu), \]

\[ \dot{w}_t = c_t Q^{T''}(l_t^\nu) l_t^\nu + Q^{T''}(l_t^\nu) \dot{c}_t, \]

\[ \dot{c}_t^p = F'(l^\nu) l_t^\nu, \]

\[ \dot{D}_t^p = S_t^p - cD_t^p, \]

\[ x \frac{S_t^p}{D_t^p} = \omega^\frac{1}{n} c_t^p - \frac{1}{\gamma} Z_t^{\frac{\gamma - 1}{\gamma}} (\rho + c) + x \left( \frac{S_t^p}{D_t^p} - c \right) (\rho + c) + \frac{x}{2} \left( \frac{S_t^p}{D_t^p} - c \right)^2 - a_2 D_t^p - \frac{1}{\gamma}, \]

where \( \chi \equiv \frac{F''(1)^n}{F'(1)l^\nu} (\partial m_c^n / \partial y^n) y^n / mc^n \) is the elasticity of the marginal cost with respect to the nontradable good output evaluated at steady states.
In case of the inflation targeting, equations (75), (77) and (82) are replaced by
\[
\frac{c_t^{T}}{c_t^{T}} = \gamma \left( \frac{(\gamma - \tau)}{\tau(1 - \gamma)} Z_t^{T} - \frac{k(1 - \beta)\alpha\pi_t}{(1 + k(r + \pi^* + \alpha(\pi_t - \pi^*)^{1 - \beta})(r + \pi^* + \alpha(\pi_t - \pi^*))^{\beta}} \right),
\]
(83)
\[
\dot{c}_t = (\alpha_{\tau} - 1)(\pi_t - \pi^*)e_t,
\]
(84)
\[
\frac{S_t^{T}}{D_t^{T}} = \omega\frac{c_t^{T}}{c_t^{T}} Z_t^{(\gamma - \tau)} \left( (\rho + c) - \frac{(\gamma - \tau)}{\tau(1 - \gamma)} \dot{Z}_t + \frac{1}{\gamma} \frac{c_t^{T}}{c_t^{T}} \right) + x \frac{S_t^{T}}{D_t^{T}} - c \right) (\rho + c)
\]
\[
+ \frac{x}{2} \left( \frac{S_t^{T}}{D_t^{T}} - c \right)^2 - a_2 D_t^{T - \frac{1}{2}}.
\]
(85)

where \( k = \frac{b}{\alpha\gamma(1 - \beta)^{1 - \gamma}} \).

In order to pin down \( \lambda \) in the process of deriving transition paths, we use the remaining two conditions related to the current account
\[
\dot{k}_t = rk_t + Q^T(t_l^T) - c_t^T - S_t^{T},
\]
(86)
\[
\lim_{t \to -\infty} \dot{k}_t = 0.
\]
(87)

Given the initial consolidated net foreign asset holding, we can derive the life-time budget-constraint of the economy
\[
f_0 + \int_0^{\infty} Q^T(t_l^T)e^{-rt}dt = \int_0^{\infty} (c_t^T + S_t^{T})e^{-rt}dt.
\]
(88)

We use the economy’s resource constraint in deriving iterated values of \( \lambda \) for each run of the inside loop.

A1. ERBS with Calvo-Celasun-Kumhof Pricing

The system (71)–(82) governing dynamics consists of three state variables \( (\psi_t, c_t, D_t^T) \) and nine jump variables \( (\{\pi_t, v_t, c_t^T, c_t^n, Z_t, S_t^T, l_t^T, l_t^n, w_t^n \}) \). At time 0, all jump variables immediately jump. Furthermore, at the time of policy reversal \( (t = t_1) \), most jump variables jump to certain points. However, since \( \pi_t, v_t \) do not respond to the foreseen shock, those variables must be continuous at time \( t_1 \). For the same reason (perfect foresight and optimizing behavior), the lagrange multiplier \( \phi_t \) associated with the law of motion for durable good is also continuous at time \( t_1 \). With the all-time continuity of state variables, those conditions comprise boundary conditions. Specifically, we adopt the following procedure.

1. Solve for initial steady states of all variables at time \( 0^- \).

2. (Outside loop starts.) Choose a time \( T^* \) corresponding the terminal steady state. Choose small perturbations for terminal steady states of \( \psi_t, c_t, D_t^T \).

3. (Inside loop starts.)
   a. Initial guess for \( \lambda \)
   b. Given the guess for \( \lambda \) and the known terminal steady state of \( \pi_t \), compute time \( T^* \) values of \( \{c_t^T, c_t^n, Z_t, S_t^T, D_t^T, e_t, l_t^T, l_t^n, w_t^n \} \)
   c. Given boundary conditions (values at time \( T^* \)), we solve for the differential equations system to compute the dynamic paths of all variables at \( t \in [t_1, T^*] \).
d. Using the continuity of $\pi_t$ and $e_t$ at time $t_1$, compute time $t_1^-$ values (discrete jumps) of \{e_t^T, c_t^0, Z_t, l_t^T, l_t^n, w_t\}.

e. Using the continuity of $\phi_t$ and $D_t^T$ at time $t_1$, compute time $t_1^-$ value of $S_t^T$.\footnote{From equation (63) and the continuity of $\phi_t$ at time $t_1$ ($\phi_{t_1^-} = \phi_{t_1^+}$), we use the following relation}

\[
\omega^\frac{1}{\gamma} c_{t_1}^T - \frac{1}{\gamma} Z_{t_1}^{\gamma}(1+\gamma) + x \left( \frac{S_{t_1}^T}{D_t^T} - c \right) = \omega^\frac{1}{\gamma} c_{t_1}^T + \frac{1}{\gamma} Z_{t_1}^{\gamma}(1+\gamma) + x \left( \frac{S_{t_1}^T}{D_t^T} - c \right).
\]

f. Given boundary conditions (values at time $t_1^-$), we solve for the differential equations system to compute the dynamic paths of all variables at $t \in [0, t_1]$.\footnote{Before we derive the paths, we calculate the steady states associated with the non-convergent paths for stabilization periods. This step is needed because there are linearized equations in our system.}

g. check whether the initial guess of $\lambda$ is consistent with the implied value of $\lambda$\footnote{With the transversality condition, the aggregate life-time resource constraint produces:}
derived using the life-time resource constraint.

h. Using the new guess of $\lambda$ for the next iteration (eg. $\lambda_{new} \equiv f(\lambda, \lambda_{implied})$, where $f$ is a weighted average of $\lambda$ and $\lambda_{implied}$), iterate until the difference lies within a pre-set tolerance level. (Inside loop ends.)

4. Check whether the derived value of $\psi_t, e_t, D_t$ at time 0 are different from the initial steady state value ($\psi_0$). If there is a discrepancy beyond a pre-set tolerance level, go to step 2 and repeat until the convergence is fulfilled. If the convergence is ensured, check the sensitivity of the choice $T^*$. (Outside loop ends.)

A2. Inflation Targeting with Calvo-Celasun-Kumhof Pricing

Since the overall procedure for inflation targeting case is almost the same as that of the ERBS case, we can apply the same method here. In the inflation targeting case, the real exchange rate ($e_t$) is a jump variable. Since a marginal utility of tradable consumption is not constant during transitions, paths of variables related to durable goods have more dynamics.