Monetary, Fiscal, and the Exchange rate Policy Interactions with Central Bank Debt

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Abstract

Some central banks issue interest-bearing debts because of policy considerations other than typical monetary policy. This paper tries to explore implications of monetary, fiscal, and the foreign exchange rate policy interactions with i) central bank debt, and ii) central bank budget autonomy in a small open economy. The results show that the exchange rate policy is not constrained by the monetary and fiscal policy in most cases. Also, the amount of central bank debt and the degree of central bank budget autonomy do not affect the price determinations, but have implications on how much adjustment of the passive policy should be for each authority. In addition, the model predicts that the sterilized foreign exchange intervention can be effective even when the central bank is an inflation targeter.

Keywords: Fiscal theory of the price level; Central bank debt; Central bank balance sheet; Foreign exchange rate policy;
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1 Introduction

Conventional wisdom about a central bank’s balance sheet is that it is just a matter of accounting, not related to policy implications such as price determination. The view was supported by the notion that central banks cannot be insolvent to domestic creditors because of their power of creating money, and the ultimate ‘back up’ of fiscal authorities’ taxing power. In this perspective, the standard fiscal theory of price level determination has assumed that central bank and government budget constraints are consolidated, and the fiscal policy decides the supply of public debts.

Recently, however, questions have been raised against the wisdom. Stella and Lönnberg (2007) surveyed 135 central banks and found that legislations didn’t always guarantee fiscal authorities’ responsibility on central banks’ liability, and fiscal authorities were not prompt in supporting central banks in needs. Indeed, as more emphasis is placed on central banks’ independence, it becomes more plausible assumption that a central bank may concern about its own balance sheet. That is, even though a central bank’s default is hardly imaginable, it may change behaviors because of the concern on the balance sheet. Sims (2003), Zhu (2003), Jeanne and Svensson (2007)’s works are based on this assumption.

There is another reason to pay attention to central bank balance sheets. That is, some central banks issue interest-bearing debts because of policy considerations other than typical monetary policy. Some central banks issued debt to maintain their currency unappreciated without altering their monetary base, and some central banks needed funds to support their financial sectors in distress. In other words, they issued debt for ‘quasi fiscal activities’, thus their balance sheets are curbed with the reasons other than their monetary policy. If this constraint make agents’ belief on monetary policy, thus equilibrium different, central bank balance sheets do matter.

Therefore, it will be natural to ask how the equilibrium and the policy interactions should be different under the circumstances that the central bank issues debts for a policy commitment other than the monetary policy and it has a budget autonomy. In particular, this paper generalizes Leeper (1991)’s fiscal theory of the price level by expanding the policy instruments to add the foreign exchange rate policy. Of course, there are existing works on the extension of the fiscal theory to the open economy set up. To mention a few but not all, Bergen (2000) applied the fiscal theory to the monetary union; Canzoneri et al. (2001) explored the implication of the fiscal theory in maintaining the exchange rate regime; Daniel (2001a) showed the fiscal policy’s role on the currency crisis in the fiscal theory framework; and Daniel (2001b) solved the indeterminacy problem in the price level and the foreign
exchange rate raised by Dupor (2000). This paper differs from the existing works in the sense that this paper introduces more realistic public sector budget constraints which take the budget autonomy of central banks and the foreign exchange rate policy seriously.

A brief summary of the result is as follows. First, the policy on the foreign exchange rate can be either ‘active’ or ‘passive’ following Leeper (1991)’s terminology in most of cases, but it should be passive when the nominal exchange rate is pegged, the fiscal policy is active, and the monetary policy is passive. Second, the degree of the budget autonomy of the central bank and the amount of central bank debt have no impact on the price determination in various policy set up. However, the required adjustment of passive policy depends upon the budget autonomy and the central bank debts. Finally, the model predicts that the sterilized foreign exchange rate intervention can affect the nominal exchange rate through the channel which are not specified so far. Moreover, the intervention is shown to be effective even when the central bank targets the price level.

The remainder of this paper is as follows. After briefly surveying the facts about central bank balance sheet issues around the world, section 2 describes agents’ optimizing behavior in a small open economy and public sector’s constraints. Then in section 3, the different set of policy regimes will be explored to show what should be the policy interactions with the expanded policy instruments, and the conclusion follows.

1.1 Stylized facts on central bank debts

Table 1 illustrates central bank domestic debts outstanding and the factors that might influence the issuance for 10 countries from Asia and Latin America. The first thing to mention is the magnitude of debt outstanding. Korea has the central bank debt issued as much as 305% of reserve money as of 2006, Taiwan has 201%, and Chile has 148%. The significant magnitude means that they had to finance resources for some policy reasons without printing more money. Then, what was the reasons? The clue for the answer lies in the usage of the resources.

Some Asian countries with rapid growth in the central bank debt outstanding have significant current account surpluses and rapid growth in the central banks’ foreign assets. Such countries include China, Korea, Malaysia, and Taiwan. Although Korea, Malaysia and Taiwan have a floating exchange rate regime, it is natural to think that they have tried to offset the impact of external surpluses on their money balances since they have accumulated foreign assets. Certainly, China is accumulating foreign assets to maintain their fixed exchange rate.
The other possible usage of the resource is to lend to private sectors including banking or nonbanking financial institutions. Compared to foreign asset accumulations, Japan, Mexico and Dominican Republic had significant portions of their assets as loans to private sectors. Of course, the direct loans to commercial banks is one of instruments of monetary policy. But if the loans were made purely for the monetary policies, they may be financed by issuing money rather than issuing debts with interest rate payments. Therefore, there had to be another reason for debts issuance such as supporting financial sector in distress rather than controlling price level.

In sum, some central banks issued a large amount of debts for quasi fiscal activities. The term ‘quasi fiscal’ was used to emphasize that the main objectives of the activities are not for stabilizing price level. Of course, nominal foreign exchange rate may have impacts on domestic price level, but it may be argued that Asian countries didn’t accumulate foreign assets to stabilize price level since the currency appreciation stabilizes the prices of imported goods.

2 Model

2.1 Equilibrium

Here I present a simple model for a rational expectation general equilibrium in a small-open endowment economy with the money in the utility function. First, the representative agent solves the following maximization problem.

\[
\max_{\{c_t^T, c_t^N, M_t, B_t^G, B_t^C, B_t^f\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{u_T(c_t^T) + u_N(c_t^N) + u_m(M_t)\}, \quad 0 < \beta < 1
\]

\[
s.t. \quad p_t^T c_t^T + p_t^N c_t^N + M_t + Q_t^G B_t^G + Q_t^C B_t^C + S_t B_t^f
\]

\[
= \quad p_t^T y_t^T + p_t^N y_t^N - P_t \tau_t + M_{t-1} + B_{t-1}^G + B_{t-1}^C + (1 + i_t^*) S_t B_t^f, \quad \forall t
\]

where \(u_{T(N)}\) is the utility function for consuming tradable (nontradable) goods, \(c_t^T\) is tradable goods, \(c_t^N\) is nontradable goods, \(\tau_t\) is the lump-sum tax, \(M_t\) is the nominal money balance, \(B_t^G(C)\) is the fiscal authority’s (central bank’s) one-period nominal debt, \(B_t^f\) is the foreign bonds denominated by the foreign currency, \(P_t\) is the price level, \(S_t\) is the nominal exchange rate, \(Q_t^G(C)\) are prices of the domestic bonds, and \(i_t^*\) is the exogenous net nominal interest rate for the foreign bonds. Let’s assume that the utility functions for the tradable and nontradable goods are log utility function with a weight \(\alpha_1\) assigned to the tradable goods.
Then, the general price level index is as follows.

\[ P_t = (p^T_t)^{\alpha_1} (p^N_t)^{1-\alpha_1} \]  \( \text{(2)} \)

First order conditions yield Euler equations as follows.

\[ \frac{p^T_t}{p^N_t} = \frac{u'_T(c^T_t)}{u'_N(c^N_t)}, \]  \( \text{(3)} \)

\[ Q_t^{G(C)} = E_t[X_{t,t+1}] \]  \( \text{(4)} \)

where \( X_{t,t+1} \equiv \beta \frac{P_t}{P_{t+1}} \frac{u'_T(c^T_{t+1})^{\alpha_1} u'_N(c^N_{t+1})^{1-\alpha_1}}{u'_T(c^T_t)^{\alpha_1} u'_N(c^N_t)^{1-\alpha_1}} \),

\[ \frac{u'_m(m_t)}{u'_T(c^T_t)^{\alpha_1} u'_N(c^N_t)^{1-\alpha_1}} + E_t[X_{t,t+1}] = 1, \]  \( \text{(5)} \)

\[ E_t[X_{t,t+1}(1 + i_t)] \frac{S_{t+1}}{S_t} = 1, \]  \( \text{(6)} \)

where \( u' \) refers to the derivative of \( u \), \( m_t \) is the real money balance \( M_t/P_t \), and \( X_{t,t+1} \) is either the pricing kernel, or a discount factor. Equation (4) is an asset pricing equation for the domestic bonds, and equation (5) is an asset pricing equation for money. In the model, domestic public debts are risk-free, so let’s denote the domestic risk-free net interest rate as \( i_t \). In that case, the followings are true from (4) and (5).

\[ Q_t^{G(C)} = \frac{1}{1 + i_t} = E_t[X_{t,t+1}], \]  \( \text{(7)} \)

\[ \frac{u'_m(m_t)}{u'_T(c^T_t)^{\alpha_1} u'_N(c^N_t)^{1-\alpha_1}} = \frac{i_t}{1 + i_t}. \]  \( \text{(8)} \)

Additionally, if the pricing kernel and the future nominal exchange rate are independent, (6) turns into the uncovered interest parity condition as in (9).

\[ (1 + i^*_t) \frac{E_t[S_{t+1}]}{S_t} = 1 + i_t. \]  \( \text{(9)} \)

The real exchange rate \( e_t \) is defined as equation (10) where the foreign price level \( P^*_t \) is assumed to be \( (p^T_t)^{\alpha_2} (p^N_t)^{1-\alpha_2} \) and \( p^T_t (N) \) is the price of tradable (nontradable) in the foreign country. Since the purchasing power parity holds for the tradable goods sector and the foreign price is exogenous, the real exchange rate is a function of domestic relative price.
of tradable goods against nontradable goods.

\[ e_t \equiv \frac{S_t p_t^*}{P_t} = \left( \frac{p_t^N}{p_t} \right)^{\alpha_2-1} \left( \frac{p_t^T}{p_t} \right)^{1-\alpha_1} = \left( \frac{p_t^*}{p_t} \right)^{\alpha_2-1} \left( \frac{\pi_t^T(c_t^T)}{\pi_t^N(c_t^N)} \right)^{1-\alpha_1}. \] (10)

For the existence of optimal condition, let’s assume further that there is a borrowing limit to the agent. That is, the agent cannot borrow greater than the present value of her future income, and the present value should be finite. These optimality conditions are expressed as follows.

\[ M_t + B_t^G + B_t^C + S_t B_t^* \geq - \sum_{T=t+1}^{\infty} E_{t+1}[X_{t+1,T}(p_t^T y_t^T + p_t^N y_t^N - P_T \tau_T)], \] (11)

\[ \sum_{T=t}^{\infty} E_t[X_{t,T}(p_t^T y_t^T + p_t^N y_t^N - P_T \tau_T)] < \infty, \] (12)

where \( X_{t,T} \equiv \prod_{s=t}^{T-1} X_{s,s+1} \). It can be shown that condition (11) is equivalent to the transversality condition (13).

\[ \lim_{T \to \infty} E_t[\beta^T u_t^T(c_t^T)^{\alpha_1} u_t^N(c_t^N)^{1-\alpha_1} (M_t + B_t^G + B_t^C + S_t B_t^*)^1 P_T] = 0. \] (13)

The transversality condition (13) states that the real value of the agent’s borrowing cannot grow faster than the real interest rate. In addition to (13), let’s assume that the domestic agent cannot borrow from the world in the faster rate than the real interest rate as in (14).

\[ \lim_{T \to \infty} E_t[\beta^T u_t^T(c_t^T)^{\alpha_1} u_t^N(c_t^N)^{1-\alpha_1} S_t B_t^* P_T] = 0. \] (14)

The domestic agent would like to make the limit term in (14) be negative, but that should be accompanied with the positive limit term of the foreign agent which is not desirable for the foreign agent. Similar versions of the transversality conditions in the open economy were derived by Daniel (2001a and 2001b).

Now, let’s impose general equilibrium conditions to Euler equations. In specific, goods, money, and bonds market should be cleared as follows:

\[ \sum_{s=0}^{\infty} \frac{c_t^T}{(1 + r_s^t)^s} = E_0 \sum_{s=0}^{\infty} \frac{y_t^T}{(1 + r_s^t)^s}, \ c_t^N = y_t^N - g_t, \forall t \] (15)

Ponzi-schemes are ruled out by these optimal conditions. 

1
\[ M_t^s = M_t, \quad \forall t \]  

\[ B_t^{Gs} = B_t^G, \quad B_t^{Cs} = B_t^C, \quad B_t^{ss} = B_t^s, \quad \forall t \]  

where \( M_t^s \) is a money supply, \( B_t^{G(Cs)} \) is government (central bank) bonds supply, and \( B_t^{ss} \) is foreign bonds supply. In order to derive the optimal consumption of the tradable goods, let’s combine the uncovered interest parity condition (6) and the domestic and foreign fisher equations to generate (18).

\[
E_t\left[ \frac{u_T'(c_{t+1}^T)^{\alpha_1} u_N'(c_{t+1}^N)^{1-\alpha_1} e_{t+1}}{u_T'(c_t^T)^{\alpha_1} u_N'(c_t^N)^{1-\alpha_1}} \right] = \frac{1}{1 + r_t^*},
\]

or,
\[
E_t\left[ \frac{u_T'(c_{t+1}^T)}{u_T'(c_t^T)} \right] = \frac{1}{1 + r_t^*},
\]

where \( r_t^* \) is the foreign real interest rate. By setting the exogenous foreign prices to be constant and \( \beta \) to be equal to \( 1/(1 + r^*) \), it can be shown that the consumption of the tradable is perfectly smoothed. Therefore, the optimal \( c_t^T \) is given by (19).

\[
c_t^T = \frac{r^*}{1 + r^*} E_0 \left[ \sum_{s=0}^{\infty} \frac{y_s^T}{(1 + r^*)^s} \right] \equiv \bar{y}_t^T, \quad \forall t.
\]

Then, we can derive the real money balance equilibrium condition from (5) with the standard assumptions on preferences, and the fisher equation from (7) as follows:

\[
\frac{M_t^s}{P_t} = f(\bar{y}_t^T, y_t^N - g_t, i_t),
\]

\[
E_t\left[ \frac{1}{1 + r_t} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1 + i_t},
\]

where \( \frac{1}{1 + r_t} \equiv \beta \frac{u_N'(y_{t+1}^N - g_{t+1})^{1-\alpha_1}}{u_N'(y_t^N - g_t)^{1-\alpha_1}}. \)

The right hand side of (20) depicts real money balance demand, and (21) is a domestic fisher equation after imposing general equilibrium conditions. The relative price of tradable goods and the real exchange rate are also the functions of exogenous sequences and policy variable.
g_t. From (3) and (10),
\[
\frac{\bar{p}_t}{p_t} = \frac{u'_T(y_T)}{u'_N(y_N - g_t)}
\]
(22)
\[
e_t = \left(\frac{u'_T(y_T)}{u'_N(y_N - g_t)}\right)^{1-\alpha_1}
\]
(23)
Note that the purchasing power parity (PPP) does not hold for the whole economy in this model. The fiscal policy is distortionary in resource allocation for tradable and nontradable goods sector, thus the real exchange rate is not a constant as in regular PPP case. The intuition of (23) is straightforward. If the government increase the government spending on the nontradable sector, it increases the price of the nontradable good, thus induces the real exchange rate appreciation. The rational expectation general equilibrium in this economy is defined as the sequence of \(\{c_t^T, c_t^N, P_t, p_t^T, p_t^N, S_t, e_t, i_t, r_t, M_t^s, B_t^{Gs}, B_t^{Cs}, B_t^s\}\) which satisfies the price aggregation (2), the uncovered interest rate parity (9), the optimality conditions (12)-(14), market clearing conditions (15)-(17), the money market equilibrium condition (20), the fisher equation (21), the relative price of tradable goods (22), and the real exchange rate determination (23) with complete specifications of fiscal and monetary policy.

So far, it has been assumed that the domestic agent has no restriction on the access to the world financial markets. That is, the agent has \(B_t^s\) in her budget constraint and the optimal choice of \(B_t^s\) is not constrained by any condition other than the transversality condition. However, if the government forbids the agent to hold the foreign bonds or to borrow from the foreign country, i.e., there is a capital control, the equilibrium should be different. In the perfect capital control case, let’s assume that \(B_t^s = 0\) for all periods. In this case, the uncovered interest rate parity (6) and (9) may not satisfied. In addition to that, the agent cannot smooth the consumption on the tradable goods, thus \(\bar{y}_t^T\) should be replaced by \(y_t^T\) in (20), (22), and (23).

### 2.2 Public sector

In this subsection, detailed budget constraints of public sector will be established. By the accounting identity, the changes in the asset account should be equal to the changes in the liability account and the capital account. Central banks raise funds from issuing debts and (or) money (liability account) and from the cumulative earnings (capital account). Then, central banks store raised funds in assets such as government bonds, loan to the private sector and foreign assets. If the central bank is only involved in open market operations, the majority of its assets will be consist of government bonds or equivalent risk-free domestic
assets. However, if the central bank is involved in quasi fiscal activities such as sterilized foreign exchange market interventions, the assets of the central bank include other form of risky assets such as foreign bonds. As illustrated in the subsection 1.1, the central bank finance its quasi fiscal activities by the central bank debts when the issuance (or withdrawal) of money is inconsistent with the monetary policy.

In addition to the existence of central bank debts, this paper incorporate the idea of central bank budget autonomy. In the conventional consolidated public sector budget constraint, it was implicitly assumed that the central bank and the fiscal authority will transfer funds to each party without constraint. However, Stella and Lönnberg (2007) surveyed legislations on the central bank to show that the transfer is not always guaranteed. The budget autonomy of central banks can be analyzed in two aspects. The first aspect is how much the fiscal authority takes the responsibility for the central bank liability issued. If the fiscal authority does not fully acknowledge the central bank’s liability as fiscal authority’s responsibility, the central bank should take its own budget constraint issues into account, albeit partially. The second aspect is how much the fiscal authority takes (supports) the central bank’s net profits (losses). In order to have a perfectly consolidated public budget constraint, the fiscal authority should be able to make use of the central bank profits and should support the bank whenever it has losses. In order to model the budget autonomy of the central bank, this paper parameterize two aspects as follows.

\[ \theta_1 \] : How much the fiscal authority is responsible for the central bank liability.

\[ \theta_2 \] : How much the fiscal authority is responsible for the central bank capital.

In the legislation for the central bank independence, either \( \theta_1 \) or \( \theta_2 \) may be specified. For example, the former Soviet Union countries specify that the fiscal authority will not be responsible for the central bank liability (\( \theta_1 = 0 \)) (Stella and Lönnberg, 2007). Argentina, Croatia, El Salvador, and Iceland’s act of central bank state that the fiscal authority will be fully responsible for the central bank liability (\( \theta_1 = 1 \)) (Stella and Lönnberg, 2007). In some other cases, \( \theta_2 \) is specified in the legislation instead of \( \theta_1 \). For example, the bank of Korea transfers 90% of net profit: that is, \( \theta_2 \) is 0.9 in Korea. Of course, the rule on the central bank capital account might be nonlinear. Specifically, even though the Bank of Korea transfers 90% of net profit to the Korean government, the Korean government may supports other than 90% of net losses because the Korean legislation is not explicit on the negative capital issue. However, for the tractability, let’s assume the linear and symmetric rule for capital.

Embodying these ideas, the generalized flow budget constraint of central bank is as
follows.

\[
(1 - \theta_1) \left\{ \frac{B^c_s}{(1 + i_t)P_t} - \frac{B^c_{t-1}}{P_t} + \frac{M^s_t - M^s_{t-1}}{P_t} \right\} + (1 - \theta_2) \left\{ \frac{(S_t - S_{t-1})F^C_t}{P_t} + \frac{i_t^s S_t F^C_t}{P_t} \right\} 
\]

\[
= \left[ \frac{B^T_t}{(1 + i_t)P_t} - \frac{B^T_{t-1}}{P_t} \right] - \frac{S_t F^C_t - S_{t-1} F^C_{t-1}}{P_t} 
\]

open market operation \quad quasi fiscal activities

where \( S_t \) is the nominal exchange rate, \( F^C_t \) is the foreign asset holding of the central bank in the foreign currency, and \( B^T_t \) is the central bank’s government bonds holding. The asset revaluation effect \( (S_t - S_{t-1})F^C_{t-1}/P_t \) appears in the change of capital account, and every entity in the right hand side is the policy instrument of the central bank. (24) can be thought as an effective budget constraint of the central bank. That is, effectively, the central bank can only use a portion of fund raised from the liability and capital because the residual portion is the fiscal authority’s responsibility. Note that the loan to private sectors is suppressed from the quasi fiscal activities in (24) for the simplicity.

The central bank budget constraint (24) is generalized in two aspects. First, (24) extends the conventional budget constraint to central bank debt and quasi fiscal activities. Without that feature, the central bank budget constraint will only state that the any changes in the money balance from the monetary policy should be fulfilled with the open market operation. However, (24) also states that the quasi fiscal activities may be financed by the debt issuance and money creation. The second generalization is from the budget autonomy of the central bank. In the conventional models with the single public sector budget constraint, \( \theta_1 \) and \( \theta_2 \) are always one and quasi fiscal activities are not modeled, thus (24) is not binding in those models. This paper acknowledges the fact that the budgets of central banks may have autonomy, and parameterized the degree of autonomy.

After receiving (or giving) transfer from (to) the central bank, the budget constraint of fiscal authority is as follows.

\[
\Delta \text{Liability from the central bank} 
\]

\[
\theta_1 \left\{ \frac{B^c_s}{(1 + i_t)P_t} - \frac{B^c_{t-1}}{P_t} + \frac{M^s_t - M^s_{t-1}}{P_t} \right\} + \theta_2 \left\{ (S_t - S_{t-1} + i_t S_t) \frac{F^C_t}{P_t} \right\} 
\]

\[
\Delta \text{Capital from the central bank} 
\]

\[
\frac{B^T_t}{(1 + i_t)P_t} - \frac{B^T_{t-1}}{P_t} - \frac{S_t F^C_t - S_{t-1} F^C_{t-1}}{P_t} 
\]

\[
\Delta \text{Liability of fiscal authority} 
\]

\[
\frac{B^G_s + B^T_t}{(1 + i_t)P_t} - \frac{B^G_{t-1} + B^T_{t-1}}{P_t} - \frac{(S_t - S_{t-1} + i_t S_t) F^C_t}{P_t} \]

\[
\Delta \text{Capital of fiscal authority} 
\]

\[
\frac{S_t F^G_t - S_{t-1} F^G_{t-1}}{P_t} 
\]

\[
(25) 
\]
where $F^G_t$ is the foreign asset holding of the fiscal authority. The first line in (25) is the effective changes in the fiscal authority’s liability account, the left hand side of second line depicts the effective changes in the fiscal authority’s capital account, and the right hand side is the changes in assets. In (25), it is assumed that the fiscal authority also participate in the quasi fiscal activities.

Although the flow budget constraints are separately satisfied in (24) and (25), it is another issue whether two flow budget constraints can be expressed in the intertemporal forms after imposing market clearing conditions. The issue is crucially related to the transversality conditions, thus to the budget separation assumption. In the agent’s optimization problem, the transversality conditions (13) and (14) were assumed to imply that the amount of public sector’s total liability to the agent cannot grow faster than the real interest rate. However, (13) and (14) do not discriminate between the central bank’s liability and the fiscal authority’s liability. That is, in the transversality condition (13), it is possible that the central bank lends to the fiscal authority in the faster rate than the real interest rate. Therefore, an additional assumption should be met as in proposition 1.

**Proposition 1** If the condition (26) is satisfied, the effective central bank liability and fiscal authority liability will satisfy transversality conditions for each separately.

\[
\lim_{T \to \infty} E_t [\beta^T u'_T(c_T^N)^{\alpha_1}u'_N(c_T^N)^{1-\alpha_1}\frac{B^G_T}{P_T}] = 0. \quad (26)
\]

**Proof.** (13) and (14) are equivalent to following condition.

\[
\lim_{T \to \infty} E_t [\beta^T u'_T(c_T^N)^{\alpha_1}u'_N(c_T^N)^{1-\alpha_1}\left\{(1 - \theta_1)(B^C_T + M_T) - B^T_T + B^G_T + B^T_T + \theta_1(B^C_T + M_T)\right\}] = 0. \quad (27)
\]

By the way, the agent cannot issue money ($M_T \geq 0$) and the central bank has no incentive to issue negative value of the central bank debt ($B^C_T \geq 0$) because it can increase the money supply by accumulating the government bonds holdings. Since $0 \leq \theta_1 \leq 1$, from (13) and (14),

\[
\lim_{T \to \infty} E_t [\beta^T u'_T(c_T^N)^{\alpha_1}u'_N(c_T^N)^{1-\alpha_1}\frac{B^G_T + \theta_1(B^C_T + M_T)}{P_T}] = 0. \quad (28)
\]

Therefore, the following two separate transversality conditions are satisfied if (26) is satisfied.

\[
\lim_{T \to \infty} E_t [\beta^T u'_T(c_T^N)^{\alpha_1}u'_N(c_T^N)^{1-\alpha_1}\frac{(1 - \theta_1)(B^C_T + M_T) - B^T_T}{P_T}] = 0, \quad (29)
\]

\[
\lim_{T \to \infty} E_t [\beta^T u'_T(c_T^N)^{\alpha_1}u'_N(c_T^N)^{1-\alpha_1}\frac{B^G_T + B^T_T + \theta_1(B^C_T + M_T)}{P_T}] = 0. \quad (30)
\]
Essentially, the condition (26) states that there is a budget autonomy of the central bank. If (26) is not hold, the central bank should issue the central bank debt without bound if the fiscal authority requires it when the demand function of real balance is bounded. In other words, since the seigniorage revenue can’t be infinite, the central bank issues debt instead of the fiscal authority without bound. Of course, the central bank can finance the government bond holdings with liquidating its assets from the quasi fiscal activities, or with the accumulated capital. However, both items are finite, thus boundless \( B_T^C / P_T \) should be financed by boundless \( B_T^C / P_T \). Therefore, (26) means that the central bank will veto the plan to increase government’s asset very fast \( (B_G^C / P_T \to -\infty) \) with the fast issuance of its debt \( (B_T^C / P_T \to \infty) \).

Now, with the budget autonomy assumption, let’s make intertemporal budget constraints from (24) and (25). If the present values are finite, by substituting equilibrium conditions into (24) and (25) and solving forward, the next equilibrium conditions are derived.

\[
\frac{(1 - \theta_1)(B^C_{t-1} + M^s_{t-1}) - B^T_{t-1}}{P_t} = (31)
\]

\[
\sum_{T=t}^{\infty} E_t[D_{t,T} \{ (1 - \theta_1) \frac{i^T_{t}}{1 + i^T_{t}} f(y_T, y^N_T - g_T, i_T) - \theta_2 \gamma^C_{1,T} + (1 - \theta_2) \gamma^C_{2,T} - \Delta a^C_T \}],
\]

\[
D_{t,T} \equiv \beta^{T-t} u'^N_{t}(y'^N_{t+1} - g_{t+1})^{1-\alpha_1} u'^N_{t}(y'^N_{t} - g_t)^{1-\alpha_1}, \quad \gamma^C_{1,t} \equiv \frac{(S_t - S_{t-1})F^C_{t-1}}{P_t}, \quad \gamma^C_{2,t} \equiv \frac{i^t_{t}S_{t-1}F^C_{t-1}}{P_t}, \quad \Delta a^C_t \equiv \frac{S_t(F^C_t - F^C_{t-1})}{P_t},
\]

where \( \gamma^C_{1,t} \) is profit or loss from the revaluation of foreign asset, \( \gamma^C_{2,t} \) is net return from the assets, and \( \Delta a^C_t \) is the central bank’s quasi fiscal policy instrument. \( \theta_2 \gamma^C_{1,T} \) and \( \theta_2 \gamma^C_{2,T} \) can be regarded as the central bank’s transfer to the fiscal authority if positive. If negative, they can be regarded as the transfer to the central bank. The seigniorage and real return from the assets can be regarded as the earnings, thus they increase the real value of liability. Meanwhile, the revaluation effect increase the asset account and capital account in the same amount, therefore the revaluation effect \( \gamma^C_{1,T} \) should not affect the real value of current liability in basic. However, as the transfer between the central bank and the fiscal authority depends on the revaluation effect, it enter into the bond valuation formula of the central bank.
\[
\frac{B_{t-1}^{G_s} + B_{t-1}^T + \theta_1 B_{t-1}^{C_s} + \theta_1 M_{t-1}^s}{P_t} = (32)
\]

\[
\sum_{T=t}^{\infty} E_t[D_{t,T} \{\tau_T - g_T + \theta_1 \frac{i_T}{1 + i_T} f(y_T^T, y_T^N - g_T, i_T) + \theta_2 (\gamma_{1,T}^C + \gamma_{2,T}^C) + \gamma_{2,T}^G - \Delta a_T^G\}],
\]

\[
\gamma_{2,t}^G \equiv \frac{i_t S_t F_{t-1}^G}{P_t}, \quad \Delta a_t^G \equiv \frac{S_t(F_t^G - F_{t-1}^G)}{P_t}.
\]

Although (31) and (32) look complicated, they are still bond valuation formulas in the fiscal theory of price level. The left hand sides are the current real value of public liability, and the right hand sides are present values of future earnings from the government surpluses, seigniorage, and transfers between two entities. If (31) is added to (32), the standard bond valuation formula is derived where the real value of public debts and money is supported by the present value of government surpluses, seigniorage, return from the assets, and changes in assets. Therefore, this model is different from the conventional fiscal theory of the price level in the sense that this paper separates real bond valuation formula into two formulas.

In the first glance, it may seem to be inconsistent to have two equilibrium conditions (31) and (32) for a \(P_t\). Specifically, the increase in the present value of transfer \(\gamma_{1,T}^C\) increases \(P_t\) in (31) and decreases \(P_t\) in (32) if other variables remain fixed. However, this argument is not valid because the other variables cannot be fixed. This model assumes that both the central bank and fiscal authority liability is risk-free, which means that the other variables should be changed according to the variation in \(\gamma_{1,T}^C\). If the other variables are not adjusting, one of (31) and (32) should be the asset pricing equation and the other equation is the equilibrium condition for the price level.

As Sims (1999) and Woodford (2001) also explained, the government budget constraint is special in the sense that only it can be regarded as an equilibrium condition for the price level. That is, since the consolidated government’s debt is denominated by the unit of its own liability (money), the bond is risk-free and the budget constraint is turned into the equilibrium condition of the general price level of the economy. On the other hand, the intertemporal budget constraints of private enterprises are the asset pricing equations for their equities which incorporate risk structures of future payoffs. The riskiness is innate to the private enterprises because the value of the liability is backed up by the uncertain future earnings, and the values are denominated by the other party’s liability which the companies cannot control. The central bank and fiscal authority in this model are still special because they control a part of their future earnings (surpluses and seigniorage), and the price of their
liability is denominated by the unit of their liability. Of course, one may imagine situations that one of (31) and (32) is declined to an asset pricing equation, i.e., with \[ 1 = 2 = 0 \]
and expected inconsistent policy mixture that makes (31) and (32) incompatible.\(^2\) However, because of the specialty of the government liability described above, it is hardly imaginable that both public liabilities are risky.

**Proposition 2** If both public sector liabilities are risk-free, the present value of future terms should satisfy (33).

\[
\begin{align*}
\Gamma_1 \sum_{T=t}^{\infty} E_t[D_{t,T} \frac{g_T}{1 + i_T} f(y^T_T, y^N_T - g_T, i_T)] + \Gamma_2 \sum_{T=t}^{\infty} E_t[D_{t,T} \gamma^C_{1,T}] + \Gamma_3 \sum_{T=t}^{\infty} E_t[D_{t,T} \gamma^C_{2,T}] + \\
\Gamma_4 \sum_{T=t}^{\infty} E_t[D_{t,T} \Delta a^C_T] = \Gamma_5 \sum_{T=t}^{\infty} E_t[D_{t,T}(\tau_T - g_T + \gamma^G_{2,T} - \Delta a^G_T)] \quad (33)
\end{align*}
\]

\[
\begin{align*}
\Gamma_1 & \equiv (1 - \theta_1)B^G_{t-1} + B^T_{t-1}, \quad \Gamma_2 \equiv -\theta_2(B^G_{t-1} + B^C_{t-1} + M^*_t), \\
\Gamma_3 & \equiv (1 - \theta_2)B^G_{t-1} + B^T_{t-1} + (\theta_1 - \theta_2)(B^C_{t-1} + M^*_t), \\
\Gamma_4 & \equiv (1 - \theta_1)(B^C_{t-1} + M^*_t) - B^T_{t-1}, \quad \Gamma_5 \equiv (1 - \theta_2)(B^C_{t-1} + M^*_t) - B^T_{t-1}
\end{align*}
\]

**Proof.** (33) is derived by simultaneously solving for \( P_t \) with (31) and (32). \( \blacksquare \)

Proposition 2 illustrates trade-off relationship between seigniorage, foreign asset revaluation, return from the asset, quasi fiscal activities, and primary surplus that should be maintained in order to make both public liability risk-free.

\[
\frac{W_t}{P_t} = \sum_{T=t}^{\infty} E_t[D_{t,T}(\tau_T - g_T + \frac{i_T}{1 + i_T} f(y^T_T, y^N_T - g_T, i_T))]
\quad (34)
\]

In the conventional model for the fiscal theory with the consolidated budgets and without any consideration on quasi fiscal activities, the real debt valuation equation is (34) as in Woodford (2001). In this case, there was no trade-off between the present value of surpluses and seigniorage. For example, if the nominal interest rate is fixed and income process is given, the changes in expected surpluses cause the price level changed. Although the surpluses are changed, there is no change in seigniorage as the interest rate is fixed. It is \( P_t \) that is adjusted to satisfy the equilibrium condition (34). However, since there are two risk-free assets and therefore two equilibrium conditions, now there is a trade-off relationship between the present values of policy variables.

\(^2\)Of course, in order to have a risky public bond, that nominal bond should be redeemed to uncertain \( q_t \) unit of money in the next period, not 1 unit of money as in the current model.
If policy regimes are specified, one can find policy implications on the price level with (31) and (32). For example, at the first glance, the positive changes in the assets from the quasi fiscal policies seem to decrease the real value of liabilities through $\Delta a_C^t$ and $\Delta a_G^t$. Intuitively, if it is expected that the policy authorities will increase the asset holdings in the future, they should be financed by the future issuance of liability, thus the current real value of liabilities decrease. However, (31) and (32) are equilibrium conditions, and there are endogenous variables in the right hand sides, thus the additional assumptions on the policy should be made. The next section will try the different variations of policy assumptions.

3 Policy experiments

Before proceeding to specific policy experiments, let’s define what the policies are. First, the monetary policy (MP) of the central bank is to specify the current and future path of the risk-free nominal interest rate process, $\{i_t\}_{t=1}^{\infty}$. Second, the fiscal policy (FP) is the path of the government spending $\{g_t\}_{t=1}^{\infty}$. The conventional studies on the policy interactions stop here. However, in this model, there is the third policy instrument: the quasi fiscal policy (QFP). This paper focuses on the effects of the foreign exchange market intervention, thus the quasi fiscal policy can be defined as the paths of foreign asset holdings, $\{F_C^t, F_G^t\}_{t=1}^{\infty}$.

Apparently, this paper opens the possibility that both the fiscal authority and the central bank are involved in the quasi fiscal policy. The actual institutional features of the foreign exchange policy confirms this set up. That is, the sterilized foreign exchange market intervention may be implemented either by the central bank or the fiscal authority. In Australia, European Union, Hong Kong, Singapore, and Switzerland, the central banks are independently responsible for the foreign exchange rate policy. Canada, Japan, Korea, United Kingdom, and United States are the countries where the fiscal authorities take the responsibility on the foreign exchange policy, but the central banks also are involved in some ways. For example, in Canada, Japan, Korea, and United States, the central banks execute day-to-day operations in the foreign exchange market interventions, discuss with the fiscal authorities on the policy. (Chiu, 2003) In the United States, the foreign exchange rate stabilization is financed equally by the central bank and the fiscal authority.

A question arise from these institutional features is, which policy authority controls QFP in the model. This question is crucially related to the issues on the policy interactions. In the simpler model only with the monetary policy and the fiscal policy, it was obvious that the central bank takes the responsibility on monetary policy albeit it may be passive. However, in this model and institutional aspect described above, it seems to be possible that the fiscal
authority actually can control the QFP of the central bank, i.e., $\Delta a^C_T$ in (31). In order to simplify the analysis, let’s assume that the central bank is responsible for its own QFP and the same for the fiscal authority.

An another clarification should be made for the meaning of "activeness". Leeper (1991) ascribed the activeness to each policy authorities’ properties. That is, the active policy authority "is not constrained by current budgetary conditions", whereas the passive policy authority "is constrained by customer optimization and the active authority’s actions" (Leeper, 1991). This paper maintains this definition, but adds one more layer. If a policy authority pursues multiple policy instruments, the passive policy is the one which is constrained by equilibrium conditions, the active policies from the other policy authority, and its own active policy.

### 3.1 Active fiscal policy, passive monetary policy (AFP/PMP)

Assume the fiscal policy is such that $\{g_t, \tau_t\}$ are exogenous sequences, and monetary policy is to peg nominal interest rate. Thus, $\{\hat{i}_t\}$ sequence is also exogenous. In this non-Ricardian policy regime, the FP is active in the sense that it does not adjust future surpluses to satisfy (31) and (32). Given the process of surpluses, the central bank adjusts the current nominal money balance to satisfy the money market clear condition (20).

First consider the case that QFP fixes the nominal exchange rate to $\bar{S}$. If there is no capital control, the domestic nominal interest rate should be equal to the exogenous foreign nominal interest rate by the uncovered interest rate parity condition (9). However, in the capital control case, $i_t$ may be deviated from $\hat{i}_t$. In both cases, the nominal interest rate is always exogenous and MP is passive. The characteristics of price level determination in this policy regime is summarized in proposition 3.

**Proposition 3 (Pegged nominal exchange rate in AFP/PMP)** In AFP/PMP regime with the pegged nominal exchange rate, both QFPs of the central bank and the fiscal authority should be passive. The price level and the real exchange rate are determined by the active fiscal policy.

**Proof.** Let the FP be active in the sense that the government spending are not adjusted to satisfy either (31) or (32). By equation (23) on the real exchange rate, the current government spending (on nontradable sector) determines the price level since every other
variables are exogenous. In order to be explicit, let’s combine (10) and (23) to generate (35).

\[ P_t = \frac{1}{S^{P^*}} \left( \frac{u'_N(y'_N - y_t)}{u'_T(y'_T)} \right)^{1-\alpha_1}. \]  

(35)

In (32), the real value of fiscal authority liability is decided by the predetermined nominal liability value and the price level from (35). Moreover, in the right hand side of (32), the current and future surpluses, seigniorage, \( \gamma_{2,T}^C \), and \( \gamma_{2,T}^G \) are determined by the active FP while \( \gamma_{1,T}^C \) is zero. Therefore, the only remain term that can be used to make (32) satisfied is \( \{\Delta F_t^G \equiv F_t^G - F_{t-1}^G\} \). In this sense, QFP of the fiscal authority should be passive. Similarly, QFP of the central bank should be passively adjusted in (31).

Canzoneri et al. (2001) argued that the fixed exchange regime is not credible in the active fiscal policy regime. However, the proposition 3 shows that FP can be active in the pegged case. The main difference from this model is that Canzoneri et al. (2001) assumed PPP but this paper does not.

**Remark 4** Assuming the FP does not change the real interest rate, the increased deficit induces either inflation as in the conventional fiscal theory or deflation when the nominal exchange rate is pegged. The mechanism that the FP affects the price is different from the conventional fiscal theory.

Assume that the FP is such that it does not introduce any changes in the real interest rate, i.e., \( D_{t,T} \) in the model. In this model set up that the government spending only occur at the nontradable sector, increased government spending cause the real exchange rate appreciation, thus the inflation. This result is same as in the conventional fiscal theory that the increased deficit always cause the inflation by the wealth effect. However, if the government spending occur at the tradable sector, increased spending will cause the deflation instead of the inflation. Unlike the conventional fiscal theory, increased deficit does not necessarily cause the wealth effect because the QFP neutralizes the effect. Another interesting point is that the expected increase in the deficit may not change the current price level. Instead, by the equation (35), the expected deficit will change the expected price level. However, if the passive QFPs in (31) and (32) perfectly isolate the changes from the future surpluses, seigniorage, and gamma terms, \( P_t \) may stay unchanged.

**Remark 5** Budget autonomy parameters \( \theta_1, \theta_2 \) and the central bank debt outstanding do not affect the price level determination, but they determine the relative assignment of the required adjustment in the QFPs between two policy authorities. Some values of \( S \) may be inconsistent with the proposed FP.
Since the price level is determined in (35), neither the budget autonomy parameters nor the central bank debt outstanding affects price level. However, if there is a larger debt outstanding, the required adjustment in $\Delta F_t^G$ and $\Delta F_t^C$ is greater when other things are equal. Note that the assignment of the adjustment in the QFPS between the central bank and the fiscal authority depend upon the budget autonomy parameters and the central bank debt outstanding. Specifically, if $\theta_1$ is greater than 50%, the larger central bank debt make QFP of the fiscal authority be more responsible for the adjustment.

Another thing to point out is that not every value of $\bar{S}$ is consistent with the current and future government financing projects. For example, let’s assume that the government plans to spend more on the nontradable sector in the current period. Besides the surpluses decreases, the seigniorage also decreases if the usual preference is assumed for the real money balance. Since the price level increases, $\gamma^C_{2,T}$, and $\gamma^G_{2,T}$ also decrease. Therefore, in both of (31) and (32), the foreign asset should be decreased. However, predetermined foreign asset holdings are bounded, thus it is possible that the QFPs cannot accommodate the proposed spending.

**Proposition 6 (Floating nominal exchange rate in AFP/PMP)** When the nominal exchange rate is floating in AFP/PFP regime, both QFPs of the central bank and the fiscal authority can be either active or passive unless both are active at the same time. Moreover, the FP affects the price level by the wealth effect unlike in the pegged case.

**Proof.** Because the nominal exchange rate is not predetermined, the FP in (35) determines $S_t/P_t$ instead of $P_t$. Therefore,

$$\gamma^C_{2,T} = \frac{i^* F^C_{T-1}}{P^*} \left( \frac{u_T'(\bar{y}^T)}{u_N'(y_T^N - g_T)} \right)^{1-\alpha_1}$$

$$\Delta a^C_T = \frac{\Delta F^C_{T-1}}{P^*} \left( \frac{u_T'(\bar{y}^T)}{u_N'(y_T^N - g_T)} \right)^{1-\alpha_1}$$

From the uncovered interest rate parity condition,

$$E_t[\gamma^C_{1,T}] = E_t[\left( \frac{u_T'(\bar{y}^T)}{u_N'(y_T^N - g_T)} \right)^{1-\alpha_1} (F^C_{T-1} - \frac{1 + i^*}{P^*(1 + i_{T-1})})].$$

Now the every entries in the right hand side of (31) and (32) are either exogenous or policy choice variables. Therefore, the price level can be adjusted by the active FP through the bond valuation formula by wealth effect. Since the price level is adjusted, the QFPs can be anything. However, because $\Delta a^C_T$ and $\Delta a^C_T$ should satisfy (33), both QFPs cannot be active at the same time. That is, if $\Delta a^G_T$ is acting actively, $\Delta a^C_T$ should accommodate the change in (33) because all other variables are fixed. \qed
Since the channel through which the FP affects the price level is unchanged from that of conventional fiscal theory, the intuition also remain same. That is, if future deficit is expected, agents feel wealthier and demand more goods, thus the price rises. What is interestingly new in this model is the effect of the QFPs on the nominal exchange rate.

**Remark 7** The active QFPs affect the nominal exchange rate through the wealth effect in AFP/PMP floating nominal exchange rate regime.

Whether the sterilized foreign exchange rate intervention is effective or not is an important issue in the international macroeconomic policy analysis. Although nothing was assumed on the relationship between the QFPs and the nominal exchange rate, remark 7 shed light on the issue. That is, together with the government surpluses, the QFPs can also have a wealth effect in the real bond valuation formula. For example, in (32), suppose the government wants to appreciate its currency, thus it decrease the foreign assets. If the government surplus is equal, decreased foreign asset increase the real value of government liability, thus induces the deflation. With the given real exchange rate, the deflation should be accompanied by the nominal exchange rate appreciation. In sum, the sterilized foreign exchange rate intervention affects the nominal exchange rate in the desired direction through the bond valuation formula.

### 3.2 Passive fiscal policy, active monetary policy (PFP/AMP)

Suppose the fiscal authority passively adjusts the surpluses to satisfy (31) and (32) for all prices, and the central bank actively sets the nominal interest rate to stabilize the price level. Then in this world, the price level should be determined out of bond valuation formula. Instead, the equilibrium price level should be from the fisher equation (21) and the monetary policy rule. For the tractability, I suggest a simple log-linearized monetary policy rule as follows.

\[
\hat{R}_t = \phi_p \hat{P}_t + \nu_t, \tag{38}
\]

where \( R_t = 1 + i_t \), \( \hat{R}_t = \ln R_t - \ln R \), \( \hat{P}_t = \ln P_t - \ln P \), arguments without time subscript are steady state values, \( \phi_p > 0 \) and \( \nu_t \) is a random shock in the policy. Assuming that the steady state inflation rate is zero and the utility function is additively separable between consumption and real money balance, a log-linearized approximation of (21) is given by following equation.

\[
\hat{R}_t = \hat{r}_t + E_t[\ln \pi_{t+1}], \tag{39}
\]
where the real interest rate $r_t$ depends on $y_t^N - g_t$ and $y_{t+1}^N - g_{t+1}$, $\hat{r}_t = \ln r_t - \ln r$ and inflation rate $\pi_t = P_t / P_{t-1}$. By substituting (38) into (39) and solving forward,

$$
\hat{P}_t = \sum_{s=0}^{\infty} (1 + \phi_p)^{-s-1} E_t[\hat{r}_{t+s} - \nu_{t+s}].
$$

(40)

Therefore, if the central bank offsets the shocks in the real return by its policy shocks, the price level will remain unchanged at the steady state level. This argument is one of the simplest expositions of the price determination around the steady state in the Taylor rule world. Proposition 8 describes the nature of the QFPs in this policy regime.

Proposition 8 (Required QFP in PFP/AMP) Both QFPs can be either active or passive in PMP/AMP regime regardless of nominal exchange rate regime. Although QFPs can be active, the QFPs can not affect the price level.

Proof. First, let’s consider the pegged case. Because the nominal exchange rate is pegged again, equation (35) is also applied to this case. However, because the price level is given by (40), (35) shows the required government spending to fix the nominal exchange rate to $\bar{S}$. Also, the government surpluses are adjusted to satisfy (32) and (33), thus there is no need for the QFPs to adjust. Besides, because the price level is determined regardless the bond valuation formula, the changes in the QFPs do not affect the price level. Rather, it mandates the changes in the required adjustment on the FP.

In the floating exchange rate case, the current government spending affects the current nominal exchange rate because the price is pinned down from (40). In the floating regime also, the FP has the freedom to be adjusted in (31) and (32), thus the QFPs are not constrained by the active MP.

The interesting feature on this regime is that the central bank can chase the policy commitment (foreign exchange rate stabilization) other than the MP. It has been believed that the inflation targeting central bank can hardly stabilize the nominal exchange rate by the sterilized intervention because the sterilized intervention cannot alter the money base. In the monetarist view on the nominal exchange rate determination, if money or interest rate is bonded by the MP, the nominal exchange rate is a residual of the MP. However, this paper shows that this conventional belief may not valid. Remark 9 illustrates the unconventional channel through which the sterilized intervention of the inflation targeting central bank.

Remark 9 The QFPs in PFP/AMP regime can affect the nominal exchange rate indirectly through the FP. However, the direction of the effect may not be the desired one.
Suppose the central bank tries to appreciate the currency by the sterilized intervention, thus decrease the foreign asset. Since $-\Delta a_C^T$ increases, then the sum of remaining terms in the right hand side of (31) should be decreased. Moreover, the only available policy instrument to decrease the right hand side of (31) is the FP. Let’s assume that the government increase the spending to accommodate (31) and (32). Because the real exchange rate is appreciated by the increased spending on nontradable sector and the price level is given by (40), the nominal exchange rate falls (currency appreciated). However, if the FP decreases the right hand side of (31) by decreasing the lump-sum tax $\tau$, there will be no effect of intervention through this channel. Moreover, if the government spending occurs at the tradable sector, the real exchange rate will be depreciated, thus the nominal exchange rate will be depreciated.

**Remark 10** The budget autonomy parameter and the central bank debt outstanding do not affect the price level, but they determine the required adjustment in the FP.

The remark 10 is analogous to the remark 5. That is, as in the AFP/PMP case, $\theta_1$, $\theta_2$ and $B_{t-1}^C$ do not enter into the equation (40), thus they do not affect the price level determination. However, $\theta$’s and $B_{t-1}^C$ affects the required changes in the FP. In specific, when the central bank actively set the nominal interest rate, the fiscal authority should change the spending in order to satisfy (31) and (32). In the higher $B_{t-1}^C$ case, the present values of seigniorage, asset revaluation effect, and the interest earning from the asset should also be greater. Then, the required changes in $g_t$ should be different for the given shock in $i_t$. The difference between Remark 5 and 10 is that, in this case, it is the single policy instrument FP that should be adjusted. Therefore $\theta$’s and $B_{t-1}^C$ do not allocate the adjustment burden between the central bank and the fiscal authority. Interestingly, if the central bank have been issued the debts because of the QFP, it requires for the FP to behave appropriately. In other words, the agents expects that the fiscal authority ultimately will back up the QFP of the central bank.

### 4 Conclusion

Should monetary authorities with significant amount of debt outstanding worry about that? The central banks may fear that the large amount of debts outstanding and the policy commitment other than the price stabilization undermine stability of the price level and (or) their independence from the fiscal authority. This paper tried to address this question by analyzing policy interactions with the foreign exchange rate policy in consideration.

The first main finding in the model is the required behaviors for the third policy instrument QFP in various situations. The only circumstance that the both QFPs should be
passive is AFP/PMP with the pegged nominal exchange rate case. This policy regime was claimed to be infeasible by Canzoneri et al. (2001), but this paper accommodates the third policy instrument to be adjusted and abandons the simple PPP assumption. In every other cases, both QFPs have a freedom to be active or passive.

Second, the model predicts that neither the central bank debt outstanding nor the degree of central bank budget autonomy affects the price level determination in both Ricardian (PFP/AMP) and Non-Ricardian (AFP/PMP) policy regime. However, they do affect the required adjustment in the passive policies. Specifically, in AFP/PMP regime, the degree of autonomy and the central bank debt outstanding affect the burden allocation of the adjustment in both passive QFPs. Therefore, the central bank do not have to worry about the price stability, but debts and budget autonomy do have implications on the relationship between two policy authorities.

Additionally, it has been shown that the QFPs may affect the nominal exchange rate determination. In the AFP/PMP regime, the current nominal exchange rate will be affected to the desired direction by the QFPs through the wealth effect in the bond valuation formula. On the other hand, in the PFP/AMP regime, the QFP of the central bank affects the nominal exchange rate only through changes in the FP, and the direction may be different from the central bank’s desired one. This result contradicts the conventional belief about the effect of the sterilized foreign exchange rate intervention of the inflation targeter.

References


# Table 1. Central bank balance sheets and fiscal authority debts

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007p</th>
</tr>
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<tbody>
<tr>
<td><strong>China</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reserve money</td>
<td>3985.2</td>
<td>4513.8</td>
<td>5284.1</td>
<td>5885.6</td>
<td>6434.3</td>
<td>7775.8</td>
<td>8804.9</td>
</tr>
<tr>
<td>Central bank debt</td>
<td>0.0</td>
<td>148.8</td>
<td>303.2</td>
<td>1107.9</td>
<td>2029.6</td>
<td>2974.1</td>
<td>3894.6</td>
</tr>
<tr>
<td>CB foreign asset</td>
<td>1986.0</td>
<td>2324.3</td>
<td>3114.2</td>
<td>4696.0</td>
<td>6344.0</td>
<td>8577.3</td>
<td>11569.7</td>
</tr>
<tr>
<td>Loan to private sector</td>
<td>2005.4</td>
<td>1973.5</td>
<td>1944.5</td>
<td>1942.6</td>
<td>2598.5</td>
<td>2858.5</td>
<td>2269.9</td>
</tr>
<tr>
<td>Fiscal authority debt</td>
<td>448.3</td>
<td>566.0</td>
<td>602.9</td>
<td>672.6</td>
<td>692.3</td>
<td>692.3</td>
<td>692.3</td>
</tr>
<tr>
<td>Current account</td>
<td>17.4</td>
<td>35.4</td>
<td>45.9</td>
<td>68.7</td>
<td>160.8</td>
<td>249.9</td>
<td>249.9</td>
</tr>
<tr>
<td>Foreign reserve</td>
<td>215.6</td>
<td>291.1</td>
<td>408.2</td>
<td>614.5</td>
<td>821.5</td>
<td>1068.5</td>
<td>1453.6</td>
</tr>
</tbody>
</table>

| **Japan** |        |        |        |        |        |        |        |
| Reserve money | 88.9   | 99.4   | 111.4  | 115.6  | 116.6  | 94.8   | 88.0   |
| Central bank debt | 3.5    | 5.2    | 2.7    | 1.6    | 0.9    | 0.3    | 0.1    |
| CB foreign asset  | 5.2    | 5.4    | 5.4    | 5.6    | 6.5    | 7.3    | 7.8    |
| Loan to private sector | 36.2   | 36.6   | 32.5   | 44.4   | 52.4   | 30.5   | 32.3   |
| Fiscal authority debt | 582.5  | 643.2  | 670.1  | 751.1  | 813.2  | 832.3  | 833.7  |
| Current account | 87.8   | 112.5  | 136.2  | 172.1  | 165.8  | 170.5  |        |
| Foreign reserve | 395.2  | 461.2  | 663.3  | 833.9  | 834.3  | 879.7  | 935.1  |

| **Korea** |        |        |        |        |        |        |        |
| Reserve money | 32.8   | 38.0   | 40.7   | 38.8   | 43.2   | 51.9   | 49.2   |
| Central bank debt | 79.1   | 84.3   | 105.5  | 142.8  | 155.2  | 158.4  | 150.3  |
| CB foreign asset  | 142.6  | 155.4  | 194.0  | 228.1  | 229.0  | 244.6  | 248.5  |
| Loan to private sector | 14.0   | 9.2    | 9.5    | 8.3    | 8.8    | 8.5    | 5.8    |
| Fiscal authority debt | 82.4   | 98.3   | 135.8  | 177.6  | 222.9  | 257.8  | 272.5  |
| Current account | 8.0    | 5.4    | 12.0   | 28.2   | 15.0   | 6.1    |        |
| Foreign reserve | 102.8  | 121.3  | 155.3  | 199.0  | 210.3  | 238.9  | 257.2  |

| **Malaysia** |        |        |        |        |        |        |        |
| Reserve money | 40.0   | 42.6   | 45.5   | 50.1   | 52.6   | 58.2   | 62.9   |
| Central bank debt | 7.5    | 12.3   | 13.4   | 16.9   | 20.3   | 24.8   | 73.9   |
| CB foreign asset  | 116.9  | 131.4  | 170.5  | 253.5  | 266.3  | 290.4  | 335.0  |
| Loan to private sector | 30.8   | 30.1   | 29.4   | 30.4   | 27.0   | 30.0   | 76.4   |
| Fiscal authority debt | 121.4  | 128.7  | 151.5  | 182.0  | 198.7  | 217.2  | 250.6  |
| Current account | 7.3    | 7.2    | 13.4   | 15.1   | 20.0   | 25.5   |        |
| Foreign reserve | 29.5   | 33.4   | 43.8   | 65.9   | 69.9   | 82.1   | 97.9   |

Source: Central banks of each country, National statistics offices, IMF IFS.

Note: Units are in billions local currencies (end of period) unless otherwise stated.

1) In billions US dollar, 2) In trillions local currencies, 3) Period average
Table 1. Central bank balance sheets and fiscal authority debts (continued)

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Source: Central banks of each country, National statistics offices, IMF IFS.
Note: Units are in billions local currencies (end of period) unless otherwise stated.
1) In billions US dollar
Table 1. Central bank balance sheets and fiscal authority debts (continued)

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Source: Central banks of each country, National statistics offices, IMF IFS.
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1) In billions US dollar