Introduction

Typically, economic models assume that changes to fiscal policy are unanticipated by the private sector and the monetary authority. However, information about when and how taxes will change is often released in advance of an actual alteration, giving agents “fiscal foresight.” Yang (2007a) documents an average time lag of seen months between a policy statement announcement and its implementation in postwar U.S. history. While economists do not doubt the presence of tax foresight, its theoretical implications still remain largely unexplored. The purpose of this research is to investigate the theoretical consequences of fiscal foresight on monetary policy and to gain an understanding of what options may be available to the monetary authority when tax foresight is present.

Past theoretical and empirical research on fiscal foresight is limited. Using a narrative approach to identify
U.S. tax changes, Romer and Romer (2007) and Mertens and Ravn (2008) found that unanticipated and anticipated tax changes have different effects on output and investment. Mertens and Ravn (2008) and Yang (2005) show that responses to anticipated and unanticipated tax changes in a standard RBC model are consistent with these empirical findings. These results suggest that foresight is a nontrivial issue that is important for understanding movements in aggregate variables. Other past research, including Branson, et. al. (1985), Poterba (1988), Leeper (1989), Yang (2007b), and Leeper, Yang, and Walker (2008), also has found evidence for tax foresight and documented some of the consequences of its presence. This paper’s contribution to the literature is to theoretically characterize the influence(s) of fiscal foresight on monetary policy.

I investigate this issue in a Calvo pricing model extended to include possible foresight about changes in distortionary labor taxes. I assume lump-sum taxes/transfers adjust each period so that the government budget constraint is met. Thus, I abstract from the fiscal consequences of fiscal foresight to focus attention on its effects on monetary policy.

I first examine how the monetary authority’s optimal policy is affected by fiscal foresight. As Woodford (2003) documents, optimal monetary policy is model specific, often taking the form of an interest rate or inflation targeting rule. Furthermore, as Giannoni and Woodford (2003a,b) show, the optimal interest rate or inflation targeting rule is robust to the nature of exogenous disturbances in an economy. Thus, fiscal foresight does not affect the monetary authority’s optimal targeting or interest rate rule. Fiscal foresight does, however, reduce the number of alternative policies that can approximate the monetary authority’s optimal response to tax news.

A well known result is that the optimal monetary policy involves a response to changes in the welfare theoretic output gap (see Woodford (2003) for more discussion). Several recent works\(^1\) have found simple “implementable” rules that are close approximations of the optimal monetary policy. These rules are “implementable” because they rely on output or output growth and not on the welfare theoretic output gap, which requires knowledge of the current state of exogenous disturbances in the economy. Approximating optimal monetary policy with such a rule is desirable since it does not require the monetary authority to know or be able to perfectly estimate the welfare theoretic output gap. However, I find that in the presence of fiscal foresight, such rules often involve the monetary authority responding to fiscal news by moving the interest rate in the opposite direction from the response under the optimal monetary policy. This suggests that the response to the welfare theoretic output gap is important under fiscal foresight.

\(^1\)Some examples include Kollmann (2004) and Schmitt-Grohe and Uribe (2004a, 2007)
I then consider additional complications that arise in the case of a particular type of asymmetry between the information available to the central bank and that available to the private sector. Specifically, I assume that the private agent has complete information about the state of the economy, while the central bank only observes a subset of all economic variables and does not observe the exogenous disturbances\(^2\). This complicates matters in that the monetary authority must estimate the disturbances in order to estimate and respond to the current output gap.

Svensson and Woodford (2004) show how the central bank can optimally extract information from a set of observables by using the Kalman filter. Instead of assuming that the central bank employs such a method to extract information, I assume that the central bank uses a VAR to recover the structural shocks in the economy. It seems reasonable to suppose the monetary authority will be able to impose identifying restrictions that allow him to recover the structural shocks, correctly estimate the output gap, and follow the optimal policy. If foresight was not present in the model, this would be the case (as long as the monetary authority uses some cross-equation restrictions to identify the structural shocks). However, foresight presents an identification issue that is rarely discussed in the literature and implies that the central bank will not be able to recover the exogenous disturbances.

The problem with using a VAR is that when fiscal foresight is present, the solution paths of variables have a VARMA\((1, q)\) representation, where \(q\) represents the number of periods of fiscal foresight\(^3\). There is no guarantee that a VARMA model has an invertible VAR representation, and as shown by Leeper, Walker, and Yang (2008), with foresight a VARMA model is not invertible. Instead, the Wold representation for the solution of a model with fiscal foresight implies that the monetary authority will not be able to recover the fiscal disturbances, i.e. fiscal news. This leads to a potential problem for setting monetary policy as the monetary authority may respond to a statistical disturbance that is in fact different from the underlying structural disturbance in the economy to which he desires to respond. By modeling a central bank that uses a VAR, I can quantify how the resulting equilibrium (from monetary policy involving a VAR) varies from the equilibrium that the monetary authority desires to reach (i.e. one where the monetary authority responds to the true structural disturbances). I find that when the monetary authority uses a VAR to set policy, he can alter the solution paths of endogenous variables and create history dependence. In addition, the resulting equilibrium reduces welfare.

\(^2\)Although one may not believe that the private sector has more information than the central bank, there is some motivation in the literature for considering this scenario. In particular, Svensson and Woodford (2004) argue that this is the only case where assuming all agents have a common information set makes sense.

\(^3\)This is the case when there is a labor/leisure trade-off in the economy. As noted by Leeper, Walker, and Yang (2008), in a RBC model with capital accumulation and no labor choice, the solution paths have a VARMA\((1,q - 1)\) representation.
With fiscal foresight, the structural disturbances span a strictly larger space than do the observables, creating an identification problem. The VAR case is just an illustration of the issues involved; the identification issue could pose problems for alternative estimation strategies as well. Klaeffing (2003) notes this type of non-identification applies to all techniques that include unobservable explanatory variables.

This paper is organized as follows. Section 2 discusses optimal monetary policy when the central bank has complete information and examines the welfare consequences of fiscal foresight. It also discusses the differences between the monetary authority’s response under optimal policy to various Taylor rules. Section 3 sets up a model of the monetary authority using a VAR to estimate structural disturbances and examines the consequences of such a monetary policy. Section 4 concludes.

2 Foresight Implications with Optimal Monetary Policy

In this section, I investigate the welfare of an economy under varying degrees of fiscal foresight and various monetary policies. I consider both discretionary policy and optimal policy from a timeless perspective, as defined in Woodford (1999b). Furthermore, I compare the responses of these policies to simple Taylor and inflation targeting rules. Following Benigno and Woodford (2005), I define a loss function for the monetary authority as

\[ L \equiv E \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{q_\pi}{2} \hat{\pi}^2_t + \frac{q_y}{2} (\hat{Y}_t - \hat{Y}^*_t)^2 \right\} | I_t \right] \] (1)

where \( \hat{Y}_t \) is output, \( \hat{\pi}_t \) is inflation, and \( \hat{Y}^*_t \) is a linear function of the exogenous disturbances at time \( t \), including the tax rate, \( \hat{\tau}_t \). Here \( E[\cdot|I_t] \) represents the expectation with respect to the central bank’s information set. For the time being, I assume that the central bank and private agent have the same information set and that both observe the path of all current and past endogenous and exogenous variables. Following Benigno and Woodford (2005), I define \( \hat{y}_t \) to be the welfare relevant output gap: \( \hat{y}_t \equiv \hat{Y}_t - \hat{Y}^*_t \).

Using this loss function, it is possible to rank policies in terms of the implied value of \( L \). Benigno and Woodford (2005) show that the loss function is a quadratic approximation to the sum of the expected utility of a representative agent in a Calvo sticky price model. The underlying model is reproduced in appendix A and the derivations of the loss function are given in appendix B.

The loss function (1) is minimized subject to the model’s structural equations: a Phillips curve and con-
where $\hat{i}_t$ is the nominal interest rate and $u_t^*$ is a supply shock. Appendix B gives derivations of these equations and several structural shocks that would qualify as a supply shock. Without loss of generality, I assume the supply shock is a shock to technological productivity, $a_t$, and, following from derivations in appendix B, $u_t^* = -\kappa \phi (1 + \nu) (\omega + \sigma^{-1})^{-1} a_t$. For future reference, note that equations (2) and (3) can be rewritten in terms of the output gap:

$$\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} + \nu^s u_t^* + \nu^\tau \hat{\tau}_t$$

$$(4)$$

$$\hat{y}_t = E_t \hat{y}_{t+1} - \sigma (\hat{i}_t - E_t \hat{\pi}_{t+1}) + \varrho^s u_t^* + \varrho^\tau \hat{\tau}_t - E_t (\varrho^s u_{t+1}^* + \varrho^\tau \hat{\tau}_{t+1})$$

$$(5)$$

where

$$\varrho^s = (\kappa q^{-1} y^{-1} (\omega + \sigma^{-1}) + \Phi (1 - \sigma^{-1}))^{-1}$$

$$\varrho^\tau = \kappa \psi (1 - q^{-1} y^{-1} (1 - \sigma^{-1}) \Phi)$$

$$\varrho = q^{-1} y^{-1} \Phi (1 - \sigma^{-1}) \psi, \quad \varrho^s = (\kappa q^{-1} y^{-1} (\omega + \sigma^{-1}) + \Phi (1 - \sigma^{-1}))^{-1}$$

### 2.1 Discretionary Policy

In this section I examine optimal discretionary policy under various degrees of fiscal foresight. Under discretion, the central bank has no control over the agent’s expectations and takes them as exogenous. Letting $\varphi$ be the Lagrange multiplier on equation (2) and $\lambda$ be the Lagrange multiplier on equation (3), the first order conditions of the central bank’s problem in this case are:

$$q_x \pi_t + \varphi_t = 0$$

$$q_y (\hat{Y}_t - \hat{Y}_t^*) + \lambda_t - \kappa \varphi_t = 0$$

$$\sigma \lambda_t = 0$$
These conditions imply that $\lambda_t = 0$ and the constraint from the consumption Euler equation, equation (3), is not binding. Using the first equation to substitute out the multiplier $\varphi_t$ from the second equation gives an inflation targeting rule:

$$\pi_t = \frac{-q_y}{\kappa q_y} \hat{y}_t$$

(6)

Substituting this into the Phillips curve, equation (4), gives:

$$\frac{-q_y}{\kappa q_y} \hat{y}_t = \kappa y_t - \frac{\beta q_y}{\kappa q_y} E_t \hat{y}_{t+1} + \nu^s u_t^s + \nu^\tau \hat{\tau}_t$$

Rewrite this equation as

$$\left(1 - \left[\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})\right]^{-1} B^{-1}\right) \hat{y}_t = -\left[\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})\right]^{-1} \left[\frac{\kappa q_\pi}{q_y}\beta\right] (\nu^s u_t^s + \nu^\tau \hat{\tau}_t),$$

Since $0 < \frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y}) < 1$, the solution path for the output gap is given by

$$\hat{y}_t = -E_t \sum_{j=0}^{\infty} \left[\frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y})\right]^{-j} \left[\frac{\kappa q_\pi}{q_y}\beta\right] (\nu^s u_t^s + \nu^\tau \hat{\tau}_t)$$

Let $u_t^s$ and $u_{t-j}^\tau$ be mean zero i.i.d. shocks and $\hat{\tau}_t = u_{t-j}$. The subscript on $u$ indicates the period in which information about the value of $u^\tau$ is received. Thus, $j$ denotes the degree of fiscal foresight present in the model. Then the solution for the output gap reduces to

$$\hat{y}_t = \frac{-\kappa q_\pi}{1 + \frac{\kappa^2 q_\pi}{q_y}} \left[\nu^s u_t^s + \nu^\tau \left(u_{t-j}^\tau + \frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y}) u_{t-j-1}^\tau + \ldots + \frac{1}{\beta^{j+1}}(1 + \frac{\kappa^2 q_\pi}{q_y}) u_0^\tau\right)\right]$$

(7)

The output gap decreases with either news of a tax increase or the realization of a tax increase. Moreover, more recent news is discounted relative to distant news. Using this solution in the targeting rule, equation (6), gives the solution for inflation:

$$\pi_t = \frac{1}{1 + \frac{\kappa^2 q_\pi}{q_y}} \left[\nu^s u_t^s + \nu^\tau \left(u_{t-j}^\tau + \frac{1}{\beta}(1 + \frac{\kappa^2 q_\pi}{q_y}) u_{t-j-1}^\tau + \ldots + \frac{1}{\beta^{j+1}}(1 + \frac{\kappa^2 q_\pi}{q_y}) u_0^\tau\right)\right]$$

(8)

These solutions can be substituted into the consumption Euler equation, equation (5), to derive the solution for the interest rate. When $j = 1$, i.e. when there is one period of fiscal foresight, the solution path for the
The interest rate is
\[
i_t = \left( \frac{\kappa q\nu^\pi}{q_y + \kappa^2 q_y} + \varphi_t \right) \sigma u_t^s + \left( \frac{\kappa q\nu^\pi}{q_y + \kappa^2 q_y} + \varphi_t \right) \sigma u_{t-1}^r + \left[ \frac{\nu^\pi (q_y - \kappa q_u \sigma)}{q_y + \kappa^2 q_y} + \beta \kappa q\nu^\pi q_y \right] u_t^r
\]

The interest rate increases with the realization of a tax increase, but it may increase or decrease with news of a tax increase. The ambiguity is due to the fact that the interest rate responds to current and expected changes in the output gap and inflation. With foresight, the monetary authority and private agent expect the output gap and inflation to change in later periods with the realization of the tax policy change. Since the expected change in the output gap and inflation are in opposite directions, the interest rate can move in either direction, depending on the relative importance the monetary authority places on minimizing inflation variability versus variability to the output gap. For reasonable parameter calibrations, the interest rate decreases with news of a tax increase.

### 2.2 The 'Timeless' Policy

In this section I examine optimal monetary policy from a timeless perspective under various degrees of fiscal foresight. In this case the central bank’s first order conditions are

\[
q_y \hat{\pi}_t - \beta^{-1} \sigma \lambda_{t-1} + \varphi_t - \varphi_{t-1} = 0
\]

\[
q_y (\hat{Y}_t - \hat{Y}_t^*) + \lambda_t - \beta^{-1} \lambda_{t-1} - \kappa \varphi_t = 0
\]

\[
\sigma \lambda_t = 0
\]

These conditions imply that \( \lambda_t = 0 \) and the constraint from the consumption Euler equation, equation (3), is not binding. Substituting the second expression into the first, to eliminate the multiplier \( \varphi_t \), gives an inflation targeting rule:

\[
\hat{\pi}_t = \frac{q_y}{q_n \kappa} [\hat{y}_{t-1} - \hat{y}_t] \tag{10}
\]

Substituting this rule into the Phillips Curve, equation (4) gives a second order difference equation in the output gap

\[
E_t \hat{y}_{t+1} - \frac{1}{\beta} \left( (1 + \beta) + \frac{\kappa^2 q\pi}{q_y} \right) \hat{y}_t + \frac{1}{\beta} \hat{y}_{t-1} = \frac{\kappa q\pi}{q_y \beta^2} [\nu_s u_t^s + \nu_r \hat{\pi}_t] \tag{11}
\]

The difference equation can be written as

\[
(B^{-2} - \alpha_1 B^{-1} + \alpha_2) \varphi_{t-1} = \frac{\kappa q\pi}{q_y \beta^2} [\nu_s u_t^s + \nu_r \hat{\pi}_t] \tag{12}
\]
This can be factored as

\[(\mu_1 - B^{-1})(\mu_2 - B^{-1})\varphi_{t-1} = \frac{kq_\pi}{q_y}\beta [v_x u_t^s + v_\tau \hat{\tau}_t] \tag{13} \]

where \(\alpha_1 = \mu_1 + \mu_2\) and \(\alpha_2 = \mu_1 \mu_2\). Let \(\mu_2 = \frac{1}{\mu_1}\). \(|\mu_1| < 1\) if

\[
\frac{q_y}{q_\pi} > \frac{-\kappa^2}{1 + \beta}
\]

For reasonable parameter calibrations, \(q_y\) and \(q_\pi\) are both positive and this condition is met. Assuming restrictions on the structural parameters satisfy this condition, the difference equation can be solved for the path of the output gap.

\[
\hat{y}_t = \mu_1 \hat{y}_{t-1} - \frac{kq_\pi}{q_y} \sum_{j=0}^{\infty} \beta^j \mu_1^{j+1} E_t[v_x u_{t+j}^s + v_\tau \hat{\tau}_{t+j}] \tag{14}
\]

Again, let \(u_t^s\) and \(u_t^\tau\) be mean zero i.i.d. shocks and \(\hat{\tau}_t = u_t^\tau - j\). Then

\[
\hat{y}_t = \mu_1 \hat{y}_{t-1} - \frac{q_\pi k \mu_1}{q_y} \left[ v_x u_t^s + v^\tau \left( u_{t-j}^\tau + \beta \mu_1 u_{t-j-1}^\tau + \ldots + (\beta \mu_1)^j u_t^\tau \right) \right] \tag{15}
\]

Similarly to the solution under discretionary policy, more recent news is discounted relative to older news. Using this solution in the targeting rule, equation (10), gives the solution for inflation:

\[
\hat{\pi}_t = \left( \frac{q_y}{q_\pi k} - \mu_1 \right) \hat{y}_{t-1} + \mu_1 \left[ v_x u_t^s + v^\tau \left( u_{t-j}^\tau + \beta \mu_1 u_{t-j-1}^\tau + \ldots + (\beta \mu_1)^j u_t^\tau \right) \right] \tag{16}
\]

As noted in Woodford (1999b), optimal monetary policy involves history dependence. This is due to the fact that the monetary authority internalizes the effects of its predictable policy on private sector expectations, which in turn affect current inflation and the output gap. Substituting the solutions for inflation and the output gap into the consumption Euler equation, equation (5), gives the solution for the interest rate. Again, the interest rate may increase or decrease with news of a tax increase, but for most parameter calibrations it will decrease.

### 2.3 Welfare Consequences of Foresight

As noted earlier, using the loss function it is possible to rank policies in terms of the implied value of \(L\). As shown in Appendix B, this is equivalent to ranking policies in terms of their implied value of utility. Following Woodford (1999a) and Giannoni (2001), I calculate welfare by taking the unconditional expectation of the
loss function over all possible histories of disturbances. To do this, I calculate

\[
Z \equiv E\{(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t \hat{z}_t^2\} = E\{(1 - \beta) \sum_{t=0}^{\infty} \beta^t (E_0[\hat{z}_t] + \text{Var}_0[\hat{z}_t])\}
\]

for \(\hat{z}_t = \hat{\pi}_t, \hat{y}_t\). The unconditional value of the loss function is a weighted sum of the \(Z\)'s. This value indicates the loss due to temporary disturbances in excess of the steady-state loss.

To calculate \(Z\), I first generate simulated time series of length 10,000 periods, discard the first 9,000 periods, and compute variances. I then repeat this procedure 50,000 times and compute the average of the variances\(^4\).

To calculate the unconditional expectations, I use the 10,000th observation of the simulated series as an initial condition, calculate the unconditional expected value of each variable, and average across the 50,000 samples. Calibration values used are reported in Table 1. These values are taken from Benigno and Woodford (2003) and are standard values used in the literature.

Table 2 displays the results. Foresight increases the variability of endogenous variables\(^5\) but gives agents more information about the expected value of future endogenous variables. This creates a trade-off in that the first point works towards lowering welfare while the second can improve welfare. However, the results suggest that foresight cannot improve welfare, and it would be socially optimal for an agent to ignore the fiscal news. The result that news lowers welfare is not specific to tax news. In this simple model, taxes are a form of a cost push shock, and any anticipated shock that is a form of a cost push shock will have the same result. For instance, Winkler and Wohltmann (2008) find a similar result for anticipated oil price shocks.

### 2.4 Comparisons with Alternative Policies

As suggested above, for reasonable calibrations of parameters, the interest rate always decreases with news of a tax increase. In stark contrast, if the monetary authority follows a simple Taylor rule where the interest rate responds to inflation and output, then the interest rate will usually increase with news of a tax increase\(^6\). Even if the Taylor rule is a function of output growth (instead of output), the interest rate response usually will be positive\(^7\).

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\(^4\)This procedure follows Schmitt-Grohe and Uribe (2004b)

\(^5\)This is clear from calculating the variances of inflation and the output gap from equations (7)-(8) or equations (15)-(16).

\(^6\)See Appendix C for analytical solutions to the model when the monetary authority follows a Taylor rule.

\(^7\)Analytical solutions are not available in this case. This result is based on numerical simulations for various calibrations of parameters.
Figure 1 compares impulse responses for the model economy when there is no fiscal foresight. The interest rate response to an unanticipated tax increase is qualitatively the same for all the monetary policies considered. However, the response under the Taylor rules is quantitatively smaller. Figures 2-3 compare impulse responses for the model economy under alternative monetary policies and varying degrees of fiscal foresight. With foresight, the interest rate response to fiscal news under optimal policy is qualitatively the same if the Taylor rule includes the output gap, but if the Taylor rule responds to output or output growth, the responses are qualitatively different. These rules involve the monetary authority responding to fiscal news by moving the interest rate in the opposite direction from the response under the optimal monetary policy.

This result has no consequential welfare implications. Table 2 displays welfare calculations (see section 2.3 for an explanation of how these results were calculated) under alternative policies. In this model, even without foresight the Taylor rule is not a good approximation of the optimal policy.

Equations (6) and (10) specify targeting rules for optimal monetary policy that are robust in the sense of Giannoni and Woodford (2003a,b). In a model where the central bank has no model uncertainty and perfectly observes $\hat{y}_t$ and $\hat{\pi}_t$, fiscal foresight is irrelevant for monetary policy; the monetary authority commits to a rule that is independent of the tax process. Furthermore, even if the monetary authority mistakenly believed agents did not respond to fiscal news, it would not matter for optimal monetary policy as long as $\hat{y}_t$ and $\hat{\pi}_t$ are perfectly observed. This result is not specific to this particular model; Woodford (2003) shows that many model specifications can be written in terms of a targeting or interest rate rule that is independent of the exogenous processes in the economy. As long as the monetary authority can observe the output gap, he will have no problem implementing the optimal policy. If the monetary authority does not observe the output gap, fiscal foresight presents some challenges for the monetary authority to implement the optimal policy. As discussed in the next section, foresight complicates the monetary authority’s ability to estimate the output gap.

3 Foresight Implications with Imperfect Information

In this section, I assume the monetary authority does not observe the structural shocks but does observe a subset of all current and past variables. I also assume that the central bank decides to estimate the current structural shocks using a VAR. With fiscal foresight, the monetary authority will not be able to recover the true structural shocks from the VAR. This is because the solution paths of the observables have a VARMA
representation, and the VARMA does not have an invertible VAR representation. The equilibrium moving average representation of the path of the observable variables is nonfundamental, i.e. the structural shocks cannot be recovered from current and past observables. The current and past structural disturbances span a strictly larger space than the observables in this case.

Using the method of Hansen and Sargent (1991) and Rozanov (1967), the nonfundamental representation can be converted to the observationally equivalent Wold representation. The statistical innovations of this representation are weighted averages of current and past structural shocks. It is these statistical innovations that the monetary authority recovers from its VAR. Thus, the monetary authority ends up responding to shocks that are different from the underlying structural shocks in the economy; instead, he responds to weighted averages of current and past structural shocks.

3.1 An Illustration of the Issue

As a simple illustration, suppose the monetary authority observes technological productivity, \( \tilde{a}_t \), and current taxes, \( \hat{\tau}_t \), and decides to use these observables to recover the underlying structural shocks. To make the example a little more interesting, assume \( \tilde{a}_t \) and \( \hat{\tau}_t \) evolve according to the AR(1) processes

\[
\begin{align*}
\tilde{a}_t &= \rho_a \tilde{a}_{t-1} + u^*_t \\
\hat{\tau}_t &= \rho_\tau \hat{\tau}_{t-1} + u^*_{t-1}
\end{align*}
\]

It is clear from these equations that \( \tilde{a}_t \) and \( \hat{\tau}_t \) do not contain information about \( u^*_{t} \), i.e. current and past structural disturbances span a strictly larger space than the observables. The moving average representation of these observables is given by

\[
\begin{bmatrix}
\tilde{a}_t \\
\hat{\tau}_t \\
X_t
\end{bmatrix} = \begin{bmatrix} 1 - \rho_a L & 0 \\ 0 & 1 - \rho_\tau L \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} u^*_t \\ u^*_{t-1} \\ U_t \end{bmatrix}
\]

(17)

where

\[
\text{det}[C(L)] = \frac{L}{(\rho_a L - 1)(\rho_\tau L - 1)}
\]

(18)
The root of \( \text{det}[C(z)] = 0 \) is \( z = 0 \). A necessary condition for a representation to be a Wold representation is for no zeros to be inside the unit circle. Thus, this is not a Wold representation. Moreover, this only happens when there is fiscal foresight; without foresight, there would not be a zero inside the unit circle.

Using the method of Hansen and Sargent (1991), I can convert equation (??) to an observationally equivalent fundamental moving average representation. I define a Blaschke matrix \( B(L) \) as

\[
B(L) = \begin{bmatrix}
1 & 0 \\
0 & \frac{1}{L}
\end{bmatrix}
\]

where \( B(L)B(L^{-1})' = I \). Multiplying \( C(L) \) by \( B(L) \) flips the zero outside the unit circle (specifically it flips the root to unity). Then I can convert equation (17) to its Wold representation with the following transformation:

\[
X_t = \underbrace{C(L)B(L)B(L^{-1})'}_{C^*(L)}U_t
\]

where

\[
C^*(L) = \begin{bmatrix}
\frac{1}{1-aL} & 0 \\
0 & \frac{1}{1-\tau L}
\end{bmatrix}, \quad U_t^* = \begin{bmatrix}
u_t^* \\
u_{t-1}^*
\end{bmatrix}
\]

and

\[
\text{det}[C^*(L)] = \frac{1}{(1-aL)(1-\tau L)}
\]

By observing \( \hat{a}_t \), the monetary authority will be able to infer \( u_t^* \). However, instead of inferring \( u_{t-1}^* \), the monetary authority will recover \( u_{t-1}^* \).

This illustration was simple for two reasons. First, the solution path of \( \hat{a}_t \) did not depend on \( u^* \) and the solution path of \( \hat{\tau}_t \) did not depend on \( u^* \). More importantly, \( \hat{a}_t \) and \( \hat{\tau}_t \) are both exogenous variables. In the more interesting (and realistic) scenario where the monetary authority’s observables include endogenous variables, namely inflation and/or output, the central bank must sort through a simultaneity problem. Observed inflation and output are forward-looking variables that depend on both the agent’s current expectations of future output and inflation and the current monetary policy. The central bank’s current expectations and monetary policy in turn depend on the estimates of structural disturbances and the output gap. These in turn depend on the observations of inflation and output. In the next sections, I show how to solve for the equilibrium in this framework.
3.2 Inflation and Technological Productivity Observables

To illustrate my approach, I present an example in this section where I assume that the monetary authority observes the path of current and past inflation and technological progress, $\tilde{a}_t$. Furthermore, for simplification I assume that $\tilde{a}_t = u_t^s$. The monetary authority uses these observables in a VAR to recover the structural disturbances. In contrast, I assume that the agent observes current and past endogenous variables and structural disturbances. Furthermore, I assume the agent knows the monetary authority uses a VAR to recover the shocks and can foresee the policy mistake of responding to statistical innovations that differ from the structural disturbances. Thus, the agent can determine the equilibrium by employing the same strategy I outline below.

This case is not realistic; I assume it for computational simplifications. In a more realistic scenario, the agent would not know what statistical innovations the monetary authority recovers and thus, would not know precisely how monetary policy is determined. In this case, the model would be complicated by "higher order expectations", i.e. the private agent’s expectations of the central bank’s expectations. Although I plan to incorporate this uncertainty into the model, this is a separate issue from the problem of solving for the equilibrium when the monetary authority uses a VAR. Thus, I abstract from this issue here to simplify the analysis and focus on the solution method in the presence of the VAR.

In order to find the equilibrium in this case, I guess that the relationship between the structural disturbances and the statistical innovations that the monetary authority recovers is

$$
U_t^* = \begin{bmatrix}
1 & 0 \\
0 & \frac{L - \theta}{1 - \theta L}
\end{bmatrix}
\begin{bmatrix}
u_t^s \\
u_t^\tau
\end{bmatrix}
$$

(21)

where $|\theta| < 1$. This implies

$$
\begin{align*}
u_t^{*s} & = u_t^s \\
u_t^{*\tau} & = \frac{L - \theta}{1 - \theta L} u_t^\tau
\end{align*}
$$
The monetary authority uses these recovered shocks to get an estimate for the current output gap, \( \hat{y}_t^* \):

\[
\hat{y}_t^* = \hat{Y}_t + g_s u_t^s + g_r u_t^r
\]

\[
= \hat{Y}_t + g_s u_t^s + g_r \frac{L - \theta}{1 - \theta L} u_t^r
\]

I assume the monetary authority commits to follow the inflation targeting rule

\[
\hat{\pi}_t = -\frac{q_y}{\kappa q_y} \hat{y}_t^*
\]  

(22)

Substituting these expressions into the Phillips curve, equation (2), gives

\[
\hat{\pi}_t = -\frac{\kappa^2 q_y}{q_y} \hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} + u_t^s + \kappa q u_{t-1}^s - \kappa q u_t^s + \kappa q \frac{L - \theta}{1 - \theta L} u_t^r
\]

(23)

Solving this difference equation forward gives

\[
\hat{\pi}_t = E_t d_1 \sum_{j=0}^{\infty} (\beta d_1)^j \left\{ (1 - \kappa q_s) u_{t+j}^s + \kappa q u_{t-1+j}^s - \kappa q u_t^s + \kappa q \frac{L - \theta}{1 - \theta L} u_{t+j}^r \right\}
\]

(24)

where

\[
d_1 = \frac{1}{1 + \frac{\kappa^2 q_y}{q_y}}
\]

Assuming \( u_t^s \) and \( u_t^r \) are mean zero i.i.d. shocks, the solution path for inflation is then

\[
\hat{\pi}_t = \theta \hat{\pi}_{t-1} + d_1 (1 - \kappa q_s) (1 - \theta L) u_t^s + d_1 \kappa q (L + \beta d_1) (1 - \theta L) u_t^r
\]

\[
- d_1 \kappa q \left[ L + \frac{\beta d_1 - \theta}{1 - \beta d_1 \theta} \right] u_t^r
\]

(25)

The derivation of this equation is given in Appendix B. The equilibrium moving average representation of the path of the observable variables is then given by

\[
\begin{bmatrix}
\hat{\pi}_t \\
u_t^s \\
u_t^r
\end{bmatrix} =
\begin{bmatrix}
1 - \theta L & 0 & 1 - \theta L \\
0 & 1 & 0 \\
\end{bmatrix}^{-1}
\begin{bmatrix}
m_1(L) & m_2(L) \\
1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
u_t^s \\
u_t^r
\end{bmatrix}
\]

(26)

Note here that I assume the monetary authority estimates \( \hat{\tau}_t = u_t^r \), which implies the monetary authority does not believe fiscal news affects endogenous variables and \( u_t^r \) is the realization of a policy change at time \( t \). Alternatively, I could assume the monetary authority estimates \( \hat{\tau}_t = u_{t-1}^r \) where \( u_{t-1}^r \) is the statistical innovation the monetary authority recovered from the VAR in period \( t - 1 \). I have solved for the equilibrium under this version of the model as well and verified my guess for the functional forms of \( u_{t-1}^r \) and \( u_t^r \). The solution is more complicated in this case (the identification “mistakes” are larger), so I focus on the current specification since it is the “better” alternative.
where

\[ m_1(L) = d_1(1 - \kappa \rho_s)(1 - \theta L) \]
\[ m_2(L) = d_1 \kappa \psi (L + \beta d_1)(1 - \theta L) - d_1 \kappa \rho_r \left[ L + \frac{\beta d_1 - \theta}{1 - \beta d_1} \right] \]

\[
\det[M(L)] = \frac{-m_2(L)}{1 - \theta L}
= \frac{\theta \psi \kappa d_1 (L - \zeta_1(\theta))(L - \zeta_2(\theta))}{-(1 - \theta L)(1 - \beta d_1 \theta)}
\]

where

\[
\zeta_{1,2}(\theta) = \frac{\psi(1 - 2\beta d_1 \theta + (\beta d_1 \theta)^2) - \vartheta_r(1 - \beta d_1 \theta) \pm \sqrt{[\psi(1 - 2\beta d_1 \theta + (\beta d_1 \theta)^2) - \vartheta_r(1 - \beta d_1 \theta)]^2 - 4\psi (d_1 \beta (\vartheta_r - \psi + d_1 \beta \vartheta) - \vartheta_r)(1 - \beta d_1 \theta)}}{2\psi(1 - \beta d_1 \theta)}
\]

For the guessed form of \( U_t^* \) to hold, there must be one root of \( \det[M(z)] = 0 \) that is greater than one in absolute value, and one root that is less than one in absolute value. This implies the absolute value of one \( \zeta \) must be greater than one, and the absolute value of the other must be less than one. To determine the magnitude of \( \zeta \), the equation\(^9\) \( \theta = \zeta_1(\theta) \) must be solved for \( \theta \). There are four possible solutions to this equation:

\[
\theta_{1,2} = \pm 1
\]
\[
\theta_{3,4} = \frac{\psi(1 - (d_1 \beta)^2) \pm \sqrt{\psi(1 + 2(d_1 \beta)^2 + (d_1 \beta)^4) - 4(d_1 \beta)^2 \vartheta_r}}{2d_1 \beta \psi}
\]

For these solutions to be consistent with the guess, \(|\theta| < 1\). Thus, the first two solutions do not satisfy the guess. I have solved for several numerical solutions, and for all of the parameter values I have tried, there exists a unique \( \theta \) that satisfies the above conditions.

Assume that a \( \theta \) consistent with the above requirements has been found and that \(|\zeta_1| < 1\) and \(|\zeta_2| > 1\).

Notice this must mean \( \theta = \zeta_1 \) for the guessed form of \( U_t^* \) to hold. I define a Blaschke matrix \( B(L) \) as

\[
B(L) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1 - \theta L}{L - \theta} \end{bmatrix}
\]

\(^9\)Solving either \( \theta = \zeta_1(\theta) \) or \( \theta = \zeta_2(\theta) \) results in the same four candidate solutions for \( \theta \) since the square root term in \( \zeta_1 \) or \( \zeta_2 \) is squared.
where \( B(L)B(L^{-1})' = I \). Then I can convert equation (26) to its Wold representation:

\[
X_t = \underbrace{M(L)B(L)B(L^{-1})'}_{M^*(L)} \underbrace{U_t^*}_{U_t^*} 
\]

where

\[
M^*(L) = \begin{bmatrix}
\frac{m_1(L)}{1-\theta L} & \frac{m_2(L)}{1-\theta L} \\
1 & 0
\end{bmatrix}
\]

(28)

\[
\det[M^*(L)] = -\frac{m_2(L)}{L-\theta} = \frac{\theta \psi \kappa d_1(L - \zeta_1)(L - \zeta_2)}{L-\theta} = \frac{\theta \psi \kappa d_1(L - \zeta_2)(L - \zeta_2)}{L-\theta} = \theta \psi \kappa d_1(L - \zeta_2)
\]

The root of \( \det[M^*(z)] = 0 \) is \( z = \zeta_2 \) so \(|z| > 1\). Thus, this is the Wold representation and I have verified that

\[
U_t^* = \begin{bmatrix}
1 & 0 \\
0 & \frac{L-\theta}{1-\theta L}
\end{bmatrix}
\begin{bmatrix}
u_t^* \\
u_t^*
\end{bmatrix}
\]

(29)

As long as \(|\zeta_1| < 1\) and \(|\zeta_2| > 1\), the guess will give the correct functional form for the statistical innovations.

### 3.3 Tax and Inflation Observables

In this section, I assume that the monetary authority observes the path of current and past taxes and inflation and uses these in a VAR to recover the structural disturbances. I assume the agent can determine the equilibrium by employing the same strategy that I outline below.

In order to find the equilibrium in this case, I guess the relationship between the structural shocks and the shocks the monetary authority recovers is

\[
U_t^* = \begin{bmatrix}
n_1 & n_2 \\
-n_2 & n_1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & L
\end{bmatrix}
\begin{bmatrix}
u_t^* \\
u_t^*
\end{bmatrix}
\]

(30)
where

\[ n_1 = \frac{k_1}{\sqrt{k_1^2 + k_2^2}}, \quad n_2 = \frac{k_2}{\sqrt{k_1^2 + k_2^2}} \]

so that matrix \( N \) is an orthonormal matrix. This implies

\[ u_t^{**} = n_1 u_t^s + n_2 u_{t-1}^\tau \]
\[ u_t^{*\tau} = n_1 u_{t-1}^\tau - n_2 u_t^s \]

The monetary authority uses these recovered shocks to estimate the current output gap

\[ \hat{y}_t^* = \hat{Y}_t + \varrho_s u_t^{**} + \varrho_u u_t^{*\tau} \]
\[ = \hat{Y}_t + \varrho_s n_1 u_t^s + \varrho_s n_2 u_{t-1}^\tau + \varrho_u n_1 u_{t-1}^\tau - \varrho_u n_2 u_t^s \]

I assume the monetary authority commits to follow the inflation targeting rule

\[ \hat{\pi}_t = \frac{-q_y}{\kappa q_\pi} \hat{y}_t^* \] (30)

Substituting these expressions into the Phillips curve, equation (2), gives

\[ \frac{-q_y}{q_\pi \kappa} \hat{y}_t^* = \kappa \hat{y}_t^* - \frac{\beta q_y}{q_\pi \kappa} E_t \hat{y}_{t+1}^{**} + u_t^s + \kappa \psi u_{t-1}^\tau - \kappa \varrho_s n_1 u_t^s - \kappa \varrho_s n_2 u_{t-1}^\tau - \kappa \varrho_u n_1 u_{t-1}^\tau + \kappa \varrho_u n_2 u_t^s \] (31)

Solving this difference equation forward gives

\[ \hat{y}_t^* = -E_t \sum_{j=0}^{\infty} \left[ \frac{1}{\beta} \left( 1 + \frac{\kappa^2 q_\pi}{q_y} \right) \right]^{-(j+1)} \frac{\kappa q_\pi}{q_y} \left\{ (1 - \kappa \varrho_s n_1 + \kappa \varrho_u n_2) u_{t+j}^s + (\kappa \psi - \kappa \varrho_u n_2 - \kappa \varrho_s n_1) u_{t-1+j}^\tau \right\} \] (32)

Using the inflation targeting rule, equation (30), to substitute for inflation gives

\[ \hat{\pi}_t = E_t d_1 \sum_{j=0}^{\infty} d_2^{-j} \left\{ (1 - \kappa \varrho_s n_1 + \kappa \varrho_u n_2) u_{t+j}^s + (\kappa \psi - \kappa \varrho_u n_2 - \kappa \varrho_s n_1) u_{t-1+j}^\tau \right\} \] (33)

where

\[ d_1 = \frac{1}{1 + \frac{\kappa^2 q_\pi}{q_y}}, \quad d_2 = \frac{1}{\beta} \left( 1 + \frac{\kappa^2 q_\pi}{q_y} \right) \]

\(^{10}\)This matrix is needed in this case to ensure the representation is causal. See Hansen and Sargent (1991) for more information and Townsend (1983) and Leeper, Walker, and Yang (2008) for examples.
Assuming $u^s_t$ and $u^\tau_t$ are mean zero i.i.d. shocks, the solution path for inflation is then

$$\tilde{\pi}_t = d_1 \{ (1 - \kappa \rho_s n_1 + \kappa \rho_\tau n_2) u^s_t + (\kappa \psi - \kappa \rho_s n_2 - \kappa \rho_\tau n_1)(u^\tau_{t-1} + \frac{1}{d_2} u^\tau_t) \} \quad (34)$$

The equilibrium moving average representation of the path of the observable variables is then given by

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{\tau}_t \end{bmatrix} = \begin{bmatrix} d_1(1 - \kappa \rho_s n_1 + \kappa \rho_\tau n_2) & d_1(\kappa \psi - \kappa \rho_s n_2 - \kappa \rho_\tau n_1)(\frac{1}{d_2} + L) \\ 0 & L \end{bmatrix} \begin{bmatrix} u^s_t \\ u^\tau_t \end{bmatrix} \quad (35)$$

where

$$\det[A(L)] = d_1(1 - \kappa \rho_s n_1 + \kappa \rho_\tau n_2)L \quad (36)$$

The root of $\det[A(z)] = 0$ is $z = 0$; this is not the Wold representation. To solve for the unknown parameters, $k_1$ and $k_2$, I define an orthogonal matrix $K$ with the following properties: (i) the product of $A(L)$ evaluated at $L = 0$ and $K$ gives a matrix with zeros in the second column and (ii) $A_{11}(L)$ evaluated at $L = 0$ equals $k_1$ and $A_{12}(L)$ evaluated at $L = 0$ equals $k_2$. Thus, to solve for $k_1$ and $k_2$, I solve the following system of equations:

$$k_1 = d_1(1 - \kappa \rho_s n_1 + \kappa \rho_\tau n_2) \quad (37)$$

$$k_2 = d_1(\kappa \psi - \kappa \rho_s n_2 - \kappa \rho_\tau n_1) \frac{1}{d_2} \quad (38)$$

Unfortunately, an analytical solution is not available. However, for given parameter values, numerical solutions can be found. For plausible parameter calibrations, I can find unique\(^{11}\) numerical solutions for $k_1$ and $k_2$ and verify the guess.

### 3.4 Output and Inflation Observables

In this section, I assume that the monetary authority observes the path of current and past output and inflation and uses these in a VAR to recover the structural disturbances. I guess the relationship between

\(^{11}\)For given parameter values, they are unique.
the structural shocks and the shocks the monetary authority recovers is

\[
U_t^* = \begin{bmatrix}
 n_1 & n_2 \\
 -n_2 & n_1
\end{bmatrix}
\begin{bmatrix}
 1 & 0 \\
 0 & \frac{L-\theta}{1-\theta L}
\end{bmatrix}
\begin{bmatrix}
 u^*_t \\
 u^*_t
\end{bmatrix}
\]  

(39)

where \(|\theta| < 1\) and

\[
n_1 = \frac{1}{\sqrt{1+k_2^2}}, \quad n_2 = \frac{k_2}{\sqrt{1+k_2^2}}
\]

In this case the solution paths for inflation and output are

\[
\hat{\pi}_t = \theta \hat{\pi}_{t-1} + \tilde{m}_1(L)u_t^* + \tilde{m}_2(L)u_t^*  
\]

\[
\hat{Y}_t = \theta \hat{Y}_{t-1} - \left[(\varrho_s n_1 - \varrho_t n_2)(1-\theta L) + \tilde{m}_1(L)\frac{\kappa q_y}{q_y} u_t^* - \left[(\varrho_s n_2 + \varrho_t n_1)(L-\theta) + \frac{\kappa q_y}{q_y} \tilde{m}_2(L) \right] u_t^* \right] u^*_t  
\]

(40)

(41)

where

\[
\tilde{m}_1 = d_1(1 - \kappa \varrho_s n_1 + \kappa \varrho_t n_2)(1-\theta L)
\]

\[
\tilde{m}_2 = d_1 \kappa \left\{ \psi(L + \beta d_1)(1-\theta L) - (\varrho_s n_2 + \varrho_t n_1) \left( L + \frac{\beta d_1 - \theta}{1-\beta d_1 \theta} \right) \right\}
\]

Analytical solutions for \(\theta\) and \(k_2\) are not available, but numerical solutions are obtained using the same method as outlined before. Again, I find for plausible parameter values there is a unique numerical solution. It is important to note that in this case the equilibrium is always indeterminate. In order to solve for the equilibrium, I had to normalize \(k_1\) to one. In general, for a guess of this functional form, such a normalization is always required. Thus, the equilibrium depends on what normalization agents choose.

### 3.5 Consequences of Using a VAR

Figures 4 and 5 give impulse response functions for the model economy when various observables are used in the VAR. These responses are calculated from the equilibrium solution paths of variables. The results are compared to the result under discretionary policy; the policy achieved when structural disturbances are correctly identified.

When taxes are an observable, the responses following an anticipated tax shock or unanticipated technology shock are quantitatively similar to the discretionary policy’s responses. This is because the solution paths of endogenous variables are similar to the paths under the discretionary policy (indeed, by comparing equations (8) and (34), it is clear that with the VAR policy only the quantitative response to disturbances changes).
This is a special case due to the fact that taxes are completely exogenous ($\tilde{\tau}_t = u\tilde{\tau}_{t-1}$). If the tax process included a feedback from endogenous variables (such as output), the VAR results would be similar to the cases with other observables.

When taxes are not included in the observables, the response following a tax disturbance changes qualitatively. This is because the central bank’s policy creates history dependence in the solution paths of variables (this can be seen by comparing equations (8), (25), and (40). The impulse response oscillates due to the fact that $\theta < 0$, implying that inflation will increase in period $t$ if it decreased in period $t - 1$. Interestingly, the response following an unanticipated technology shock does not change qualitatively, even when the monetary authority does not correctly identify the technology shock. This is because the statistical innovations that the monetary authority observes involve only the current technology shock, as opposed to a linear combination of current and past technology shocks (as he does with the anticipated tax shock). Although it appears the impulse response functions for a technology shock are the same for the different policies (see Figure 5), the responses do vary slightly (however the difference is practically nonexistent).

Table 3 displays welfare calculations (see section 2.3 for an explanation of how these results were calculated) under policies with various observables in the VAR. The central bank’s policy always reduces welfare (compared to the discretionary policy). Interestingly, when taxes are included in the observables, the welfare loss is very small. In addition, it is interesting to note that the welfare losses are still less than those from the simple Taylor rule policies (see Table 2).

Although the above results were for specific scenarios, several results generalize. In general, the equilibrium will be indeterminate if all observables are endogenous variables (due to the normalization required to solve for the VAR fixed point). When taxes are exogenous, identification mistakes will be minimized when taxes are included in the observables. In addition, in general, the central bank’s VAR policy induces history dependence in the model. Qualitatively, these results hold for longer periods of fiscal foresight as well (although, in general welfare worsens with additional periods of fiscal foresight).

## 4 Conclusion

This paper investigated the theoretical consequences of fiscal foresight on monetary policy. When the central bank can observe the current output gap, foresight does not affect the optimal policy. However, the resulting optimal response to fiscal news can be qualitatively different from the response under alternative monetary
policy rules.

When the central bank does not observe the current output gap and must estimate the structural disturbances using a VAR, the central bank’s policy can change the structure of the equilibrium. This is due to the fact that the resulting moving average representation of the economy does not have an invertible VAR representation, and thus the statistical innovations the monetary authority recovers are different from the underlying structural disturbances. The central bank can induce history dependence and reduce the welfare of the economy in this case.

There are several directions in which I would like to expand this research. Currently, I am investigating how the results change when the central bank uses a VAR to recover structural disturbances to implement the timeless policy (instead of discretionary policy). In particular, I plan to compare how the inflation targeting rule under discretionary policy compares to that under the timeless policy in this case. In addition, I plan to relax the information set of agents (to a more realistic assumption) and consider how the results change. For this case I plan to assume that agents do not know how the central bank recovers structural disturbances and must form expectations of the central bank’s expectations. Thus, higher order expectations may need to be modeled.
References


A The Model

This appendix describes the theoretical model. The model is a version of the Benigno and Woodford (2005) model, extended to allow advance news of tax policy changes. The economy consists of a representative household, a representative final good producing firm, a continuum of intermediate goods producing firms (each indexed by \( i \in [0, 1] \)), a monetary authority, and the government. Each intermediate good is supplied by a monopolistically competitive producer. I assume there are an infinite number of industries (each indexed by \( j \in [0, 1] \)), producing many differentiated intermediate goods, and that labor is specific to an industry.

A.1 Households

A representative household seeks to maximize the expected utility of its consumption and leisure:

\[
U \equiv \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [u(C_t; \xi_t) - \int_0^1 v(H_t(j); \xi_t) dj]
\]

where \( H_t(j) \) is the amount of labor of type \( j \) supplied, \( \xi_t \) is a vector of the realization of exogenous variables at time \( t \), and \( C_t \) is the aggregate consumption of a continuum of differentiated goods, due to Dixit and Stiglitz (1977),

\[
C_t \equiv \left[ \int_0^1 c_t(i) \frac{\theta-1}{\theta} di \right]^{\frac{\theta}{\theta-1}}
\]

where \( \theta > 1 \) is the elasticity of substitution. Following Benigno and Woodford (2005), the following functional forms are assumed for the utility function:

\[
u(C_t; \xi_t) \equiv \frac{C_t^{1-\theta} \bar{C}_t^\theta}{1-\theta}
\]

\[
v(H_t; \xi_t) \equiv \frac{\lambda}{1+\nu} \bar{H}_t^{1+\nu} \bar{H}_t^{-\nu}
\]

The household's nominal flow budget constraint is

\[
P_tC_t + E_t[Q_{t,t+1}A_{t+1}] = A_t + P_t \int_0^1 w_t(j)H_t(j) dj - T_t
\]

where \( w_t(j) \) is the real wage in industry \( j \), \( T_t \) is a lump sum transfer, \( P_t \) is the price index, and \( Q_{t,t+1} \) is a stochastic discount factor such that the price of any bond portfolio at period \( t \) with the random value \( A_{t+1} \) in period \( t + 1 \) is

\[
W_t = E_t[Q_{t,t+1}A_{t+1}]
\]

where \( W_t \) is the household's end-of-period bond portfolio. The household's first order conditions from maximizing its expected utility (A.1) subject to its budget constraint (A.5) give

\[
Q_{t,t+1} = \frac{\beta u_c(C_{t+1}; \xi_{t+1})}{u_c(C_t; \xi_t)} \frac{P_t}{P_{t+1}}
\]

\[
w_t(j) = \frac{v_h(H_t(j); \xi_t)}{u_c(C_t; \xi_t)}
\]

\footnote{Specifically, \( \xi_t \) includes shocks to tastes and preferences, \( \bar{C}_t \) and \( \bar{H}_t \), government spending, \( G_t \), taxes, \( \tau_t \), and a technology shock \( A_t \).}
A.2 The Final Good Producer

The representative final good producer uses \( y_t(i) \) units of each intermediate good \( i \) to produce the final good, \( Y_t \), according to the constant returns to scale technology due to Dixit and Stiglitz (1977)

\[
\int_0^1 y_t(i)^{(\theta-1)/\theta} \, di \geq Y_t \tag{A.9}
\]

Denote the price of the intermediate good \( p_t(i) \). Then the final good firm’s problem is to choose \( Y_t \) and \( y_t(i) \) to maximize

\[
P_t Y_t - \int_0^1 p_t(i) y_t(i) \, di \tag{A.10}
\]

subject to equation (A.9). From the first order conditions, we get

\[
y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} \tag{A.11}
\]

Substituting equation (A.18) into equation (A.9) leads to an expression for \( P_t \) that must hold at equilibrium

\[
P_t = \left[ \int_0^1 p_t(i)^{1-\theta} \, di \right]^{1/(1-\theta)} \tag{A.12}
\]

The aggregate resource constraint for the economy is given by

\[
Y_t = C_t + G_t \tag{A.13}
\]

where \( G_t \) is exogenously varying government spending.

A.3 Intermediate Goods Producers

The intermediate goods producers are monopolistic competitors in their product market and take factor prices as given. Given the price \( p_t(i) \) that a firm charges for its product, it is assumed the firm must produce enough to meet the demand for its good. Assuming a common technology for the production of all goods, firms hire \( h_t(i) \) units of labor to produce \( y_t(i) \) according to the technology

\[
y_t(i) = A_t f(h_t(i)) = A_t h_t(i)^{1/\phi} \tag{A.14}
\]

where \( \phi > 1 \) and \( A_t \) is exogenously varying technology. Following Calvo (1983), producers in each industry fix the prices of their goods for a random interval of time. Each period, \( 0 \leq \alpha < 1 \) fraction of producers in an industry are unable to change their price. The \( 1 - \alpha \) fraction of suppliers who do change their price in period \( t \) choose a new price, \( p_t(i) \), that maximizes

\[
\mathbb{E}_t \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t:T} \Pi(p_t(i), p_j^{T_{t}, P_T; Y_T, \xi_T}) \right\} \tag{A.15}
\]

The profit function is given by

\[
\Pi(p_t(i), p_j^{T_{t}, P_T; Y_T, \xi_T}) \equiv (1 - \tau_t) p_t Y_t \left( \frac{p_t}{P_t} \right)^{-\theta} - \frac{\nu_t f^{-1} \left( \frac{Y_t}{A_t} \left( \frac{p_t}{P_t} \right)^{-\theta} \right) ; \xi_t}{u_t(Y_t - G_t; \xi_t)} P_t f^{-1} \left( \frac{Y_t}{A_t} \left( \frac{p_t}{P_t} \right)^{-\theta} \right) \tag{A.16}
\]

where \( \tau_t \) is a tax on nominal profits. Note that I have substituted the industry wage, \( w_t(j) \), into equation (A.16) using equation (A.8).
Letting $p_t = p_t^j = p_t^*$, the solution to equation (A.15) is

$$E_t \left\{ \prod_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ (1 - \tau_T)(1 - \theta)Y_T \left( \frac{p_t^*}{F_T} \right)^{-\theta} + \frac{\nu_T}{u_T} Y_T \left( \frac{p_t^*}{F_T} \right)^{-\theta-1} \right] \right\}$$  \hspace{1cm} (A.17)

Substituting for $Q_{t,T}$ gives

$$E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} u_T(Y_T; \xi_T)(1 - \tau_T)Y_T \left( \frac{P_T}{F_T} \right)^{\theta-1} \left( \frac{p_t^*}{F_T} \right)^{-\theta} \right\}$$

$$= E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} Y_T(u_T(Y_T; \xi_T)) \frac{\theta}{\theta-1} P_T^\theta P_t^\theta \right\}$$  \hspace{1cm} (A.18)

Equation (A.18) can be written as

$$\left( \frac{p_t^*}{P_t} \right)^{1+\omega\theta} = \frac{K_t}{F_t}$$  \hspace{1cm} (A.19)

where

$$K_t \equiv E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{\theta}{\theta-1} u_T(Y_T; \xi_T)Y_T \left( \frac{P_T}{F_T} \right)^{\theta(1+\omega)} \right\}$$  \hspace{1cm} (A.20)

$$F_t \equiv E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t}(1 - \tau_T)Y_Tu_T(Y_T; \xi_T) \left( \frac{P_T}{F_T} \right)^{\theta-1} \right\}$$  \hspace{1cm} (A.21)

and $\omega = \phi(1 + \nu) - 1 > 0$. With Calvo pricing, the price index evolves according to

$$P_t = [(1 - \alpha)p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}]^{1/1-\theta}$$  \hspace{1cm} (A.22)

Combining equation (A.19) with the price index gives the aggregate supply equation

$$\frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} = \left( \frac{K_t}{F_t} \right)^{1-\theta}$$  \hspace{1cm} (A.23)

A.4 Monetary and Fiscal Policy

I assume the central bank has control of the riskless short-term nominal interest rate, $i_t$, which can be defined as

$$1 + i_t = [E_t Q_{t,t+1}]^{-1}$$  \hspace{1cm} (A.24)

I assume lump-sum taxes/ transfers adjust each period so that the government budget constraint is met. Thus, I abstract from the fiscal consequences of fiscal foresight and alternative monetary policies.
B Derivations of Equations in the Paper

This Appendix gives derivations of the equations in the text. Many of these derivations closely follow those of Benigno and Woodford (2005).

B.1 Derivation of Equation (3)

Substituting the aggregate resource constraint, (A.13), and equation (A.24) into equation (A.7) and log linearizing yields

\[ \dot{\pi}_t = \sigma^{-1}[E_t(\dot{Y}_{t+1} - g_{t+1}) - (\dot{Y}_t - g_t)] \]

where \( \sigma^{-1} \equiv \dot{\sigma}^{-1}Y, g_t \equiv G_t + C_t, \) and \( \dot{G}_t \equiv (G_t - G)/Y. \) Rearranging the equation yields equation (3) in the text where

\[ u_t^d \equiv \dot{G}_t + \frac{C_t}{Y} \dot{C}_t \]

B.2 Second Order Approximation of AS Equation, (A.23)

I derive a second order approximation of the Phillips Curve equation, (A.23), since this will be needed to derive the quadratic welfare measure. Reducing this result to a first order approximation gives equation (2) in the text. From equation (A.23),

\[ \log \left( \frac{1 - \alpha \Pi_{t+1}^d}{1 - \alpha} \right) = \frac{\theta - 1}{1 + \omega \theta} \log F_t - \log K_t \]  

(B.1)

A second order Taylor series expansion of the left-hand side with respect to \( \Pi_t \) around \( \bar{\Pi} = 1 \) yields

\[ \log \left( 1 - \frac{\alpha \Pi^d_{t+1}}{1 - \alpha} \right) = -\frac{\alpha(\theta - 1)}{1 - \alpha} \left( \hat{\pi}_t + \left( 1 + \frac{\theta - 2 + \alpha}{1 - \alpha} \right) \frac{\dot{\pi}_t}{2} \right) \]

\[ = -\frac{\alpha(\theta - 1)}{1 - \alpha} \left( \hat{\pi}_t + \frac{1}{2} \left( \frac{\theta - 1}{1 - \alpha} \dot{\pi}_t^2 + O(||\xi||^3) \right) \right) \]  

(B.2)

where, following Benigno and Woodford (2005), \( O(||\xi||^3) \) will be used throughout as shorthand for \( O(||\xi||^3, \dot{X}_{t_0}/X_{t_0}, \bar{X}_{t_0}/X_{t_0}) \) where \( \dot{X}_{t_0} \) are state-contingent commitments for period \( t_0 \). Now note that \( \log \bar{F} = \log \bar{K}. \) Then equation (B.1) can be written as

\[ \hat{\pi}_t + \frac{\theta - 1}{2} \dot{\pi}_t^2 = \frac{\alpha - 1}{\alpha(1 + \omega \theta)} (\hat{K}_t - \bar{F}_t) + O(||\xi||^3) \]  

(B.3)

I now proceed to derive a second order approximation of the right-hand side of equation (B.1). Note that

\[ \hat{K}_t + \frac{1}{2} \dot{K}_t^2 + O(||\xi||^3) = (1 - \alpha \beta) E_t \sum_{T=t}^\infty (\alpha \beta)^{T-t} [\hat{k}_{t,T} + \frac{1}{2} \dot{k}_{t,T}^2] + O(||\xi||^3) \]  

(B.4)

where

\[ \dot{k}_{t,T} = \frac{v_{\theta u}}{v_{\theta y}} \dot{Y}_T + \frac{v_{\theta h}}{v_{\theta y}} \bar{H}_T + \frac{v_{\theta a}}{v_{\theta y}} \bar{A}_T + \bar{Y}_T + \theta(1 + \omega) \sum_{s=t+1}^T \hat{\pi}_s \]

\[ = (1 + \omega) \bar{Y}_T - \omega q_T + \theta(1 + \omega) \sum_{s=t+1}^T \hat{\pi}_s \]  

(B.5)

and \( \bar{h}_t \equiv \ln \bar{H}_t/\bar{A}, \) \( a_t \equiv \ln A_t/\bar{A}, \) and \( \omega q_t \equiv \nu \dot{h}_t + \phi(1 + \nu) a_t. \) Also

\[ \dot{F}_t + \frac{1}{2} \dot{F}_t^2 + O(||\xi||^3) = (1 - \alpha \beta) E_t \sum_{T=t}^\infty (\alpha \beta)^{T-t} [\dot{f}_{t,T} + \frac{1}{2} \dot{f}_{t,T}^2] + O(||\xi||^3) \]  

(B.6)
\[ \dot{f}_{t,T} = -\frac{\tau}{1-\tau} \dot{r} + \dot{Y}_T + \frac{u_{\text{sec}}}{u_{\text{C}}} \ddot{C}_T + \frac{u_{\text{sec}}}{u_{\text{C}}} \ddot{\bar{c}}_T + (\theta - 1) \sum_{s=t+1}^{T} \dot{\pi}_s \]

\[ = -\frac{\tau}{1-\tau} \dot{r} + \dot{Y}_T - \sigma^{-1}(\ddot{C}_T - \ddot{\bar{c}}_T) + (\theta - 1) \sum_{s=t+1}^{T} \dot{\pi}_s \]  
\[ = -\frac{\tau}{1-\tau} \dot{r} + (1 - \sigma^{-1})\dot{Y}_T + \sigma^{-1} g_T + \frac{\sigma^{-1}(1 - \frac{\varphi}{\theta})}{2} \dot{Y}_T^2 + \sigma^{-1} \frac{\varphi}{\theta} \ddot{Y}_T \dot{G}_T + (\theta - 1) \sum_{s=t+1}^{T} \dot{\pi}_s \]  

(B.7)

The last expression follows by taking a second order Taylor approximation of \( Y_T = C_T + G_T \) and using this to substitute out \( \ddot{C}_T \). For future reference, I further define

\[ k_T \equiv (1 + \omega)\dot{Y}_T - \omega q_T \]

\[ f_T \equiv -\frac{\tau}{1-\tau} \dot{r} + (1 - \sigma^{-1})\dot{Y}_T + \sigma^{-1} g_T + \frac{\sigma^{-1}(1 - \frac{\varphi}{\theta})}{2} \dot{Y}_T^2 + \sigma^{-1} \frac{\varphi}{\theta} \ddot{Y}_T \dot{G}_T \]

I use equations (B.4) and (B.6) to get a second order expansion of the right hand side of (B.1):

\[ K_t - F_t = (1 - \alpha \beta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [(\dot{k}_{t,T} - \dot{f}_{t,T} + \frac{1}{2} (\ddot{k}_{t,T} - \ddot{f}_{t,T})^2) - \frac{1}{2} (K_t^2 - F_t^2) + \mathcal{O}(||\xi||^3)] \]

\[ = (1 - \alpha \beta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [(\dot{k}_{t,T} - \dot{f}_{t,T} + \frac{1}{2} (\ddot{k}_{t,T} - \ddot{f}_{t,T})^2) - \frac{1}{2} (K_t - F_t)^2 + (K_t + F_t) + \mathcal{O}(||\xi||^3)] \]  
\[ = (1 - \alpha \beta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [(\dot{k}_{t,T} - \dot{f}_{t,T} + \frac{1}{2} (\ddot{k}_{t,T} - \ddot{f}_{t,T})^2) - \frac{1}{2} (1 - \alpha \beta) \frac{\alpha(1 + \omega \theta)}{1 - \alpha} \dot{\pi}_t Z_t + \mathcal{O}(||\xi||^3)] \]

where

\[ Z_t = \sum_{\tau=t}^{\infty} (\alpha \beta)^{T-t} [\dot{k}_{t,T} + \dot{f}_{t,T}] \]

The last expression was derived by substituting \((\dot{K}_1 - \dot{F}_1)\) from equation (B.3), to a first order. Note we can further expand the first term of equation (B.8):

\[ E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [(\dot{k}_{t,T} - \dot{f}_{t,T})] = E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [(\dot{k}_{t,T} - \dot{f}_{t,T}) + (1 + \omega \theta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \sum_{s=t+1}^{t} \dot{\pi}_s] \]

\[ = E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [(\dot{k}_{t,T} - \dot{f}_{t,T}) + \frac{(1 + \omega \theta)}{1 - \alpha \beta} \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-t} \dot{\pi}_T] \]  
(B.9)

The last part follows since

\[ E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \sum_{s=t+1}^{T} \dot{\pi}_s = E_t \{ (\alpha \beta) \dot{\pi}_{t+1} + (\alpha \beta)^2 [\dot{\pi}_{t+1} + \dot{\pi}_{t+2}] + \ldots \} \]

\[ = \frac{1}{1 - \alpha \beta} E_t \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-t} \dot{\pi}_T \]

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Also, note that
\[
\frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (k_{1,T} - f_{1,T}^2)] = \frac{1}{2} E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} (k_{1,T} - f_{1,T}^2)]
\]

Then it follows that
\[
\begin{align*}
\hat{K}_t - \hat{F}_t &= (1 - \alpha \beta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \hat{k}_{T} - \hat{f}_T \hat{f}_T + (1 + \theta \omega) E_t \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-t} \hat{\pi}_T - \frac{1}{2} (1 - \alpha \beta) \frac{\alpha (1 + \omega \theta)}{1 - \alpha} \hat{\pi}_t Z_t \\
+ \frac{1}{2} (1 - \alpha \beta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [k_{1,T}^2 - f_{1,T}^2] + (1 - \alpha \beta) E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \hat{\pi}_T N_T \\
+ \frac{1}{2} (2 \theta + \theta \omega - 1)(1 + \theta \omega) E_t \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-t} \hat{\pi}_T \hat{\pi}_T \hat{\pi}_T (\hat{\pi}_T + 2 V_T) + O(||\xi||^3)
\end{align*}
\]

This can be written recursively as
\[
\begin{align*}
\hat{K}_t - \hat{F}_t &= (1 - \alpha \beta) \frac{\alpha (1 + \omega \theta)}{1 - \alpha} \hat{\pi}_t Z_t = (1 - \alpha \beta) \hat{k}_{T} - \hat{f}_T + \frac{1}{2} (k_{1,T}^2 - f_{1,T}^2)] + \alpha \beta (1 + \omega \theta) E_t \hat{\pi}_{t+1} \\
+ (1 - \alpha \beta) \alpha E_t \hat{\pi}_{t+1} N_{t+1} \\
+ \frac{1}{2} (2 \theta + \theta \omega - 1)(1 + \theta \omega) \alpha \beta E_t \hat{\pi}_{t+1} \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2 V_{t+1}) \\
+ \alpha \beta [\hat{K}_{t+1} - \hat{F}_{t+1} + \frac{1}{2} (1 - \alpha \beta) \frac{\alpha (1 + \omega \theta)}{1 - \alpha} \hat{\pi}_t Z_{t+1} + O(||\xi||^3)]
\end{align*}
\]

Substitute out \( \hat{K}_t - \hat{F}_t \) using equation (B.3):
\[
\begin{align*}
\hat{\pi}_t + \frac{\theta}{2} (1 - \alpha) \hat{\pi}_t = (1 - \alpha) \frac{(1 - \alpha \beta)}{\alpha (1 + \omega \theta)} [k_T - \hat{f}_T + \frac{1}{2} (k_{1,T}^2 - f_{1,T}^2)] \\
+ (1 - \alpha) \beta E_t \hat{\pi}_{t+1} + \frac{1}{2} (2 \theta + \theta \omega - 1)(1 - \alpha) \beta E_t \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2 V_{t+1}) \\
+ \alpha \beta E_t [\hat{\pi}_{t+1} + \frac{\theta}{2} (1 - \alpha) \hat{\pi}_t Z_{t+1} + O(||\xi||^3)]
\end{align*}
\]
where the second term of the last expression comes from using equation (B.3), to the first order. Substituting

\[
\dot{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \ddot{\pi}_t^2 + \frac{1}{2} (1 - \alpha) \dot{\pi}_t Z_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha (1 + \omega \theta)} [k_T - \dot{f}_T + \frac{1}{2} (k_T^2 - f_T^2)] + (1 - \alpha) \beta E_t \ddot{\pi}_{t+1} + \frac{1}{2} (1 - \alpha) \beta (1 - \alpha \beta) E_t Z_{t+1} \ddot{\pi}_{t+1}^2
\]

\begin{equation}
= \frac{1}{2} \left( \dot{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \ddot{\pi}_t^2 + \frac{1}{2} (1 - \alpha) \dot{\pi}_t Z_t \right) = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha (1 + \omega \theta)} [k_T - \dot{f}_T + \frac{1}{2} (k_T^2 - f_T^2)] + (1 - \alpha) \beta E_t \ddot{\pi}_{t+1} + \frac{1}{2} (1 - \alpha) \beta (1 - \alpha \beta) E_t Z_{t+1} \ddot{\pi}_{t+1}^2 + \frac{1}{2} (1 - \alpha) \beta (1 - \alpha \beta) E_t Z_{t+1} \ddot{\pi}_{t+1}^2 + O(\|x\|^3)
\end{equation}

(A.16)

Define \( V_t \equiv \dot{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \ddot{\pi}_t^2 + \frac{1}{2} (1 - \alpha) \dot{\pi}_t Z_t \). Then equation (A.16) can be written as

\[
V_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha (1 + \omega \theta)} [k_T - \dot{f}_T + \frac{1}{2} (k_T^2 - f_T^2)] + \frac{1}{2} (1 - \alpha) \beta \dot{E}_t \ddot{\pi}_{t+1}^2 + \frac{1}{2} (1 - \alpha) \beta (1 - \alpha \beta) E_t Z_{t+1} \ddot{\pi}_{t+1}^2 + O(\|x\|^3)
\]

(B.17)

Substituting for \( k_T \) and \( f_T \), we arrive at the second order approximation of equation (A.23):

\[
V_t = \kappa \left\{ \dot{Y}_t + (\omega + \sigma^{-1})^{-1} \left[ \frac{\tau}{1 - \tau} \dot{\pi}_t - \sigma^{-1} g_t - \omega q_t \right] + \frac{1}{2} (d_{\omega} \ddot{\pi}_t^2 - \dot{Y}_t d_{\omega} \xi_t + \frac{1}{2} d_{\omega} \dot{z}_t^2) \right\} + \beta E_t V_{t+1} + \text{i.t.p.} + O(\|x\|^3)
\]

(B.18)

where

\[
\kappa \equiv \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha (1 + \omega \theta)} (\omega + \sigma^{-1})^{-1}
\]

\[
d_{\omega} \equiv (2 + \omega - \sigma^{-1}) + \sigma^{-1} \left( 1 - \frac{\hat{Y}}{C} \right) (\omega + \sigma^{-1})^{-1}
\]

\[
d_{\omega} \xi_t \equiv (\omega + \sigma^{-1})^{-1} \left[ (1 + \omega) q_t (1 - \sigma^{-1}) g_t - \sigma^{-1} \frac{\hat{Y}}{C} \dot{G}_t - (1 - \sigma^{-1}) \frac{\tau}{1 - \tau} \dot{\pi}_t \right]
\]

\[
d_{\sigma} \equiv \frac{\theta (1 + \omega)}{\alpha}
\]

and t.i.p. refers to terms that are not dependent on policy (i.e. terms that only involve the exogenous variables). Note that to a first order approximation, equation (B.18) reduces to equation (2), the Phillips curve, in the text:

\[
\dot{\pi}_t = \kappa [\dot{Y}_t + \psi \dot{\pi}_t] + \beta E_t \ddot{\pi}_{t+1} + u_t^s
\]
where
\[ u_t^s \equiv -\kappa(\omega + \sigma^{-1})^{-1}[\sigma^{-1}g_t + \omega \eta_t] \]
and
\[ \psi \equiv (\omega + \sigma^{-1})^{-1} \frac{\tau}{1 - \tau} \]
Note the form of the Phillips curve is not specific to the assumption of a tax on profits. If I had assumed instead a tax on labor income, the above equation would still hold (although the second order approximation would differ slightly).

**B.3 Derivation of Quadratic Loss Function, (1)**

Note that
\[ Z_1 \equiv \int_0^1 v(H_t(j); \xi_t) dj = \int_0^1 \frac{\lambda}{1 + \nu} \left( \frac{Y_t(j)}{A_t} \right)^{\phi(1+\nu)} \Delta_t dj \]
\[ \equiv \frac{\lambda}{1 + \nu} \frac{Y_t^{1+\omega} - 1}{H_t^{\tau + 1}} \Delta_t \]
where \( \Delta_t \) is the measure of price dispersion:
\[ \Delta_t \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\theta(1+\omega)} di \]
Then the utility function (A.1) can be written as
\[ U \equiv E_t \sum_{t=0}^{\infty} \beta^t [u(Y_t; \xi_t) - v(Y_t(j); \xi_t) \Delta_t] \]
A second order Taylor approximation of the first term gives
\[ u(Y_t; \xi_t) = \bar{Y} \bar{u}_c \left[ 1 + \frac{1}{2} \bar{Y}^2 \right] + \bar{u} \xi_t \bar{Y} + \frac{1}{2} \bar{u}_c \bar{Y}^2 + \bar{u} \xi_t \bar{Y} + O(\|\xi\|^3) \]
Now note that
\[ u_{cc} = -\frac{\sigma^{-1}}{c} \bar{u}_c = -\frac{\sigma^{-1}}{\bar{Y}} \bar{u}_c \]
\[ u_{cG} = -u_{cc} \]
\[ u_{cE} = -u_{cc} \]
Using these expressions, \( u(Y_t; \xi_t) \) can be written as
\[ u(Y_t; \xi_t) = \bar{Y} \bar{u}_c \left[ 1 + \frac{1}{2} (1 - \sigma^{-1}) \bar{Y}^2 + \sigma^{-1}g_t \bar{Y} \right] + O(\|\xi\|^3) \]
where t.i.p. stands for terms that are independent of policy, specifically \( \bar{u}, \bar{u}_c \xi_t \), and \( \frac{1}{2} \xi_t^2 \bar{u} \xi_t \xi_t \). These terms can be ignored since they are not relevant for the welfare ranking of alternative policies.
A second order expansion of the second term of equation (B.19) gives

\[
v(Y_t; \xi_t) \Delta_t = \bar{v} + \bar{v}(\Delta_t - 1) + \bar{v}_y(Y_t - \bar{Y}) + \bar{v}_y(\Delta_t - 1)(Y_t - \bar{Y}) + (\Delta_t - 1)\bar{v}_\xi \xi_t
\]
\[
+ \bar{v}_x \xi_t + \frac{1}{2} \bar{v}_{yy}(Y_t - \bar{Y})^2 + (Y_t - \bar{Y}) \bar{v}_{x,y} \xi_t + \frac{1}{2} \bar{v}_{x,y} \xi_t + O(||\xi||^3)
\]
\[
= \bar{v}(\Delta_t - 1) + \bar{v}_y(Y_t + \frac{1}{2} \bar{Y})^2 + \bar{v}_y \bar{Y}(\Delta_t - 1)\bar{Y} + \bar{v}_\xi (\Delta_t - 1)\xi_t
\]
\[
+ \frac{1}{2} \bar{v}_{yy} \bar{Y}^2 \bar{Y}^2 + \bar{Y} \bar{v}_{x,y} \xi_t + O(||\xi||^3)
\]

Now note that

\[
\bar{v}_y = \frac{\bar{v}}{Y} \bar{v}_y, \quad \bar{v}_{yy} = \frac{\bar{v}}{Y} \bar{v}_{yy}
\]
\[
\bar{v}_x = -\nu \bar{v}_x, \quad \bar{v}_{x,y} = -(1 + \nu) \bar{v}_y
\]
\[
\bar{v}_\xi = -(1 + \omega) \bar{v}_\xi, \quad \bar{v}_{x,y} = -v \bar{v}_y
\]

Using these expressions, \(v(Y_t; \xi_t) \Delta_t\) can be written as

\[
v(Y_t; \xi_t) \Delta_t = \bar{v}_y \bar{Y} \left[ \frac{\Delta_t - 1}{1 + \omega} + \bar{Y} + \frac{1}{2} (1 + \omega) \bar{Y}^2 + (\Delta_t - 1) \bar{Y} - \bar{Y}_t \omega \theta - \frac{\Delta_t - 1}{1 + \omega} \omega \theta \right] + O(||\xi||^3)
\]

(B.21)

It would be useful to express \(\Delta_t - 1\) in terms of \(\hat{\Delta}_t\). Towards this end, note that using the price index, the price dispersion measure can be written as

\[
\Delta_t = \alpha \Delta_{t-1} \Pi_t \theta^{(1+\omega)} + (1-\alpha) \left( 1 - \alpha \Pi_t \theta^{(1+\omega)} \right) \frac{\theta(1+\omega)}{1-\alpha}
\]

Taking a Taylor series expansion of this equation around \(\Pi = 1\) and \(\hat{\Delta} = 1\) gives

\[
\Delta_t = \alpha \Delta_{t-1} - \frac{1}{2} \left[ \alpha \theta(1+\omega)(\theta - 2) - \frac{\alpha^2 \theta(1+\omega)(1 + \theta \omega)}{1-\alpha} - \alpha \theta(1+\omega)(\theta + \theta \omega - 1) \right] \bar{\Delta}_t^2 + O(||\xi||^3)
\]

(B.22)

This equation has no linear inflation terms, thus \(\Delta_t = O(||\xi||^2)\) for all \(t > t_0\) if \(\Delta_{t_0 - 1} = O(||\xi||^2)\). It follows that \(\Delta_t - 1 = O(||\xi||^2)\) for all \(t > t_0\). Substituting this into equation (B.21) gives

\[
v(Y_t; \xi_t) \Delta_t = \bar{v}_y \bar{Y} \left[ \frac{\Delta_t}{1 + \omega} + \bar{Y}_t + \frac{1}{2} (1 + \omega) \bar{Y}_t^2 - \bar{Y}_t \omega \theta \right] + O(||\xi||^3)
\]

From the household’s first order condition for labor, \(v_y\) can be expressed as a function of \(u_c\):

\[
v_h = (1 - \tau)(\theta - 1) \frac{u_c}{\theta}
\]

Substituting this in the above equation gives

\[
v(Y_t; \xi_t) \Delta_t = (1 - \Phi) \bar{u}_c \bar{\bar{Y}} \left[ \frac{\Delta_t}{1 + \omega} + \bar{Y}_t + \frac{1}{2} (1 + \omega) \bar{Y}_t^2 - \bar{Y}_t \omega \theta \right] + O(||\xi||^3)
\]

(B.23)

Combining equations (B.20) and (B.23) gives a second order expression for the utility function

\[
U = E_t \sum \beta^t \left[ \bar{Y} \bar{u}_c \left( \bar{Y}_t + \frac{1}{2} (1 - \sigma^{-1}) \bar{Y}_t^2 + \sigma^{-1} g_t \bar{Y}_t \right) \right] + O(||\xi||^3)
\]

\[
= E_t \sum \beta^t \left[ (1 - \Phi) \bar{Y} \bar{u}_c \left( \bar{Y}_t + \frac{\Delta_t}{1 + \omega} + \frac{1}{2} (1 + \omega) \bar{Y}_t^2 - \bar{Y}_t \omega \theta \right) \right] + O(||\xi||^3)
\]

\[
= \bar{Y} \bar{u}_c E_t \sum \beta^t \left[ \Phi \bar{Y}_t - \frac{1}{2} \left( \omega - \sigma^{-1} - \Phi (1 + \omega) \right) \bar{Y}_t^2 + \bar{Y}_t \left( \sigma^{-1} g_t + (1 - \Phi) \omega \theta \right) - \frac{1}{1 + \omega} \Delta_t \right] + O(||\xi||^3)
\]

(B.24)
Since $\alpha < 1$, using equation (B.22), $\Delta_t$ can be written in terms of $\hat{\pi}_t$:

$$
\hat{\Delta}_t = \alpha^{t+1} \Delta_{t-1} + \frac{\alpha}{1-\alpha} \theta (1 + \omega)(1 + \omega \theta) \sum_{s=0}^{t} \alpha^{t-s} \frac{\delta_t^2}{2} + O(||\xi||^3)
$$

Multiplying this equation by $\beta^t$ and summing over $t$ gives

$$
\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\alpha}{1-\alpha \beta} \hat{\Delta}_{t-1} + \frac{\alpha}{(1-\alpha)(1-\alpha \beta)} \theta (1 + \omega)(1 + \omega \theta) \sum_{t=0}^{\infty} \beta^t \frac{\delta_t^2}{2} + O(||\xi||^3)
$$

Substituting this expression into equation (B.24) gives

$$
U = \bar{Y} \bar{u}_c E_t \sum_{s=0}^{\infty} \beta^t \left[ \Phi \bar{Y}_t - \frac{1}{2} \left( (\omega + \sigma^{-1}) - \Phi (1 + \omega) \right) \bar{Y}_t^2 + \bar{Y}_t \left\{ \sigma^{-1} g_t + (1 - \Phi) \omega q_t \right\} \right] - \bar{Y} \bar{u}_c \frac{(1 - \Phi) \alpha \theta (1 + \omega \theta)}{(1 - \alpha)(1 - \alpha \beta)} E_t \sum_{t=0}^{\infty} \beta^t \frac{\delta_t^2}{2} + \text{t.i.p.} + O(||\xi||^3)
$$

(B.25)

Multiplying the second order approximation of the aggregate supply equation, (B.18), by $\Phi \bar{Y} \bar{u}_c$ and subtracting the resulting equation from equation (B.25) gives:

$$
U = -\bar{Y} \bar{u}_c E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{q^*_e}{2} \hat{\pi}_t^2 + \frac{q^*_y}{2} (\bar{Y}_t^2 - \bar{Y}_t^*) \right\}
$$

(B.26)

where

$$
q^*_e = \frac{\theta}{\kappa} (\omega + \sigma^{-1}) + \Phi (1 - \sigma^{-1})
$$

$$
q^*_y = (\omega + \sigma^{-1}) + \Phi (1 - \sigma^{-1}) + \frac{\Phi \sigma^{-1} (1 - \bar{Y}^*)}{\omega + \sigma^{-1}}
$$

$$
\bar{Y}_t^* = q^{-1} g_t + (1 - \Phi) \omega \eta_t + (\omega + \sigma^{-1})^{-1} \Phi \left\{ \sigma^{-1} \bar{Y} G_t + \sigma^{-1} (1 - \sigma^{-1}) g_t + (1 + \omega) \omega \eta_t - (1 - \sigma^{-1}) \frac{\tau}{1-\sigma^{-1}} \hat{\pi}_t \right\}
$$

which is a form of the loss function given by equation (1) in the text.

### B.4 Derivation of Equation (25)

Assuming $u^*_t$ and $u^r_t$ are mean zero iid shocks, equation (24) can be written as:

$$
\hat{\pi}_t = d_1 (1 - \kappa q_t) u^*_t + d_1 \kappa \psi (L + \beta d_1) u^r_t - E_t d_1 \sum_{j=0}^{\infty} (\beta d_1)^j \left\{ \kappa q_t \frac{L - \theta}{1 - \theta L} u^*_{t+j} \right\}
$$

(B.27)

Thus, the last term needs to be simplified in order to derive equation (25) in the text. Towards this end, write this term as:

$$
E_t d_1 \kappa q_r \sum_{j=0}^{\infty} (\beta d_1)^j \frac{1}{1 - \theta L} u^*_{t+j-1} - E_t d_1 \theta \kappa q_r \sum_{j=0}^{\infty} (\beta d_1)^j \frac{1}{1 - \theta L} u^r_{t+j}
$$

(B.28)

Notice that this can be rewritten as

$$
d_1 \kappa q_r E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u^r_{t+j-1-s} - d_1 \theta \kappa q_r E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u^r_{t+j-s}
$$

(B.29)
Consider term 2, which can be written as:

\[
E_t \sum_{j=0}^{\infty} \sum_{s=0}^{\infty} \theta^s u_t^j \kappa_{j-s} \\
= \sum_{s=0}^{\infty} \theta^s u_{t-s}^j + \beta d_1 E_t \sum_{s=0}^{\infty} \theta^s u_{t+1-s}^j + (\beta d_1)^2 E_t \sum_{s=0}^{\infty} \theta^s u_{t+2-s}^j + \ldots
\]

(B.30)

Now notice that \(u_t^j\) is an iid mean zero shock, so that \(E_t[u_{t+j}] = 0\) for \(j > 0\). This means that

\[
E_t \sum_{s=0}^{\infty} \theta^s u_{t+1-s}^j = \theta u_t^j + \theta^2 u_{t-1}^j + \theta^3 u_{t-2}^j + \ldots
\]

(B.31)

Using this fact, expression (B.30) can be rewritten as

\[
\sum_{s=0}^{\infty} \theta^s u_{t-s}^j + \beta d_1 \theta E_t \sum_{s=0}^{\infty} \theta^s u_{t-s}^j + (\beta d_1 \theta)^2 E_t \sum_{s=0}^{\infty} \theta^s u_{t-s}^j + \ldots
\]

\[
= (1 + \beta d_1 \theta + (\beta d_1 \theta)^2 + \ldots) \sum_{s=0}^{\infty} \theta^s u_{t-s}^j
\]

\[
= (1 + \beta d_1 \theta + (\beta d_1 \theta)^2 + \ldots) \frac{1}{1 - \theta L} u_t^j
\]

\[
= \frac{1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^j
\]

(B.32)

Similarly, term 1 can be written as:

\[
E_t \sum_{j=0}^{\infty} (\beta d_1)^j \sum_{s=0}^{\infty} \theta^s u_{t+j-s}^j
\]

\[
= \sum_{s=0}^{\infty} \theta^s u_{t-1-s}^j + \beta d_1 E_t \sum_{s=0}^{\infty} \theta^s u_{t-s}^j + (\beta d_1)^2 E_t \sum_{s=0}^{\infty} \theta^s u_{t+1-s}^j + (\beta d_1)^3 E_t \sum_{s=0}^{\infty} \theta^s u_{t+2-s}^j + \ldots
\]

\[
= \frac{L}{1 - \theta L} u_t^j + \beta d_1 \frac{1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^j
\]

(B.33)

Substituting expressions (B.32) and (B.33) into expression (B.29) gives

\[
d_1 \kappa_{\theta} \left[ \frac{L}{1 - \theta L} u_t^j + \beta d_1 \frac{1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^j - \frac{1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^j \right]
\]

(B.34)

Thus,

\[
E_t d_1 \sum_{j=0}^{\infty} (\beta d_1)^j \left\{ \kappa_{\theta} \frac{L}{1 - \theta L} u_t^j + \beta d_1 \frac{1}{1 - \beta d_1 \theta} \frac{1}{1 - \theta L} u_t^j \right\}
\]

\[
= d_1 \kappa_{\theta} \left[ \frac{L}{1 - \theta L} + \beta d_1 - \theta \right] u_t^j
\]

(B.35)

Substituting this into equation (B.27) gives equation (25) in the text.
C Derivations for the Model with a Taylor Rule

This appendix gives derivations for the model when the central bank sets the short-term interest rate, \( i_t \), via a simple Taylor rule:

\[
\dot{i}_t = \phi_y \dot{Y}_t + \phi_y \pi_t \tag{C.1}
\]

I assume taxes are set exogenously according to

\[
\dot{r}_t = \rho_r \dot{r}_{t-1} + \alpha_r u^r_{t-1} + (1 - \nu) u^r_t \tag{C.2}
\]

Equations (2), (3), (C.1), and (C.2) characterize an equilibrium for this economy. In this case the equilibrium is determinate if and only if

\[
\phi_y + \frac{1 - \beta}{\kappa} \phi_y > 1
\]

The proof is given in Woodford (2003). In what follows I will assume this condition is satisfied.

C.1 No Foresight

In this section I assume the standard timing assumption of the fiscal literature where a change in tax policy at period \( t \) alters tax rates at period \( t \), i.e. a tax "shock." Using the method of undetermined coefficients, the unique analytical solution to equations (C.1) - (C.2) is

\[
\begin{align*}
\dot{Y}_t &= a_1 \dot{r}_{t-1} + a_2 u^r_t + a_3 u^r_t \\
\dot{\pi}_t &= b_1 \dot{\pi}_{t-1} + b_2 u^r_t + b_3 u^r_t
\end{align*}
\]

where

\[
\begin{align*}
a_1 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & b_1 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0 \\
a_2 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & b_2 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0 \\
a_3 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & b_3 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0
\end{align*}
\]

Without foresight, a tax shock acts like a supply shock, causing output to decrease and prices to increase with a tax increase, since production is less profitable.

C.2 One Period Foresight

In this section I assume one period fiscal foresight. Using the method of undetermined coefficients, the unique analytical solution to equations (C.1) - (C.2) is

\[
\begin{align*}
\dot{\pi}_t &= \tilde{a}_1 \dot{\pi}_{t-1} + \tilde{a}_2 u^r_t + \tilde{a}_3 u^r_{t-1} + \tilde{a}_4 u^r_{t-1} \\
\dot{Y}_t &= b_1 \dot{\pi}_{t-1} + b_2 u^r_t + b_3 u^r_t + b_4 u^r_{t-1}
\end{align*}
\]

where

\[
\begin{align*}
\tilde{a}_1 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & \tilde{b}_1 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0 \\
\tilde{a}_2 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & \tilde{b}_2 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0 \\
\tilde{a}_3 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & \tilde{b}_3 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0 \\
\tilde{a}_4 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} > 0, & \tilde{b}_4 &= \frac{\kappa \phi (1 - \beta + \kappa \phi)}{1 + \beta \rho + \kappa \phi \rho + \phi \rho (1 + \beta + \kappa \phi + \beta \phi)} < 0
\end{align*}
\]

The realization of a tax increase causes inflation to increase and output to decrease. If \( \rho_r = \phi_y = 0 \), then news of a tax increase will always cause inflation to increase and, thus, the interest rate to increase. If \( \rho_r > 0 \) and \( \phi_y > 0 \),
then the effects of tax news on inflation and output are ambiguous. However, it is possible to characterize how the news will affect the interest rate. Notice in this case the effect of tax news on the interest rate is given by

\[
\phi_y \kappa \psi [\kappa \sigma (1 - \phi_u + \sigma \phi_y) - \beta (\rho \tau - 1 - \sigma \phi_y)(1 + \sigma \phi_y)] - \phi_y \kappa \sigma \psi [\phi_u (1 + \beta (1 - \rho \tau) + \beta \sigma \phi_y) - 1 - \sigma \phi_y] u_t \]  

\[= \frac{\kappa \psi [\kappa (\kappa \phi_u + \phi_y) (1 - \phi_u + \sigma \phi_y) + \beta \phi_u (1 + \sigma \phi_y - \rho \tau)]}{[1 + \sigma (\phi_u + \phi_u \kappa)] [1 + \beta \phi \tau + \kappa \sigma \phi_u + \sigma \phi_y - \rho \tau (1 + \beta + \kappa \sigma + \beta \sigma \phi_y)]} u_t \]  

The denominator is always positive while the numerator is positive if \((1 + \sigma \phi_y) > \phi_x\) (which is plausible since for most parameter values \(\sigma\) is greater than 1) or if

\[
1 - \phi_u + \sigma \phi_y < \frac{\beta \phi_x (1 + \sigma \phi_y - \rho \tau)}{\sigma (\kappa \phi_u + \phi_y)},
\]

which will hold for most parameterizations as long as \(\kappa\) is not too large (i.e. roughly if \(\kappa < 5\)). Thus, for most parameter values the interest rate will increase when there is news of a tax increase. Notice that if \(\phi_y = 0\) (and hence \(\phi_u > 1\)), then news of a tax increase will always cause inflation to increase and, thus, the interest rate to increase.
Table 1: Calibrated parameter and steady-state values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0236</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$1/3$</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.473</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.0236</td>
</tr>
<tr>
<td>$\bar{\tau}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\sigma_s^2$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma_s^2$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 2: Welfare Consequences of Various Degrees of Fiscal Foresight.

<table>
<thead>
<tr>
<th>Loss Function Values</th>
<th>No Foresight</th>
<th>1 Period Foresight</th>
<th>4 Period Foresight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretion</td>
<td>0.0272</td>
<td>0.0298</td>
<td>0.0332</td>
</tr>
<tr>
<td>Timeless</td>
<td>0.0207</td>
<td>0.0222</td>
<td>0.0225</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}_t$</td>
<td>0.2238</td>
<td>0.2335</td>
<td>0.2601</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}<em>t - \hat{Y}</em>{t-1}$</td>
<td>0.2234</td>
<td>0.2327</td>
<td>0.2597</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{y}_t$</td>
<td>0.2009</td>
<td>0.02099</td>
<td>0.2336</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variances</th>
<th>No Foresight</th>
<th>1 Period Foresight</th>
<th>4 Period Foresight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>$5.6846 \times 10^{-6}$</td>
<td>$6.1933 \times 10^{-5}$</td>
<td>$6.8533 \times 10^{-5}$</td>
</tr>
<tr>
<td>Timeless</td>
<td>$4.1287 \times 10^{-6}$</td>
<td>$4.2127 \times 10^{-5}$</td>
<td>$4.1467 \times 10^{-5}$</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}_t$</td>
<td>$5.8729 \times 10^{-4}$</td>
<td>$6.3395 \times 10^{-4}$</td>
<td>$7.5347 \times 10^{-4}$</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}<em>t - \hat{Y}</em>{t-1}$</td>
<td>$5.827 \times 10^{-4}$</td>
<td>$6.3837 \times 10^{-4}$</td>
<td>$3.9654 \times 10^{-4}$</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{y}_t$</td>
<td>$2.8332 \times 10^{-4}$</td>
<td>$3.0620 \times 10^{-4}$</td>
<td>$3.6505 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output Gap</th>
<th>No Foresight</th>
<th>1 Period Foresight</th>
<th>4 Period Foresight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretion</td>
<td>0.0059</td>
<td>0.0064</td>
<td>0.0071</td>
</tr>
<tr>
<td>Timeless</td>
<td>0.0056</td>
<td>0.0065</td>
<td>0.0075</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}_t$</td>
<td>0.1192</td>
<td>0.1192</td>
<td>0.1193</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}<em>t - \hat{Y}</em>{t-1}$</td>
<td>0.1190</td>
<td>0.1190</td>
<td>0.1191</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{y}_t$</td>
<td>0.1024</td>
<td>0.1024</td>
<td>0.1026</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output</th>
<th>No Foresight</th>
<th>1 Period Foresight</th>
<th>4 Period Foresight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretion</td>
<td>0.1805</td>
<td>0.1810</td>
<td>0.1818</td>
</tr>
<tr>
<td>Timeless</td>
<td>0.1678</td>
<td>0.01699</td>
<td>0.1715</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}_t$</td>
<td>$1.4476 \times 10^{-5}$</td>
<td>$1.639 \times 10^{-5}$</td>
<td>$2.6613 \times 10^{-5}$</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{Y}<em>t - \hat{Y}</em>{t-1}$</td>
<td>$1.6908 \times 10^{-5}$</td>
<td>$1.9703 \times 10^{-5}$</td>
<td>$3.8167 \times 10^{-5}$</td>
</tr>
<tr>
<td>Taylor Rule w/ $\hat{y}_t$</td>
<td>$8.4493 \times 10^{-4}$</td>
<td>$8.7998 \times 10^{-4}$</td>
<td>$9.7738 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
Table 3: Welfare Consequences of Various Observables in the VAR.

<table>
<thead>
<tr>
<th></th>
<th>Discretion</th>
<th>Inflation &amp; Technology</th>
<th>Inflation &amp; Taxes</th>
<th>Inflation &amp; Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss Function Value</td>
<td>0.0298</td>
<td>0.0348</td>
<td>0.0310</td>
<td>0.0368</td>
</tr>
<tr>
<td>$\text{Var}(\hat{\pi})$</td>
<td>$6.1933 \times 10^{-5}$</td>
<td>$6.6278 \times 10^{-5}$</td>
<td>$6.2009 \times 10^{-6}$</td>
<td>$6.6465 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\text{Var}(\hat{y})$</td>
<td>0.0064</td>
<td>0.0101</td>
<td>0.0101</td>
<td>0.0128</td>
</tr>
<tr>
<td>$\text{Var}(\hat{y}^*)$</td>
<td>-</td>
<td>0.0069</td>
<td>0.0059</td>
<td>0.0069</td>
</tr>
<tr>
<td>$\text{Var}(i)$</td>
<td>7.5432</td>
<td>7.4489</td>
<td>8.2132</td>
<td>7.2424</td>
</tr>
</tbody>
</table>

Figure 1: Impulse responses to an unanticipated 1% tax increase under various monetary policies. The response under the Taylor rule responding to output is not included as it is almost quantitatively the same as the Taylor rule responding to output growth.
Figure 2: Impulse responses to news of a 1% tax increase one period prior to the tax change. The response under the Taylor rule responding to output is not included as it is almost quantitatively the same as the Taylor rule responding to output growth.
Figure 3: Impulse responses to news of a 1% tax increase four periods prior to the tax change. The response under the Taylor rule responding to output is not included as it is almost quantitatively the same as the Taylor rule responding to output growth.
Figure 4: Impulse responses to news of a 1% tax increase one period prior to the tax change.
Figure 5: Impulse responses to an unanticipated 1% decrease in technology productivity.