Markov Switching Terms of Trade

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Preliminary

Abstract

The paper studies the effect of terms of trade fluctuations on equilibrium debt position of a small open economy. I develop a simple model of a small open economy producing exportable and nontradable goods to investigate the effects of terms of trade shock assuming that terms of trade follow two state Markov switching process. Under this process, permanent terms of trade shock means change of regime. With switching probability, rational agent optimizes debt position according to the current state and the probability of transition. I define switching effect using transition probability and state dependent parameters. Unlike traditional argument, permanent terms of trade shock now has either positive or negative effect depending on the magnitude of state-contingent parameter.

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1 Introduction

The impact of terms of trade shock on a country’s current account balance is a contended issue in international economics. In two classic articles, Harberger (1950) and Laursen and Metzler (1950) postulated that a decrease in current income arising from an adverse terms of trade shock would lower both private savings and the current account balance (The Harberger-Laursen-Metzler effect). A deterioration in the terms of trade decreases a country’s real income level therefore lowers the purchasing power of the country. If we assume the marginal propensity to consume to be less than unity, private savings will decrease in a static open economy model. However, if agents optimize utilities from forward-looking dynamic decisions, Obstfeld (1982), Sachs (1981) and Svensson and Razin (1983) have explained the response of private savings to terms of trade shocks depends on the duration of the shock. In case of permanent improvements in the terms of trade being expected, economic agents will revise upward their estimate of national income in current as well as future periods. Unlike the HLM effect, the expected permanent higher level of income would result in higher level of consumption without changing savings. On the other hand, if improvements are expected to be temporary, economic agents will smooth this one-time additional income over all future periods by raising savings. Hence the HLM effect exists in the presence of only transitory terms of trade shocks.

These frameworks have been extended in different ways and figured out various channels in which terms of trade affect the current account balance. Later studies (Dornbusch, 1983; Edwards, 1989) investigated the fact that transitory shocks to the terms of trade have unambiguous effect on private savings. Using a three good (imports, exports, non-tradable goods) model, these studies showed that an adverse terms of trade shock have three different ways to affect private savings. First, an adverse terms of trade shock will lower the current national income relative to future national income therefore agents will adjust their savings to smooth consumption (the consumption smoothing or HML effect). Second, an adverse terms of trade shock will increase the price of current imports relative to future imports. This cause consumers to postpone their consumption of imports and save more (the consumption tilting effect). At last, an adverse terms of trade shock will increase the current price of imports relative to the current price of the non-tradable goods and it generates
an appreciation of the real exchange rate. This, in turn, will make agents to postpone the relatively expensive current consumption of imports and increase savings (the real exchange rate effect). Hence, in response to an adverse transitory terms of trade shock, private savings will increase if the consumption smoothing effect dominates the saving-enhancing effects of the consumption-tilting and real exchange rate effects. Within this context, Cashin and McDermott (2002) empirically examined the relationship between temporary terms of trade shock and private savings, using data series of five OECD countries for a two good (tradables and non-tradables) model. They found that intertemporal and intratemporal substitution of consumption between tradables and non-tradables both have large effect on private saving. In a recent paper, Agenor and Aizenman (2003) have suggested that if households subject to a borrowing constraint terms of trade shocks can also lead to an asymmetric response in savings. Under a borrowing constraint, households may not be able to smooth consumption when faced with adverse shocks to the terms of trade. As a result, households should reduce savings to maintain a smooth consumption path. Also if agents anticipate facing restrictive borrowing constraints during economic slumps, they may also consume less and save more in economic booms according to the possibility of being constrained.

In addition to the discussion about the relationship between terms of trade and private savings, a number of studies indicate that a country’s terms of trade are as important as its productivity in determining capital investment and production. Empirical analyses covering the relationship between the terms of trade and country’s output growth find that an adverse terms of trade shock depresses real demand and real incomes, leading in lower growth.1 On the other hand, a rise in the terms of trade would induce greater capital investment and an increase in output. In a dynamic setting, these lead to an increase in the growth rate for as long as the capital stock to the new equilibrium is maintained by new investment from high terms of trade. In practice, for sizeable changes in the terms of trade, this may yield a prolonged positive relationship between the level of the terms of trade and the country’s growth rate.

In addition to the level of the terms of trade, volatility in the terms of trade may be important. Turnovsky and Chattopadhyay (2003) investigate that various sources of volatility can affect the

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The volatility may reduce capital intensity, especially where a country faces an upward sloping supply of capital. Mendoza (1997) finds that higher terms of trade volatility has a negative impact on a country’s economic growth rate. Across his sample of 9 industrial and 31 developing countries, he finds that growth is slower in countries with higher terms of trade volatility. This result is robust to a number of different specifications. Mendoza postulates that the terms of trade volatility affects savings behavior and therefore growth. As with Turnovsky and Chattopadhyay, he finds also that the growth in the terms of trade affects the mean growth rate. This collection of results suggests that a country with volatile terms of trade will tend to have lower investment rates, lower capital stock and lower economic growth than a country with less terms of trade volatility. This paper focuses on the process of terms of trade and how the assumed process changes the relationship between terms of trade and equilibrium debt position. Specifically, the conventional process for terms of trade is first order autoregressive. In this paper I suggest a

\footnote{Their empirical work finds that terms of trade volatility has a statistically significant negative impact on the growth rate across a sample of 61 developing countries. The mean growth rate of the terms of trade has a weak positive impact on the mean economic growth rate.}

\footnote{Agenor et al. (2002) also finds terms of trade disturbances to current accounts to be highly correlated with output fluctuations.}
modest alternative to this approach, exploring the terms of trade follow a nonlinear process rather than a linear process and the source of nonlinearity is shift in regime of terms of trade. Terms of trade have both low-frequency and high-frequency fluctuations. If this is inherited from different regime, ignoring the nonlinearity would induce bias. In this context, I assume terms of trade follow 2 state markov switching process and investigate the effect of terms of trade on a country’s debt level.

The plan of the paper is as follows. In the next section, I provide some empirical facts supporting switching in terms of trade process. Section 2 specifies two kinds of models with three goods and compares markov switching terms of trade process with AR(1) process by deriving the effect on current account or debt level. Brief conclusion is offered in section 3.

1.1 Empirical motivation

Fig.(1) plots the logarithm of quarterly terms of trade from 1960:I to 2006:I for Canada and New Zealand. The graph indicates that the terms of trade volatility was high during the 1970s reflecting

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4 The discussion about nonlinearity cause by discrete shift in regime can be found Hamilton(1989).
5 The terms of trade are calculated using export unit price over import unit price.
the oil crises and reduced slightly in the 1980s and more so in the 1990s both countries. To see how the level and the volatility of terms of trade developed, I investigate the changes in the sample mean and the sample variance of terms of trade for different sample periods.

Fig.(2) illustrates that the mean and volatility of terms of trade of Canada has been changed over the periods. This shows the possibility of nonlinearity in terms of trade process. The mean value displays more complicated behavior and the volatility clearly shows there are periods of high volatility and low volatility.

In addition to graphical investigation, I attempt to estimate 2-state markov switching terms of trade process using Canadian data. Before estimating the process, I test the stationarity of terms of trade process. Since unit root is the most popular type of non-stationarity, Augmented Dickey-Fuller test is used to test of existence of unit root in Canadian terms of trade data. Like many other macro variables, test statistics cannot reject the existence of unit root. As a remedy for unit root problem, I take first order differences and test result shows that there is no unit root in the first differences of the log of terms of trade process. Table(1) shows the test results for the log of terms of trade($ln(TOT)$) and the first differences of the log of terms of trade($\Delta ln(TOT)$).

Markov switching terms of trade process to be estimated are defined by

$$tt_t = \rho(s_t)tt_{t-1} + \epsilon_t \quad (1)$$

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6These graphs are using 5 year-rolling windows for Canada. For different periods(2 year or 10 year) and New Zealand’s graphs are available in the appendix.

7Refer to Hamilton(1989) or Kim and Nelson(1999).
Table 2: Estimated parameters of switching process

<table>
<thead>
<tr>
<th>state</th>
<th>ρ(s_t)</th>
<th>ϵ_t(s_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0162 (0.0283)</td>
<td>0.0262(0.0009)</td>
</tr>
<tr>
<td>2</td>
<td>0.3681(0.2166)</td>
<td>0.0087(0.0036)</td>
</tr>
</tbody>
</table>

where

\[ ϵ_t \sim N(0, σ^2(s_t)) \]
\[ ρ(s_t) = ρ_1(1 - S_t) + α_2 S_t \]
\[ σ^2(s_t) = σ_1^2(1 - S_t) + σ_2^2 S_t \]
\[ S_t = 0 \text{ or } 1 \]

There are two different states \((S_t = 1 \text{ or } 2)\). Terms of trade follow the first order autoregressive process within state and the coefficient and variance are state dependent. The switch between states governed by transition matrix \(P\). For example, the terms of trade may either be in a fast growth or a slow growth phase by the realization of state. Or terms of trade may either be highly volatile or low volatile depending on state. Table(2) reports the parameter estimates and standard errors in the parenthesis. The estimates of \(ρ\) are statistically not significant but estimates \(σ\) are significant. The simple observation and estimation done here are suggesting terms of trade process has nonlinearity and I adopt markov switching process to explore the effect of terms of trade shock on equilibrium.

2 Model

2.1 Endowment economy

The analysis considers a small open economy populated by a large number of identical infinitely lived households. Each agent maximizes a utility function given by

\[ E_0 \sum_{t=0}^{∞} β^t u(c_{m,t}, c_{n,t}). \]
The concave utility function $u(\cdot,\cdot)$ is defined over the household’s consumption of an importable good($m$) and a non-tradable good($n$). The parameter $\beta(<1)$ is the subjective discount factor. Each household is endowed with exogenous, perishable flows of $\{y^x_t, y^n_t\}$ units of the export good and the non-tradable good respectively. Households can borrow from the rest of the world. Since this economy is small, the world interest rate is given exogenously and assumed to be a constant $r$. Households face a budget constraint given by

$$p^m_t (1 + r_t) d_{t-1} + p^m_t c_{m,t} + p^n_t c_{n,t} = p^n_t d_t + p^n_t y^n_t + p^x_t y^x_t$$

(3)

where $p^m$, $p^x$ and $p^n$ is the price of an importable, an exportable and a non-tradable good respectively and $d_t$ is the debt position. Households are also subject to a no-Ponzi scheme constraint of the form

$$\lim_{j \to \infty} E_t \frac{d_{t+j}}{(1 + r_t)^j} \leq 0$$

(4)

Hence the household’s problem is to choose $\{c_{m,t}, c_{n,t}, d_t\}$ to maximize (2) subject to (3) and (4). Necessary conditions for this problem are given by

$$u_m(c_{m,t}, c_{n,t}) = \lambda_t$$

$$u_n(c_{m,t}, c_{n,t}) = \lambda_t p^n_t$$

$$E_t \lambda_{t+1} (1 + r_t) \beta = \lambda_t$$

If we use the definition of terms of trade (the relative price of exports in terms of imports), with market clearing condition for the non-tradable good($c_{n,t} = y^n_t$), the budget constraint (3) becomes

$$(1 + r) d_{t-1} = d_t - c_{m,t} + tt_t y^x_t$$

(5)

where $p^x_t / p^m_t \equiv tt_t$. I make additional assumptions to facilitate the analysis. The output of an exportable good remains constant throughout all the time period and the gross interest rate at which domestic agents borrow from the rest of the world is constant and equal to the inverse of
subjective discount factor.\(^8\) Also the functional form of utility function is quadratic.

\[
y_t^x = 1 \quad \forall t
\]

\[
\beta(1 + r) = 1
\]

\[
u(c_m, c_n) = -\frac{(c_m^2 + c_n^2)}{2}
\]

Hence the Euler equation becomes,

\[
c_{m,t} = E_t c_{m,t+1}
\]

Furthermore, I iterate \(d_t\) forward from the resource constraint with the transversality condition to have

\[
(1 + r)d_{t-1} = tt_t - c_{m,t} + d_t
\]

\[
= E_t \sum_{j=0}^{\infty} \frac{tt_{t+j} - c_{m,t+j}}{(1 + r)^j} \tag{7}
\]

\[
= \frac{1 + r}{r} c_{m,t} + E_t \sum_{j=0}^{\infty} \frac{tt_{t+j}}{(1 + r)^j} \tag{8}
\]

\[

The consumption of an importable good \((c_{m,t})\) is given by

\[
c_{m,t} = rd_{t-1} - \frac{r}{1 + r} E_t \sum_{j=0}^{\infty} \frac{tt_{t+j}}{(1 + r)^j} \tag{10}
\]

\(^8\)With constant interest rate, the model displays no stationary. The model induces a random walk component in the equilibrium marginal utility of consumption and the net foreign asset position. Schmitt-Grohe and Uribe(2002) presents four different stationary small open economy models and a quantitative comparison of those alternatives including non-stationary case. The main finding was the four specified models deliver similar business cycle features in terms of unconditional second moments and impulse-response. They suggested the researchers to choose model specifications according to computational convenience. In this section, I focus on investigating markov-switching terms of trade shocks on macroeconomic variables while ignoring stationary issue but three sector production model, I have debt adjustment with constant world interest rate to induce stationarity.
Therefore the current account for this economy is given by

\[ CA_t = -(d_t - d_{t-1}) \]  
\[ = -rd_{t-1} - c_{m,t} + tt_t \]  
\[ = -2rd_{t-1} - \frac{r}{1 + r} E_t \sum_{j=0}^{\infty} \frac{tt_{t+j}}{(1 + r)^j} + tt_t. \]  

To summarize, in no investment, endowment, small open economy, the agent exports and borrows to consume the importable good. Therefore the current account depends solely on the relative price of exports and imports, which is the terms of trade. I will trace out the response of imports consumption and current account to different terms of trade shocks using equations (10) and (13) under different assumed terms of trade process.

### 2.1.1 Effects of Terms of trade process

Some empirical works have been modeled the terms of trade process by the first order autoregressive process.\(^9\) The AR(1) terms of trade process is given by

\[ tt_t = \rho tt_{t-1} + \epsilon_t \]  

where \( \epsilon_t \) is white noise error term and \( 0 \leq \rho < 1 \). Therefore (10) and (13) become

\[ c_{m,t} = rd_{t-1} - \frac{r}{1 + r - \rho} tt_t \]  
\[ CA_t = -2rd_{t-1} + \frac{1 - \rho}{1 + r - \rho} tt_t. \]  

How do \( c_{m,t} \) and \( CA_t \) vary with \( tt_t \)?

\[ \frac{\partial c_{m,t}}{\partial tt_t} = \frac{-r}{1 + r - \rho} \]  
\[ \frac{\partial (CA_t)}{\partial tt_t} = \frac{1 - \rho}{1 + r - \rho} \]

\(^9\) Mendoza (1995) is one example.
Using above, we can find the effects of terms of trade. If the terms of trade shock is transitory ($\rho = 0$) and adversary, it makes the agent to increase her consumption for imports because of low price. However she needs to borrow more from abroad to support consumption and this leads current account to decrease. Therefore the current account has positive relationship with terms of trade shock (HLM effect). In general, for $0 < \rho < 1$, consumption of imports has negative relationship with terms of trade and current account has positive relationship with them.\(^{10}\)

When a positive but permanent terms of trade shock occurs ($tt_t = tt_{t+1} = \cdots = tt_{t+j}$), the import consumption and current account are given by,

\[
c_{m,t} = r d_{t-1} - tt_t
\]
\[
CA_t = -2 r d_{t-1}.
\]

This is the case that the agent decreases the consumption of imports as much as the terms of trade improved. The agent expects an increase their income permanently hence no HLM effect exist.

\[
\frac{\partial c_{m,t}}{\partial tt_t} = -1
\]
\[
\frac{\partial (CA_t)}{\partial tt_t} = 0
\]

### 2.1.2 Markov switching terms of trade process

As introduced in introduction, certain characteristics of terms of trade cannot be captured with simple AR(1) process. In this section, I return to markov switching terms of trade process. Terms of trade process are given by

\[
\begin{align*}
tt_{1t} &= \rho_1 tt_{t-1} + \epsilon_{1t} \\
tt_{2t} &= \rho_2 tt_{t-1} + \epsilon_{2t}.
\end{align*}
\]

\(^{10}\)Note that with $\rho < 0$, we have pervert result. I assume $0 < \rho < 1$ from now on.
Under Markov switching terms of trade process, the consumption of imports is given by,

\[
\begin{pmatrix}
c_{1m,t} \\
c_{2m,t}
\end{pmatrix} = \begin{pmatrix}
rd_{t-1} \\
r_{t-1}
\end{pmatrix} - \frac{r}{1+r} \left[ I - \left( \frac{1}{1+r} \right) P \right]^{-1} \begin{pmatrix}
tt_{1t} \\
tt_{2t}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
rd_{t-1} \\
r_{t-1}
\end{pmatrix} - \frac{1}{2 + r + p_{11} + p_{22}} \begin{pmatrix}
(1 + r - p_{22})tt_{1t} + (1 - p_{11})tt_{2t} \\
(1 - p_{22})tt_{1t} + (1 + r - p_{11})tt_{2t}
\end{pmatrix}
\]

\[P\] is a transition matrix specified by

\[
P = \begin{pmatrix}
p_{11} & 1 - p_{11} \\
1 - p_{22} & p_{22}
\end{pmatrix}
\]

where \[p_{ij}\] is the probability of \(S_t = j|S_{t-1} = i\). The consumption of imports becomes state dependent because of terms of trade process. Notice that consumption at state 1 \((c_{1m,t})\) also depends on terms of trade at state 2 \((tt_{2,t})\) through transition matrix. Hence the definitions of permanent and transitory shocks need to be corrected. The terms of trade process is assumed to follow an AR(1) process within each regime. Transitory shock has the conventional definition within the regime and it is related to the magnitude of the AR(1) coefficients \(\rho_1\) and \(\rho_2\). However once the state is changed, the whole process has to be changed so permanent shock within regime does not exist. Therefore a shock from shift in state is a permanent shock in this case. I explore the permanent terms of trade effect in next section more closely.

### 2.2 Three sector production Economy

In this section, I incorporate productions into a small open economy model. Unlike the two-good open economy model, the introduction of non-tradable sector will make it possible to analyze the terms of trade shock on the real exchange rate (the relative price of non-tradable in terms of imports). The goods market are consist of three sectors, non-tradable, exportable and importable goods. It is assumed that imports are produced entirely abroad and consumed domestically and exports are produced but not consumed domestically. Thus private agents derive utilities from the
consumption of the non-tradable and the imported goods and from leisure. Imported goods are also used for production as capital goods.  

2.2.1 Household

The economy consists of a large number of identical households with preferences given by

\[ E_t \sum_{t}^{\infty} = \beta^t u(C_{m,t}, C_{n,t}, H_t) \] (20)

where \( C_{m,t} \) denotes the consumption of imports, \( C_{n,t} \) denotes the consumption of non-tradable goods, \( H_t \) denotes labor effort, and \( u(\cdot) \) is a period utility function. Households offer labor services for wages \( W_{n,t} \) and \( W_{x,t} \) and borrow from the rest of the world. The interest rate faced by domestic agents in world financial markets is assumed to be constant (\( r \)) and the price of imports (\( P_{m,t} \)) is chose as the numeraire. Agents face convex costs of holding assets in quantities different from steady state level (\( \psi_2 (D_t - D)^2 \)).  

Therefore households face a budget constraint given by

\[ P_{n,t}C_{n,t} + C_{m,t} + (1 + r)D_{t-1} + \frac{\psi}{2} (D_t - D)^2 \leq D_t + W_{x,t}H_{x,t} + W_{n,t}H_{n,t}. \] (21)

In addition, the functional form of utility function is assumed to be additively separable for simplicity.

\[ E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{m,t}^{1-\gamma_m}}{1-\gamma_m} + \frac{C_{n,t}^{1-\gamma_n}}{1-\gamma_n} \right) - \frac{H_{n,t}^{1+\varphi_n}}{1+\varphi_n} - \frac{H_{x,t}^{1+\varphi_x}}{1+\varphi_x} \] (22)

\[ P_{n,t}C_{n,t} + C_{m,t} + (1 + r)D_{t-1} + \frac{\psi}{2} (D_t - D)^2 \leq D_t + W_{x,t}H_{x,t} + W_{n,t}H_{n,t}. \] (23)

The households’ maximization problem is that households choose processes \{\( C_{n,t}, C_{m,t}, H_{x,t}, H_{n,t}, D_t \)\} given prices \{\( P_{n,t}, P_{x,t}, W_{x,t}, W_{n,t} \)\} so as to maximize (22) subject to (23) and a no-Ponzi constraint (4). Letting \( \lambda_t \) denotes the Lagrange multiplier on (23), the first-order conditions of the household’s

\[ ^{11} \text{According to Mendoza(1995), most developing countries are using imports as capital goods for production. This would be the general model contrast to this model.} \]

\[ ^{12} \text{With constant world interest rate, adjustment cost will induce stationarity.} \]
maximizing problems are

\[ C_{m,t}^{-\gamma_m} = \lambda_t \] (24)

\[ C_{n,t}^{-\gamma_n} = \lambda_t P_{n,t} \] (25)

\[ \lambda_t [1 - \psi(D_t - D)] = \beta(1 + r)E_t \lambda_{t+1} \] (26)

\[ H_{x,t}^{x} = W_{x,t} \lambda_t \] (27)

\[ H_{n,t}^{n} = W_{n,t} \lambda_t \] (28)

As in the endowment economy, at an optimum, household decides consumptions of each goods by equating their marginal rate of substitution between non-tradable and importable to the relative price of non-tradable goods in terms of importable goods.

\[ \frac{u_m(m, n, h)}{u_n(m, n, h)} = \frac{C_{m,t}^{-\gamma_m}}{C_{n,t}^{-\gamma_n}} = \frac{1}{P_{n,t}} \]

### 2.2.2 Firms

Goods are categorized into exportable, importable and non-tradable goods. All goods are produced only by labor using linear technology. Households supply labor input for exportable and non-tradable goods sector. The economy do not produce any of importable goods. These assumptions allow analytical solutions of log-linearized systems for this economy.

\[ Y_{i,t} = A_{i,t} H_{i,t}, \text{ for } i = n, x \] (29)

Necessary conditions for producers’ maximization problems are

\[ W_{n,t} = A_{n,t} P_{n,t} \] (30)

\[ W_{x,t} = A_{x,t} P_{x,t} \] (31)

where \( P_{x,t} \equiv tt_t \) by construction.
2.2.3 Equilibrium

There are three market clearing conditions. The supply of non-tradable goods is the total consumption of non-tradable goods and labor markets are also cleared.

\[ C_{n,t} = Y_{n,t} \] (32)
\[ H^a_{x,t} = H^d_{x,t} \] (33)
\[ H^a_{n,t} = H^d_{n,t} \] (34)

Addition to market clearing conditions, in equilibrium, the wages across sector should be the same.

\[ W_{x,t} = W_{n,t} \] (35)

I solve this systems by log-linearizing variables around their steady state. The lower case characters stand for variables in deviations from steady states. \((x = dx/X)\).\(^{13}\) Log-linearized equations have solved using MSV methods. Equilibrium allocations in terms of deviations from steady state are :

\[ p_{m,n,t} = a_{x,t} - a_{n,t} + \Delta t_{it} \] (36)
\[ c_{m,n,t} = \frac{\varphi_n(\gamma_n - 1)}{\gamma_m(\gamma_n + \varphi_n)} a_{n,t} + \frac{1}{\gamma_m} a_{x,t} + \frac{1}{\gamma_m} \Delta t_{it} \] (37)
\[ c_{n,t} = \frac{1 + \varphi_n}{\gamma_n + \varphi_n} a_{n,t} \] (38)
\[ h_{n,t} = \frac{\varphi_n(1 - \gamma_n)}{\varphi_x(\gamma_n + \varphi_n)} a_{n,t} \] (39)
\[ h_{x,t} = \frac{1 - \gamma_n}{\gamma_n + \varphi_n} a_{n,t} \] (40)

where \(i = 1\) or \(2\). Notice that terms of trade directly affect real exchange rate\((p_{m,n,t})\) and the consumption of imported goods\((c_{m,t})\). However the consumption of non-tradable goods and labor supplies are not affected by terms of trade because the real exchange rate soaks up the shock immediately. This adjustment is passed to the labor supply decisions through wages hence there are no changes in labor supplies for both exportable and non-tradable goods sectors. Under simple

\(^{13}\)Log-linearized systems of equations are in the appendix.
two state markov switching process, using the approach in Davig and Leeper (2007), conditional expectations can be rewritten using a smaller information set excluding the current state, $\Omega_t^{-s}$, where $\Omega_t = \Omega_t^{-s} \cup \{s_t\}$. The state-contingent expected value of terms of trade are as follows: 

For $s_t = 1$

$$E_t[tt_{t+1}] = E[tt_{t+1}|s_t = 1, \Omega_t^{-s}]$$

$$= p_{11}E[tt_{1,t+1}|\Omega_t^{-s}] + (1 - p_{11})E[tt_{2,t+1}|\Omega_t^{-s}]$$

$$= (p_{11}\rho_1 + (1 - p_{11})\rho_2)tt_{1t}$$

(41)

and for $s_t = 2$

$$E_t[tt_{t+1}] = E[tt_{t+1}|s_t = 2, \Omega_t^{-s}]$$

$$= p_{22}E[tt_{2,t+1}|\Omega_t^{-s}] + (1 - p_{22})E[tt_{1,t+1}|\Omega_t^{-s}]$$

$$= (p_{22}\rho_2 + (1 - p_{22})\rho_1)tt_{2t}$$

(42)

According to the realized state at $t$, the expected terms of trade have different values generally. The expected values of consumption of imported goods given state at $t$ can be derived from equation (38), (43) and (46).

$$E_t[c_{m,t+1}] = \frac{\varphi_n(\gamma_n - 1)}{\gamma_m(\gamma_n + \varphi_n)}p_na_{n,t} + \frac{1}{\gamma_m}\rho_xa_{x,t} + \frac{(p_{11}\rho_1 + (1 - p_{11})\rho_2)}{\gamma_m}tt_{1t}, \text{ for } s_t = 1$$

(47)

$$E_t[c_{m,t+1}] = \frac{\varphi_n(\gamma_n - 1)}{\gamma_m(\gamma_n + \varphi_n)}p_na_{n,t} + \frac{1}{\gamma_m}\rho_xa_{x,t} + \frac{(1 - p_{22})\rho_1 + p_{22}\rho_2}{\gamma_m}tt_{2t}, \text{ for } s_t = 2$$

(48)

From the Euler equation (27), the equilibrium debt is determined by current and expected consumption of imported goods.

$$d_t = \frac{1}{\psi D}(c_{m,t} - E_t[c_{m,t+1}])$$

(49)
Under state dependent consumption above, the equilibrium debt is also state dependent and we can decide it by (27) with (38)~ (48).

For \( s_t = 1 \)
\[
d_{1t} = \frac{1}{\psi D} \left[ \frac{\varphi_n(\gamma_n - 1)}{\gamma_m(\gamma_n + \varphi_n)}(1 - \rho_n)a_{n,t} + \frac{1}{\gamma_m}(1 - \rho_x)a_{x,t} + \frac{1 - (p_{11}\rho_1 + (1 - p_{11})\rho_2)}{\gamma_m} t_{1t} \right].
\]

For \( s_t = 2 \)
\[
d_{2t} = \frac{1}{\psi D} \left[ \frac{\varphi_n(\gamma_n - 1)}{\gamma_m(\gamma_n + \varphi_n)}(1 - \rho_n)a_{n,t} + \frac{1}{\gamma_m}(1 - \rho_x)a_{x,t} + \frac{1 - (p_{22}\rho_2 + (1 - p_{22})\rho_1)}{\gamma_m} t_{2t} \right].
\]

### 2.2.4 Comparative Statics

Using the equilibrium debt equation above, we can explore the response of equilibrium debt to terms of trade shock.

\[
\frac{\partial d_{1t}}{\partial t_{1t}} = \frac{1 - p_{11}\rho_1 - (1 - p_{11})\rho_2}{\psi \gamma_m D} \text{, for } s_t = 1
\]
\[
\frac{\partial d_{2t}}{\partial t_{2t}} = \frac{1 - p_{22}\rho_2 - (1 - p_{22})\rho_1}{\psi \gamma_m D} \text{, for } s_t = 2
\]

That is,

\[
\text{Response of debt} = \frac{1 - \text{effect from within state} - \text{effect from change in state}}{\text{constant}}.
\]

Let’s consider that the terms of trade follow markov switching process but agent assumes them following AR(1) process. If \( s_t = 1 \),

\[
\text{Response from AR(1) process} = \frac{1 - \rho_1}{\text{constant}}
\]
\[
\text{Response from MS process} = \frac{1 - p_{11}\rho_1 - (1 - p_{11})\rho_2}{\text{constant}}
\]

The response of debt depends on the magnitude of autoregressive coefficient \( \rho \). With \( 0 < \rho < 1 \), the responses are non-negative for both processes. I define the switching effect by subtracting agent’s optimal debt response under AR(1) process from that under markov switching terms of trade.
trade process given $s_t = i$.  

$$\text{Switching effect} = \frac{(1 - p_{11})(\rho_1 - \rho_2)}{\text{constant}}, \text{ for } s_t = 1$$

The switching effect is the response of debt only to the change of regime. In other words, this is the effect by the permanent change in terms of trade. If $\rho_1 > \rho_2$ and $0 < p_{11} < 1$, the switching effect is positive. Agent borrows less in response to adverse current terms of trade shock. On the other hand, if $\rho_1 < \rho_2$ and $0 < p_{11} < 1$, the switching effect is negative. That is, agent increase the debt level response to an adverse terms of trade shock. Similarly, for $s_t = 2$, the switching effect is positive when $0 < p_{22} < 1$ and $\rho_1 < \rho_2$ and negative when $0 < p_{22} < 1$ and $\rho_1 > \rho_2$.\footnote{In case of absorbing state, $p_{11} = p_{22} = 1$, there is no switching effect.}

Once agent knows that there is regime change, she will optimize with taking the regime change into account. That is

$$\text{Switching effect} = \text{prob}(\text{change regime}) \cdot \text{difference of coefficient}.$$  

Empirical studies reports there is positive but small relationship between terms of trade and trade balance. This suggests that there is a relationship between terms of trade and debt though it might be small. Because terms of trade effect cannot be separated from the effect of productivity shock in the traditional model, it was not clear whether the relationship exists or not. However, the result shows that if there is switching between regimes and agent optimize with knowing the probabilities of transition, there is a response to the permanent terms of trade change. If two regimes are close in the sense of having similar autoregressive parameters, this effect might be small like we observe practically.

### 3 Conclusion

Several empirical facts of terms of trade make it to be a good candidate of following markov switching process. By adopting markov switching process, this paper redefine permanent shock in terms of change in regime. Traditionally shocks to the terms of trade have investigated in
two ways, transitory and permanent. I define 'switching effect' by taking difference of effects
with and without switching. This is the permanent effect under markov switching process. In
standard models, current account response in positive way to the transitory terms of trade shock
and do not response to the permanent terms of trade shock.(OSR effect) Under markov switching
process, the effect of permanent terms of trade shock depends on state contingent parameters and
transition probabilities and the effect can be either positive or negative according to the difference
of parameters. Therefore HLM effect is present for both temporary and transitory terms of trade
shock. This paper has potential extensions. First of all, introducing capital into the model might
have exaggerate or dampen the result. Also one can develop a model which can explore the effect of
state contingent volatility of terms of trade. As mentioned in introduction, the volatility of terms of
trade might have important role in country’s growth. Markov switching terms of trade and model
with capital might give interesting results.
References


4 Appendix

4.1 Log-linearization

\[ \hat{\lambda}_t = -\gamma_m c_{m,t} \]
\[ \hat{\lambda}_t + p_{n,t} = -\gamma_n c_{n,t} \]
\[ \hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \psi Dd_t \]
\[ \varphi_{x, t} = w_{x,t} + \hat{\lambda}_t \]
\[ \varphi_{n, t} = w_{n,t} + \hat{\lambda}_t \]
\[ w_{x,t} = tt_t + a_{x,t} \]
\[ w_{n,t} = p_{n,t} + a_{n,t} \]

The law of motion of the productivity shocks are given by

\[ a_{j,t+1} = \rho_j a_{j,t} + \epsilon_{t+1} \quad (51) \]
\[ \epsilon_{t+1} \sim NIID(0, \sigma^2) \quad (52) \]

Transition matrix is given by

\[ P = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix} \quad (53) \]

where \( p_{ij} \) is the probability of \( (S_t = j | S_{t-1} = i) \). Log-linearized terms of trade process are given by

\[ tt_{1t} = \rho_1 tt_{t-1} + \epsilon_1 t \]
\[ tt_{2t} = \rho_2 tt_{t-1} + \epsilon_2 t. \]
Figure 3: Changes in terms of trade-Canada

Figure 4: Changes in terms of trade-Canada
Figure 5: Changes in terms of trade—New Zealand