Entrepreneurial Activities of Returning Migrants  
under Credit Constraint in Rural China  

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1 Introduction

This paper shall take the angle of entrepreneurship among returning migrants, who face borrowing constraints, to examine whether and to what extent the entrepreneurial activities of returnees affect the rural area development. In the literature there has been some debate about financial sector reforms in developing countries, and there has also been studies on internal migration and its influence. I combine the two seemingly separate but indeed in many cases closely related issues in a general equilibrium model.

After the household registration system reform in China in 1984, the urbanization has leaped from 24.52% in 1986 to 42.99% in 2005, and the estimated rural migrants were 140 millions in 2003 (BSC, 2004, 2006). According to the China Rural Development Research Center, one third of migrants started to go back to native homes in late 1990s (Murphy, 1999). There are extensive empirical studies showing a wide array of reasons for the return migration, to name a few, congestion cost in cities, high living expenditure, housing cost, emotional preference of hometown over guest area, uncertainty of living in guest place, marriage match, etc. More recently, a series of studies started to pay attention to the occupational choices of returnees (Mesnard, 2004, McCormick, 2003, Murphy, 1999, Rapoport, 2002, Ilahi, 1999). A common finding is that, among all the returnees, those who become entrepreneurs tend to have higher level of saving through their working in guest place and higher human capital. However, none of them have studied the general equilibrium effect of the entrepreneurial activities of returning migrants.
on rural non-farm sector, farm sector and urban sector.

The lack of perfect credit market in rural area of developing countries prevents people who do not have certain collateral level from borrowing sufficient money to start out own business (Ellis, 1998, Rapoport, 2002, Gine, 2007). On the other hand, migrants who have saved a substantial amount of money through hard working in cities show advantage in the source of usable capital from their own saving. The returning entrepreneurs generally open small non-farm business while hiring local labor (Murphy, 1999). The reasons why those migrants do not open business in cities are various: high fixed cost, unaffordable launch cost, exclusive labor cost hindering profit, and most importantly, the pressure from highly competitive corporations (Quadrini (1999).

This paper is expected to achieve three goals: First, to study how much the entrepreneurship can explain the phenomenon of returning migration; Second, to analyze to what extent the returnee enterprises contribute to rural economic growth and employment; Third, discuss policies related with rural financial sector liberalization and internal migration obstacle removing.

2 Background of Rural China after 1980s

After China’s government loosened the household registration system in 1984, the excess labor in agricultural sector turned to urban area looking for opportunities. In 2003, there are 140 million rural migrants in China. Started from the mid 1990s, the wave of returning migration has formed and became noticeable. Most of the returnees are the early migrants who have spent their golden working age in urban area. Murphy (1999, 2002) conducted a study in rural China, and she found that manufacturing business in rural area by returnees were important to the local economic diversification and growth. One fifth of individual enterprises in the surveyed county were owned by returnees. The industrial product value of returnee enterprises accounts for almost 13% of the total industrial product value of industries in Xinfeng. In another county, Yudu, over 4000 migrants have returned to set up around 1450 private and individual business engaged in production or manufacturing. In 1996, out of 109 new projects with annual prod-
uct values of around one million yuan, 63% were created by returnees (Murphy, 1999). From Table 1 we can see that even "large" manufacturing business hires around 40 employees. Notwithstanding, returnees have contributed in creating new jobs for the rural area than otherwise would be.

<table>
<thead>
<tr>
<th>Type and scale</th>
<th>No. in Survey</th>
<th>No. of Employees</th>
<th>Financial resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large scale</td>
<td>27</td>
<td>16 to 860, median 40</td>
<td>Some have formal loans, some partly owned by the gov.</td>
</tr>
<tr>
<td>Small Scale</td>
<td>25</td>
<td>1 to 15, median 4</td>
<td>Personal saving, informal loans</td>
</tr>
<tr>
<td>Services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Scale</td>
<td>22</td>
<td>1 to 13, median 4</td>
<td>Personal Saving, informal loans</td>
</tr>
</tbody>
</table>

Table 1: Job Creation by the Returning Migrants

Table 2 describes the human capital portfolio of non-migrants, migrants and returnees. The human capital of migrants and returnees tend to be higher than non-migrants, whereas there is no significant difference in the human capital level between continuing migrants and returnees. Another data is from Murphy (2002), which documents a survey among 60,000 rural migrants conducted by the Statistical Bureau of Yudu County in 1992. The data is as following: illiterate, 2.1%, primary, 47%, lower middle school, 50.9%; the 1995 government figures for the total rural labor force of Ganzhou are: illiterate, 12.92%; primary, 40.41%; lower middle school, 37.49%. The returnees generally have either better education or better special skills. The percentages of returnees with special skills prior to migration are 53% in Yudu and 47% in Xinfeng. Two thirds of the entrepreneurs in Yudu and three quarters in Xinfeng possessed vocational skills. It is not only that rural migrants have better human capital prior to migration, but they also obtain different extent of gain from migration. In a survey conducted by Chinese Agricultural Survey Team (Nongcun Gongzuo Diaochaozu), 95.1% of the 737 returned migrants reported a gain in skills during their
migration stage.

Table 2: Return Migrants, Non-migrants, and Migrants in China, 1999

<table>
<thead>
<tr>
<th>No. of workers</th>
<th>All</th>
<th>Non-migrants</th>
<th>Continuing migrants</th>
<th>Return migrants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2137</td>
<td>1673</td>
<td>289</td>
<td>175</td>
</tr>
<tr>
<td>%</td>
<td>100</td>
<td>78.3</td>
<td>13.4</td>
<td>8.3</td>
</tr>
</tbody>
</table>

- **Personal characteristics**
  - Male(%) 52.0 47.8 63.3 73.6
  - Married(%) 83.3 89.5 48.6 80.9
  - Age(years) 39.6 42.0 27.9 35.6
  - Schooling(years) 6.0 5.5 7.7 7.1
  - Illiterate(%) 12.4 14.9 3.8 2.2
  - Primary school (%) 38.9 42.9 18.5 33.7
  - Junior high(%) 41.3 35.1 68.5 54.9
  - Senior high(%) 6.9 6.6 7.6 8.6
  - Technical school or higher(%) 0.6 0.4 1.0 1.1
  - Months of migration(months) 8.4 0.0 41.7 39.4
  - Married only Spouse non-migrant (%) 87.4 90.2 69.2 67.0

Data Source: Survey, Ministry Of Agriculture(1999)

After 1980s, there have been several government attempts to liberalize the rural financial system in rural China, and the results are, however, not as encouraging as expected. Over the past nearly three decades, there are dual financial system in China: the one in the urban area has experienced fast modernization which has joined the world competition now; whereas the one in rural area has been largely stagnant. Literature has well summarized its characteristics: formal credit programs are highly centralized; the "cheap" credits are earmarked to certain agricultural investment; private lending is strictly regulated and usually illegal; the rural credit market is fragmented (Cheng, 2004; Jia, 2007). A glance of the table 3 might help us see more about China’s rural financial market.
Table 3: Deposits of Rural Financial Institutions\(^1\) in China

<table>
<thead>
<tr>
<th>Year</th>
<th>(ABC^{1}) (Bi.Yuan)</th>
<th>(ABC) Agri. Deposit (Bi.Yuan) (%)</th>
<th>(RCCs^{2}) (Bi.Yuan) (%)</th>
<th>(PS^{3}) (Bi.Yuan) (%)</th>
<th>Total Rural Deposits (Bi.Yuan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>91.2</td>
<td>47.5</td>
<td></td>
<td></td>
<td>47.5</td>
</tr>
<tr>
<td>1986</td>
<td>122.4</td>
<td>58.8</td>
<td></td>
<td></td>
<td>58.8</td>
</tr>
<tr>
<td>1987</td>
<td>148.7</td>
<td>65.7</td>
<td></td>
<td></td>
<td>65.7</td>
</tr>
<tr>
<td>1988</td>
<td>171.4</td>
<td>67.2</td>
<td></td>
<td></td>
<td>67.2</td>
</tr>
<tr>
<td>1989</td>
<td>205.5</td>
<td>72.3</td>
<td>29.9</td>
<td>166.9</td>
<td>69.1</td>
</tr>
<tr>
<td>1990</td>
<td>264.0</td>
<td>85.4</td>
<td>28.0</td>
<td>214.5</td>
<td>70.4</td>
</tr>
<tr>
<td>1991</td>
<td>331.9</td>
<td>102.8</td>
<td>26.9</td>
<td>270.3</td>
<td>70.8</td>
</tr>
<tr>
<td>1992</td>
<td>413.1</td>
<td>18.3</td>
<td>24.7</td>
<td>347.8</td>
<td>72.7</td>
</tr>
<tr>
<td>1993</td>
<td>518.4</td>
<td>149.2</td>
<td>24.9</td>
<td>429.1</td>
<td>71.5</td>
</tr>
<tr>
<td>1994</td>
<td>697.2</td>
<td>173.1</td>
<td>22.4</td>
<td>567.0</td>
<td>73.3</td>
</tr>
<tr>
<td>1995</td>
<td>694.0</td>
<td>n/a</td>
<td>n/a</td>
<td>717.0</td>
<td>92.9</td>
</tr>
<tr>
<td>1996</td>
<td>910.7</td>
<td>n/a</td>
<td>n/a</td>
<td>879.0</td>
<td>92.2</td>
</tr>
<tr>
<td>1997</td>
<td>1132.0</td>
<td>26.4</td>
<td>2.2</td>
<td>1062.0</td>
<td>90.3</td>
</tr>
<tr>
<td>1998</td>
<td>1333.0</td>
<td>30.0</td>
<td>2.2</td>
<td>1219.0</td>
<td>89.8</td>
</tr>
<tr>
<td>1999</td>
<td>1549.0</td>
<td>34.2</td>
<td>2.3</td>
<td>1336.0</td>
<td>89.3</td>
</tr>
<tr>
<td>2000</td>
<td>1752.0</td>
<td>37.2</td>
<td>2.2</td>
<td>1513.0</td>
<td>88.3</td>
</tr>
<tr>
<td>2001</td>
<td>2025.0</td>
<td>37.7</td>
<td>1.9</td>
<td>1726.0</td>
<td>87.8</td>
</tr>
<tr>
<td>2002</td>
<td>2410.7</td>
<td>42.4</td>
<td>1.9</td>
<td>1987.5</td>
<td>87.1</td>
</tr>
<tr>
<td>2003</td>
<td>2900.5</td>
<td>41.8</td>
<td>1.5</td>
<td>2371.0</td>
<td>87.2</td>
</tr>
<tr>
<td>2004</td>
<td>3417.3</td>
<td>44.1</td>
<td>1.4</td>
<td>2728.9</td>
<td>86.6</td>
</tr>
</tbody>
</table>

Note: 1) ABC: Agricultural Bank of China; RCC: Rural Credit Cooperatives; PS: Postal Savings. 2) Include deposits in urban areas. 3)4) Only include rural deposits. 5) The sum of rural deposits of ABC, RCC, and PS.
From Table 3 we can see that deposits in rural China has been increasing over the two decades. However at the mean time, the loan in rural area does not show the similar trend of growth.

Table 4: Loan Portfolio of Rural Financial Institutions in China

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Rural Loan(^1) (Bi.Yuan)</th>
<th>Rural Loans (Bi.Yuan)</th>
<th>ABC (%)</th>
<th>RCC (%)</th>
<th>ADBC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>41.0</td>
<td>168.8</td>
<td>41.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>56.8</td>
<td>199.8</td>
<td>56.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>68.9</td>
<td>232.3</td>
<td>68.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>80.1</td>
<td>263.2</td>
<td>80.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>197.9</td>
<td>305.8</td>
<td>88.4</td>
<td>44.7</td>
<td>109.5</td>
</tr>
<tr>
<td>1990</td>
<td>243.5</td>
<td>377.4</td>
<td>102.5</td>
<td>42.1</td>
<td>141.0</td>
</tr>
<tr>
<td>1991</td>
<td>303.9</td>
<td>457.8</td>
<td>123.0</td>
<td>40.5</td>
<td>180.9</td>
</tr>
<tr>
<td>1992</td>
<td>388.4</td>
<td>546.8</td>
<td>143.0</td>
<td>36.8</td>
<td>245.4</td>
</tr>
<tr>
<td>1993</td>
<td>503.4</td>
<td>652.9</td>
<td>177.2</td>
<td>35.2</td>
<td>326.2</td>
</tr>
<tr>
<td>1994</td>
<td>953.8</td>
<td>591.2</td>
<td>181.5</td>
<td>19.0</td>
<td>415.9</td>
</tr>
<tr>
<td>1995</td>
<td>991.8</td>
<td>655.0</td>
<td>n/a</td>
<td>n/a</td>
<td>523.0</td>
</tr>
<tr>
<td>1996</td>
<td>1261.2</td>
<td>856.7</td>
<td>n/a</td>
<td>n/a</td>
<td>636.0</td>
</tr>
<tr>
<td>1997</td>
<td>1771.6</td>
<td>981.0</td>
<td>174.9</td>
<td>9.9</td>
<td>733.0</td>
</tr>
<tr>
<td>1998</td>
<td>1752.6</td>
<td>1367.0</td>
<td>209.1</td>
<td>11.9</td>
<td>834.0</td>
</tr>
<tr>
<td>1999</td>
<td>2563.2</td>
<td>1553.0</td>
<td>912.7</td>
<td>35.6</td>
<td>923.0</td>
</tr>
<tr>
<td>2000</td>
<td>2583.4</td>
<td>1450.0</td>
<td>794.3</td>
<td>30.7</td>
<td>1049.0</td>
</tr>
<tr>
<td>2001</td>
<td>2757.4</td>
<td>1605.0</td>
<td>817.2</td>
<td>29.6</td>
<td>1197.0</td>
</tr>
<tr>
<td>2002</td>
<td>2572.1</td>
<td>1857.9</td>
<td>441.7</td>
<td>17.2</td>
<td>1393.8</td>
</tr>
<tr>
<td>2003</td>
<td>2845.0</td>
<td>2211.8</td>
<td>456.9</td>
<td>16.1</td>
<td>1697.9</td>
</tr>
<tr>
<td>2004</td>
<td>3106.4</td>
<td>2514.6</td>
<td>463.6</td>
<td>14.9</td>
<td>1923.8</td>
</tr>
</tbody>
</table>

Source: Almanac of China’s Finance and Banking(1986-2005); ADBC internal report; Jia (2007)

Notes: 1) Outstanding loans of ABC, RCC, and ADBC. Rural loans consist of loans to agricultural households for both production and consumption, loans to TVE (Town and Village Enterprises).

At the meantime of a huge enlargement in rural deposits, the loan amount is stable and decreasing. This plummet in the ratio of total rural loans to deposits indicates that the rural loanable funds are either channeled outside of rural China or are left unused (Jia, 2007).

Not like the case in other developing countries, the constraint facing rural Chinese is unbalanced, in that, people have the access to save but have trouble in finding credit. I thus set up the model according to this unbalanced financial sys-
tem, in that rural people are constrained in taking out loans, which creates the comparative advantage for those who have migrated and brought back capital with them. In this line, returning migrants serve the function of channeling credit back from cities to rural area.

3 The Model

In order to model the two-stage decision sequences - migrate or not, return or stay in cities - I use the two periods overlapping generation model with heterogenous agents. Agents only live two periods and work in both, thus I abstract from modeling the childhood and retirement. Each individual has one child, i.e. there is no population growth. There is no monetary bequest. The paper models two areas, rural and urban, and three sectors, farm production, rural non-farm production and urban industry. In this paper, I focus on the economic activities of rural agents, who are heterogenous in human capital endowment, which is a random variable drawn from a known distribution. In the first period, a young rural resident chooses whether to migrate or not. In the second period of life, those who did not migrate in the first period still stay in rural area, whereas a migrant faces two choices: to be a permanent migrant, or to return to rural hometown. The returning migrants can either be a worker or be an entrepreneurs in the second period. Whether a return entrepreneur’s investment back home is profitable is conditional partly on her management skill. In general, a person’s management skill is not exactly the same with her human capital, but I abstract from modeling the difference between the two.

Following Galor and Zeria (1993), I assume people only consume in the second period of life and derive utility from it, thus an individual who is born in period $t$ has lifetime utility:

$$u_t = \ln c_{f,t+1} + \ln c_{e,t+1} + \ln c_{u,t+1}$$

(1)

Everyone takes utility from three types of goods: agricultural goods $c_f$, non-farm sector product $c_e$, and urban manufacturing goods $c_u$. Each agent has one unit of non-leisure time in each period, which is supplied inelastically to either of the three production sectors.

There is asymmetry in credit market and capital market between rural and urban area. In the rural area, there is imperfect credit market, i.e. everyone can save at the equilibrium world interest rate $r$ but cannot borrow; there is no capital market either. This is a quite strong assumption, however, it characterizes the reality of China’s rural economy after 1980s. Therefore, temporary migration offers a unique channel to finance the non-farm sector in rural area. Potential entrepreneurs purchase physical capital through migration income and invest in their own business in the second period of life. On the very opposite of rural entrepreneurs, urban manufacturing corporations do not face any credit constraints.

There are two types of production in rural area, farm and non-farm. Farm production uses Cobb-Douglas technology with land and raw labor as inputs:

$$F_1(L_{f,t}) = A_f(L_{f,t})^\eta,$$

(2)
where $L_{f,t}$ is the raw labor input in farm production. Land is owned by the state and equal size of land is assigned to each household who works on the farm, thus quantity of land is normalized to one.

A returnee has the choice between to work for other entrepreneurs or to be entrepreneurs themselves in their second period of life. The rural non-farm enterprises have the production function as following:

$$F_2(K_{e,t+1}, L_{e,t+1}, h_t) = A_e K_{e,t+1}^{\alpha_1} L_{e,t+1}^{\alpha_2} h_t^{\alpha_3}$$

in which, $\alpha_1 + \alpha_2 + \alpha_3 = 1$. $h_t$ is human capital endowment an entrepreneur has. Facing the difficulty of borrowing, the physical capital input a returning migrant uses cannot exceed his own saving $s_t$ from migration; the returning entrepreneur hires labor $L_{e,t+1}$ and physical capital $K_{e,t+1}$ and produce through a constant returns to scale production function.

The urban manufacturing production uses capital and effective labor as inputs:

$$F_3(K_{u,t}, H_{u,t}) = A_u K_{u,t}^{\gamma_u} H_{u,t}^{1-\gamma_u}$$

in which $A_u$ is the total factor productivity level in urban manufacturing sector. $K_{u,t}$ and $H_{u,t}$ are capital and effective labor inputs. The urban production requires relatively higher human capital, thus here $H_{u,t}$ stands for human capital included effective labor. I follow Quadrini (1999) in which he differentiates the business activities into two groups: small activities run in the form of non-corporate organizations, and big firms organized as corporations. The non-corporate sector is run by households engaging in entrepreneurial activities, which is opposed to the "impersonality" of big corporate organizations.

In the following sections I will describe the heterogeneity in rural residents.

### 3.1 The Household Problem

Individual agents work in both periods but only consume at the end of the second period. Given his lifetime wealth at the end of second period $W(h_t)$, the exogenous prices for three types of consumption goods, agricultural goods, $c_f$, non-farm products, $c_e$ and urban manufacturing goods, $c_u$, the agent chooses the optimal consumption to maximizes his lifetime utility subject to lifetime budget constraint:

$$\max_{\{c_{f,t+1}, c_{e,t+1}, c_{u,t+1}\}} u_t = \ln c_{f,t+1} + \ln c_{e,t+1} + \ln c_{u,t+1}$$

s.t. $p_f c_{f,t+1} + p_e c_{e,t+1} + p_u c_{u,t+1} \leq W(h_t)$

given $\{p_f, p_e, p_u, r\}$

where $p_f$, $p_e$, $p_u$ are the exogenous prices for products from agricultural, non-farm and manufacturing sectors. $W(h_t)$ is lifetime income of the agent with human capital $h_t$. The ultimate wealth level depends on the agent’s occupational choices, which I proceed to study in the later sections.
The lifetime wealth level depends on private agents’ occupational choices. The urban sector pays \( w_{u,t} \) for one unit of human capital, and the rural sector pays \( w_{r,t} \) for one unit of labor. If a rural agent decides not to migrate and work in rural area for two periods, his lifetime wealth is given by the two periods wage income \( w_{r,t}(1 + r) + w_{r,t+1} \); If he decides to temporarily migrate and work in rural area in the second period, his lifetime wealth is given by \( w_{u,t} h_t(1 + r) + w_{r,t+1} \); If he decides to temporarily migrate and be an entrepreneur in the second period, his lifetime wealth is given by \( w_{u,t} h_t(1 + r) + \pi_{e,t+1} \), in which, \( \pi_{e,t+1} \) is the net profit he obtains from his own business; If he decides to permanently migrate, then he needs to pay dwelling cost, high urban living cost, which I denote as \( J \), a permanent migrant’s lifetime wealth is \( (w_{u,t}(1 + r) + w_{u,t+1})h_t - J \).

3.2 Firm’s Problem

3.2.1 Urban Manufacturing Production

The urban firms choose optimal level of capital \( K_{u,t} \) and effective labor input \( H_{u,t} \) to maximize their profit, given the price of manufacturing goods, urban wage \( w_{u,t} \), and interest rate \((1 + r)\):

\[
\pi(K_{u,t}, H_{u,t}) = \max_{\{K_{u,t}, H_{u,t}\}} \{ p_u A_u K_{u,t}^{\gamma_u} H_{u,t}^{1-\gamma_u} - w_{u,t} H_{u,t} - (1 + r) K_{e,t}\}
\]

given \( \{w_{u,t}, r, p_u\} \)

3.2.2 Rural Non-farm Production

In the model, there is no capital market in the rural area. Thus, an potential entrepreneur needs to migrate and purchase the capital using his saving. Since he does not consume in the first period, the amount of capital that he can purchase is constrained by his income from migration, \( w_{u,t} h_t \). Therefore, \( K_{e,t+1} \leq w_{u,t} h_t \). The net profit of a returning entrepreneur is defined as the value of products netting cost of production factors and fixed cost \( D \):

\[
\pi_{t+1}(K_{e,t+1}, h_t) = \max_{\{K_{e,t+1}, L_{e,t+1}\}} \{ p_e A_e K_{e,t+1}^{\alpha_e} L_{e,t+1}^{\alpha_e} h_t^{\alpha_t} - w_{r,t+1} L_{e,t+1} - (1 + r) K_{e,t+1} - D \}
\]

s.t. \( F_2(K_{e,t+1}, L_{e,t+1}, h_t) = A_e K_{e,t+1}^{\alpha_e} L_{e,t+1}^{\alpha_e} h_t^{\alpha_t} \)

\( K_{e,t+1} \in (0, w_{u,t} h_t] \)

given \( \{w_{u,t}, w_{r,t+1}, r, p_e, h_t, D\} \)
Since the returning entrepreneurs have the option of depositing their saving in the banking system, the net profit is defined as after subtracting the opportunity cost of capital. A returning migrant does not have to be an entrepreneur, so an alternative is to deposit his saving and earn interest income. Therefore, the opportunity cost of the capital \((1 + r)K_{e,t+1}\) is subtracted in the profit definition.

4 Solving the Model

4.1 Solving Urban Manufacturing Firm’s Problem

Because the urban firms have access to the complete credit market, they can borrow as much as they demand at the equilibrium interest rate \(r\), which equals to the exogenous world interest rate. The equilibrium urban wage does not depend on the urban labor supply, which provides me a very handy way of abstracting from modeling the impact of rural migrants on urban wages.

\[
\frac{H_{u,t}}{K_{u,t}} = \left[ \frac{1 + r}{A_{u,t}} \right]^{\frac{1}{1-\gamma}}
\]

\[
w_{u,t} = A_{u,t}^{\frac{1}{1-\gamma}} \gamma^{\frac{1}{1-\gamma}} (1 - \gamma) \frac{p_{u,t}}{(1 + r)^{\frac{1}{1-\gamma}}}
\]

4.2 Solving Rural Non-farm Enterprise’s Problem

I rewrite the optimal level of the hired labor, \(L_{e,t+1}\), as a function of physical capital input and the entrepreneur’s human capital level:

\[
L_{e,t+1}(K_{e,t+1}, h_t) = \left[ \frac{p_e A_e \alpha_2 K_{e,t+1}^{\alpha_1} h_t^{\alpha_3}}{w_{r,t+1}} \right]^{\frac{1}{\alpha_1 + \alpha_3}}
\]

Similarly, I can also express the rural non-farm’s profit as a function of \(K_{e,t+1}\) and \(h_t\):

\[
\pi_{t+1}(K_{e,t+1}, h_t) = \Gamma_1 K_{e,t+1}^{\frac{\alpha_1}{\alpha_1 + \alpha_3}} h_t^{\frac{\alpha_3}{\alpha_1 + \alpha_3}} - (1 + r)K_{e,t+1} - D,
\]

where,

\[
\Gamma_1 \equiv (A_e p_e)^{\frac{1}{\alpha_1 + \alpha_3}} \left( \alpha_2^{\frac{\alpha_3}{\alpha_1 + \alpha_3}} - \alpha_2^{\frac{1}{\alpha_1 + \alpha_3}} \right) w_{r,t+1}^{-\frac{\alpha_3}{\alpha_1 + \alpha_3}}
\]

Maximizing the profit by choosing the optimal physical capital without any constraint will give the unconstrained capital input level \(K_{e,t+1}^u\):

\[
K_{e,t+1}^u(h_t) = \left[ \frac{\Gamma_1^{\frac{\alpha_1}{\alpha_1 + \alpha_3}}}{1 + r} \right]^{\frac{1}{\alpha_3}} h_t
\]
I use the above unconstrained capital input as a standard to make partition in between of constrained and unconstrained entrepreneurs. If the saving of a returning entrepreneur is lower than the desired optimal capital input level, i.e. $w_{u,t}h_t < K_{e,t+1}^u$, the returning entrepreneur is constrained.

\[
\begin{align*}
\text{Constrained, } & \quad \text{if } \left[ \frac{\Gamma_1}{1+r} \right]^{\frac{\alpha_1+\alpha_3}{\alpha_3}} > w_{u,t} \\
\text{Unconstrained, } & \quad \text{if } \left[ \frac{\Gamma_1}{1+r} \right]^{\frac{\alpha_1+\alpha_3}{\alpha_3}} < w_{u,t}
\end{align*}
\]  

(9)

Given the partition between constrained and unconstrained entrepreneurs, the physical capital input and labor input under the two cases can be calculated as following:

\[
K_{e,t+1} = \begin{cases} 
  w_{u,t}h_t, & \text{if constrained} \\
  \left( \frac{\Gamma_1}{1+r} \right)^{\frac{\alpha_1+\alpha_3}{\alpha_3}} h_t, & \text{if unconstrained}
\end{cases}
\]

(10)

\[
L_{e,t+1} = \begin{cases} 
  \left( \frac{p_e A_e}{w_{r,t+1}} \right)^{\frac{\alpha_1+\alpha_3}{\alpha_3}} h_t, & \text{if constrained} \\
  \left( \frac{p_e A_e}{w_{r,t+1}} \right)^{\frac{\alpha_1+\alpha_3}{\alpha_3}} \left( \frac{\Gamma_1}{1+r} \right)^{\frac{\alpha_1+\alpha_3}{\alpha_3}} h_t, & \text{if unconstrained}
\end{cases}
\]

(11)

Finally, the net profit earned by a returning entrepreneur follows from the results of capital and labor inputs shown above. Notice that the urban wage only affects the capital, labor and profit of the constrained entrepreneurs, whereas for unconstrained entrepreneurs, the only thing matters is his human capital level.

\[
\pi_{e,t+1} = \begin{cases} 
  \Gamma_2 w_{r,t+1}^{\frac{\alpha_2}{\alpha_1+\alpha_3}} w_{u,t}^{\frac{\alpha_1}{\alpha_1+\alpha_3}} h_t - (1+r)w_{u,t}h_t - D, & \text{if constrained} \\
  \left( \frac{\Gamma_1}{1+r} \right)^{\frac{\alpha_1+\alpha_3}{\alpha_3}} \left( \frac{\pi_2}{\alpha_2} \right) h_t - D, & \text{if unconstrained}
\end{cases}
\]

(12)

where

\[
\Gamma_2 \equiv (A_e p_e)\left( \frac{1}{\alpha_1+\alpha_3} \right) \left( \alpha_2 \frac{1}{\alpha_2+\alpha_3} - \frac{1}{\alpha_1+\alpha_3} \right)
\]

(13)

### 4.3 Solving Household’s Problem

Given the exogenous prices $\{p_f, p_e, p_u\}$ and lifetime wealth $W(h_t)$, the agents choose optimal level for the three consumption goods. The agent’s indirect utility function only depends on his lifetime wealth level $W(h_t)$:

\[
V(h_t) = 3\ln W(h_t) - B
\]

where,

\[
B \equiv \ln(3p_f) + \ln(3p_e) + \ln(3p_u)
\]

11
The utility maximization problem is now equivalent to the lifetime wealth maximization problem through occupational choices. For an agent with a given human capital $h_t$ and prices $\{w_{r,t}, w_{r,t+1}, w_{u,t}, w_{u,t+1}, r\}$, the choice of occupation melts down to the following four categories:

$$W(h_t; w_{r,t}, w_{r,t+1}, w_{u,t}, w_{u,t+1}, r) =$$

$$\begin{cases} 
 w_{r,t}(1 + r) + w_{r,t+1} & \text{Non-Migrants} \\
 w_{u,t}h_t(1 + r) + w_{r,t+1} & \text{Returning workers} \\
 w_{u,t}h_t(1 + r) + \pi_{t+1} & \text{Returning entrepreneurs} \\
 (w_{u,t}(1 + r) + w_{u,t+1})h_t - J & \text{Permanent migrants} 
\end{cases} \quad (15)$$

Figure 1: Human Capital Profile and Occupational Choices
5 Updating, Aggregating and Equilibrium

At the beginning of each period, young agents observe their own human capital level \( h \), which is drawn from a known log-normal distribution (cdf) \( \Phi(h; \mu, \sigma) \). Private agents differ over the individual states and share the aggregate states. The individual state variable is human capital \( h \), which is not under the control of the agent. The aggregate states of the economy consist the distribution of current old agents’ occupation represented by the probability measure \( \lambda(s) \), the population of last generation \( N \), and the distribution of young agents’ human capital \( \Phi(h; \mu, \sigma) \). Private agents make decisions according to the individual state, and aggregating across different individuals gives the distribution of occupation \( \lambda(s') \), which is one of next period’s aggregate states.

Because the capital and labor input in urban area are exogenous, I only need to consider the updating and aggregating for rural agents. First, the human capital distribution changes over time, since the children of permanent migrants are born in cities and become "urban residents", thus, the population of rural area shrink if there were stayers last period:

\[
N(t_1) \geq N(t_2), \quad \forall \ 0 < t_1 < t_2
\]  

Second, each period, the young people are making lifetime plan, however at the same time, the old generation only act according to the decision they made at young age. Therefore, the fraction of people working in the three sectors need to be updated each period.

Let \( s_t(h; w_{r,t}, w_{r,t+1}, w_{u,t}, w_{u,t+1}, r) \) be occupational choice an individual endowed with human capital \( h \) makes at \( t \):

\[
s_t(h; w_{r,t}, w_{r,t+1}, w_{u,t}, w_{u,t+1}, r) = \begin{cases} 
1, & V_{\text{max}} = V_{t}^{{\text{Non-Mig}}} \\
2, & V_{\text{max}} = V_{t}^{{\text{R-Worker}}} \\
3, & V_{\text{max}} = V_{t}^{{\text{R-Entr}}}
4, & V_{\text{max}} = V_{t}^{{\text{Con-Mig}}}
\end{cases}
\]

where,

\[
V_{\text{max}} = \max \{ V_{t}^{{\text{Non-Mig}}}, V_{t}^{{\text{R-Worker}}}, V_{t}^{{\text{R-Entr}}}, V_{t}^{{\text{Con-Mig}}} \}
\]

In order to track how many people are choosing each of the occupation, it is necessary to define the fraction of choices made by young agents at each period. Denote \( \lambda_{i,t} \) as the proportion of rural young agents at time \( t \) who choose occupation \( i \).

\[
\text{Fraction of } \begin{cases} 
\text{Non-Migrant} \quad \lambda_{1,t} \\
\text{Return Worker} \quad \lambda_{2,t} \\
\text{Return Entrepreneur} \quad \lambda_{3,t} \\
\text{Permanent Migrant} \quad \lambda_{4,t}
\end{cases}
\]

Before I give the definition of equilibrium, I want to clarify some demographic notations in the model. For this two period OLG model, the total population of
those who were born in rural area is \( N_t \); The young and old rural agents are \( N_t^y \) and \( N_t^o \) respectively, and satisfies:

\[
N_t^y + N_t^o = N_t \\
N_t^y = N_t^o(1 - \lambda_{4,t-1}) \\
= N_{t-1}^o(1 - \lambda_{4,t-1})
\]  

(20)

5.1 Definition of Equilibrium

A competitive equilibrium for this economy consists of

1. A sequence of set of decision rules \( \{s_{i,t}\}_{t=1}^{\infty}, i \in \{i|h_{i,t} \in \Omega_t\}. \Omega_t \) is the support of rural human capital at time \( t \);
2. A sequence of distribution of occupations \( \lambda_{i,t}, i = 1, 2, 3, 4 \);
3. A sequence of rural wages \( \{w_{r,t}\}_{t=1}^{\infty} \);
4. A sequence of exogenous urban wages \( \{w_{u,t}\}_{t=1}^{\infty} \);
5. A sequence of exogenous interest rate \( 1 + r \);
6. A sequence of rural population \( \{N_t\}_{t=1}^{\infty} \);
7. The proportion of people leaving the rural area for cities each period \( \zeta_t \);

such that,

- Given the price \( \{w_{r,t}, w_{r,t+1}, w_{u,t}, w_{u,t+1}, 1 + r\}_{t=1}^{\infty} \), the occupational choices solve the rural household’s utility maximization problem; the rural non-farm’s profit maximization problem is solved; the rural farm’s profit maximization problem is solved;
- The rural labor market clear:

\[
L_{d,f,t} + L_{d,e,t} = N_t^y \lambda_{1,t-1} + N_t^o(\lambda_{1,t-1} + \lambda_{2,t-1})
\]  

(21)

where, \( L_{d,f,t} \) and \( L_{d,e,t} \) are labor demand from agricultural sector and from returning entrepreneur’s rural non-farm firms. The aggregate labor demand from the agricultural sector is given by:

\[
L_{d,f,t} = \left( \frac{A_f \eta}{w_{r,t}} \right) \frac{1}{1-\eta}
\]  

(22)

The aggregate labor demand from returning entrepreneurs \( L_{d,e,t} \) is calculated by integrating the individual labor demand \( l_{e,t} \) across all the returning entrepreneurs:

\[
L_{d,e,t} = N_t^o \int_{s=3} l_{e,t}(h_{t-1})d\Phi(h_{t-1})
\]  

(23)
• The rural population evolves according to:
\[
N_y^t = N_y^{t-1} (1 - \lambda_{4,t-1})
\]
  \[= N_y^{t-1} (1 - \lambda_{4,t-1}) \quad (24)\]

• The leaving rate of rural households satisfies:
\[
\zeta_t = \lambda_{2,t} + \lambda_{3,t} + \lambda_{4,t}
\]
  \[= (25)\]

• The rural capital input equals the saving of returning entrepreneurs.

5.2 Stationary Equilibrium

A stationary competitive equilibrium for this economy is a competitive equilibrium, which, in addition to the condition defined for competitive equilibrium, also satisfies: \textit{The fraction of people who leave the countryside for urban area is constant}, i.e. \(\zeta_t \equiv \zeta\) for all \(t\):
\[
\lambda_{2,t} + \lambda_{3,t} + \lambda_{4,t} = \zeta
\]
  \[= (26)\]
in which \(\zeta\) is the constant leaving rate, and \(1 - \zeta\) is the invariant fraction of people who never migrate.

6 Characterization of Stationary Equilibrium

**Proposition 1:** In the stationary equilibrium where the exogenous urban wage \(w_u\) is constant, there is no permanent migrants. In another word, the fraction of returning workers plus returning entrepreneurs is constant.

\textbf{Proof:} Suppose there are four types of people in the stationary equilibrium, and thus the distribution in the occupational choices is given by \(\lambda_{i,t}, i = 1, 2, 3, 4\). Since human capital has the same distribution of \(\Phi(h; \mu, \sigma)\), the fraction of non-migrants is given by
\[
\lambda_1 = \Phi\left(\frac{w_{r,t}}{w_{u,t}}\right),
\]
  \[= (27)\]
where \(\frac{w_{r,t}}{w_{u,t}}\) is the threshold of human capital in between of choosing to be a non-migrant and a migrant. Thus, the fraction of people who leave the countryside at \(t\) is the sum of fraction of last three groups in the above category, which is given by
\[
\zeta = 1 - \Phi\left(\frac{w_{r,t}}{w_{u,t}}\right)
\]
  \[= (28)\]
According to the definition of stationary equilibrium that the leaving rate \(\zeta\) is constant, therefore, rural wage \(w_{r,t}\) will be constant if urban wage \(w_{u,t}\) is also constant.
Next step, let us look at the rural labor market clearing condition at any time $t$:

$$\text{Labor Supply} = N_{t}^{y} \lambda_{1,t} + N_{t}^{o}(\lambda_{1,t} + \lambda_{2,t})$$

$$\text{Labor demand} = L_{c,t} + L_{f,t}.$$  \hfill (29)

Notice that the fractions of each occupation are only functions of the rural wage $w_{r,t}$ and the exogenous urban wage $w_{u,t}$, which are now constant, therefore $\lambda_{i,t}$ are constants. From equations (11), (22), (23), (24), the labor market clearing condition is given by:

$$N_{t-1}^{y}[1 - \lambda_{4}(w_{r})] \lambda_{1}(w_{r}) + N_{t-1}^{y}(\lambda_{1}(w_{r}) + \lambda_{2}(w_{r}))+$$

$$= N_{t-1}^{y} \int \left[ p_{r} \alpha_{2} w_{u}^{\alpha_{1}} \right] \frac{1}{\alpha_{1}+\alpha_{2}} h d\Phi(h) + \left( \frac{A_{f} \eta}{w_{r}} \right)^{\frac{1}{\alpha_{1}+\alpha_{2}}} \forall t.$$  \hfill (30)

In the above equation, rural new-born population $N_{t}^{y}$ has to be constant in the stationary equilibrium if the rural wage is constant and the fraction of the sum of returning workers and returning entrepreneurs is constant in steady state.

**Proposition 2**: In the stationary equilibrium where urban wage is rising at a constant rate $\rho$, the fraction of people leaving rural area (including temporarily and permanently) is constant; the fraction of migrants who staying in cities is growing, and the rural population asymptotically goes to zero.

**Proof**: I am going to prove proposition 2 in several steps, including claim 1, 2, 3.

The exogenous urban wage $w_{u,t}$ rises at a constant rate $\rho$, i.e.

$$w_{u,t} = w_{u,0} \rho^{t}.$$  \hfill (31)

According to the definition of stationary equilibrium, the fraction of people who leave the rural area is constant, therefore, rural wage is growing at the same rate with the growth rate of urban wage in the steady state, i.e.

$$w_{r,t} = w_{r,0} \rho^{t}.$$  \hfill (32)

In the economy right after the reform starts, there are observed to have non-migrants, returning workers, returning entrepreneurs, and permanent migrants. That is to say, the economy at the beginning of the reform need to satisfy the following inequality:

$$\frac{w_{r,0}}{w_{u,0}} < \frac{w_{r,0} \rho + D}{C w_{r,0}^{\alpha_{1}} \rho^{\alpha_{2}} - w_{u,0}(1+r)\rho} < \frac{J - D}{w_{u,0}(1+r) + w_{u,0} \rho - C w_{r,0}^{\alpha_{1}} \rho^{\alpha_{2}}}.$$  \hfill (33)
where $C$ is a constant:

$$C \equiv (A e^{-p c})^{\frac{1}{\alpha_1 + \alpha_3}} (\alpha_2^{\frac{1}{\alpha_1 + \alpha_3}} - \alpha_2^{\frac{1}{\alpha_1 + \alpha_3}})$$

(34)

At the beginning of the reform, the population of people leaving rural area is $N_0[1 - \Phi(w_{r,0}/w_{u,0})]$, and the rest stay in rural. Among those who migrate at $t = 0$, the mass of each group is given by

$$\begin{cases}
\text{Returning workers,} & \Phi\left(\frac{w_{r,0} \rho + D}{C w_{r,0}^{\frac{1}{\alpha_1 + \alpha_3}} \rho^{\frac{1}{\alpha_1 + \alpha_3}} - w_{u,0}(1 + r)\rho}\right) - \Phi\left(\frac{w_{r,0}}{w_{u,0}}\right) \\
\text{Returning Entrepreneurs,} & \Phi\left(\frac{J - D}{w_{u,0}(1 + r) + w_{u,0}\rho - C w_{r,0}^{\frac{1}{\alpha_1 + \alpha_3}} \rho^{\frac{1}{\alpha_1 + \alpha_3}} w_{r,0}^{\frac{1}{\alpha_1 + \alpha_3}} \rho^{\frac{1}{\alpha_1 + \alpha_3}} - w_{u,0}(1 + r)\rho}\right) - \Phi\left(\frac{w_{r,0} \rho + D}{C w_{r,0}^{\frac{1}{\alpha_1 + \alpha_3}} \rho^{\frac{1}{\alpha_1 + \alpha_3}} - w_{u,0}(1 + r)\rho}\right) \\
\text{Permanant migrants,} & 1 - \Phi\left(\frac{J - D}{w_{u,0}(1 + r) + w_{u,0}\rho - C w_{r,0}^{\frac{1}{\alpha_1 + \alpha_3}} \rho^{\frac{1}{\alpha_1 + \alpha_3}} w_{r,0}^{\frac{1}{\alpha_1 + \alpha_3}} \rho^{\frac{1}{\alpha_1 + \alpha_3}} - w_{u,0}(1 + r)\rho}\right)
\end{cases}$$

(35)

The rural wage keeps rising at the same pace with urban wage. However, this ever rising wage does not retain rural people, on the opposite, the rising urban wage eventually depletes the rural population. There are both direct and indirect reasons for it. First, given the same distribution of innate ability over time, to permanently migrate becomes attractive to more and more people; Second, the ever rising wage in rural area seriously damages the profitability of returning entrepreneurs. Thus the fraction of stayers enlarges while that of the returning entrepreneurs shrinks. Over time, the group of entrepreneurs disappears, replaced by more higher fraction of returning workers and permanent migrants. Furthermore, as urban wage keeps rising, the group of returning workers will be completely replaced by permanent migrants. Next consideration is that whether the non-migrants group will be replaced by permanent migrants. The answer is no, and the proof follows.

**Claim 1:** In a stationary equilibrium, the fraction of non-migrants will always stays at $\Phi(w_{r,0}/w_{u,0})$.

**Proof:** I only need to show that, for people with human capital level lower than $w_{r,0}/w_{u,0}$, the wealth from being a non-migrants always exceeds the wealth if they are permanent migrants, i.e. the following inequality always holds:

$$w_{r,t}(1 + r) + w_{r,t+1} > (w_{u,t}(1 + r) + w_{u,t+1}) h - J, \quad h \in (0, w_{r,0}/w_{u,0}]$$

(36)

which is true if the following is true:

$$\frac{w_{r,t}(1 + r) + w_{r,t+1} + J}{w_{r,t}} > \frac{w_{u,t}(1 + r) + w_{u,t+1}}{w_{u,t}}$$

(37)
which is obvious true given \( w_{r,t+1}/w_{r,t} = w_{u,t+1}/w_{u,t} = \rho \).

**Claim 2**: In a stationary equilibrium, the return workers and return entrepreneurs will eventually be replaced by permanent migrants.

**Proof**: The lifetime wealth of the four groups of agents are given by equation system (15), which can be rewritten in a stationary equilibrium as following:

\[
W(h_t; w_{r,t}, w_{r,t+1}, w_{u,t}, w_{u,t+1}, r) =
\begin{cases}
(w_{r,0}(1 + r) + w_{r,0}\rho)\rho^t & \text{Non-Migrants} \\
w_{u,0}(1 + r)\rho^t + w_{r,0}\rho^t & \text{Returning workers} \\
\Gamma_2(\frac{w_{u,0}}{(w_{r,0})^{\alpha_2}})^{1 - \alpha_2} \rho^{\frac{1}{\alpha_1 + \alpha_2}} h - D & \text{Returning entrepreneurs} \\
(w_{u,0}(1 + r) + w_{u,0}\rho)\rho^t h - J & \text{Permanent migrants}
\end{cases}
\]

Let us first check the threshold in human capital between permanent migrants and return entrepreneurs, which is given by:

\[
J - D \left( w_{u,0}(1 + r) + w_{u,0}\rho \right) \rho^t - \Gamma_2(\frac{w_{u,0}}{(w_{r,0})^{\alpha_2}})^{1 - \alpha_2} \rho^{\frac{1}{\alpha_1 + \alpha_2}} t
\]

It is obvious that as time goes by, the above threshold goes to zero as \( \rho > 1 \). Second, let us check the threshold in human capital between permanent migrants and return workers, which is given by:

\[
\frac{J - w_{r,0}\rho^t}{w_{u,0}\rho^t}
\]

The above threshold asymptotically goes to \( w_{r,0}/w_{u,0} \) because

\[
\frac{J - w_{r,0}\rho^t}{w_{u,0}\rho^t} \equiv \frac{J}{\rho^t + w_{r,0}} \rightarrow w_{r,0}/w_{u,0}
\]

Therefore, when time goes to infinity, there will only be non-migrants and permanent migrants, whereas return entrepreneurs disappear in finite time and return workers disappear when \( t \rightarrow \infty \). Below I am going to calculate the time at which the entrepreneurs’ group disappears.

The time it takes for entrepreneurs’ group to disappear depends on how fast the wages are rising and the technology the returning entrepreneurs possess. As seen in the graph of occupational choices, as rural and urban wage rise at the same pace, the slope of the black line always rises, the slope of the green line either increases but not as fast as the black line (if \( \alpha_1 > \alpha_2 \)) or decreases (if \( \alpha_1 < \alpha_2 \)). The time when the group of entrepreneurs disappear is when the orange, green and black line cross each other at the same point. This can be expressed as:

\[
C(\frac{w_{u,t}}{w_{r,t}^{\alpha_1}})^{1 - \alpha_2} \frac{w_{r,t+1} + J}{w_{u,t+1}} - D = (1 + r)w_{u,t} \frac{w_{r,t+1} + J}{w_{u,t+1}} + w_{r,t+1}
\]
because at the point of time when the last entrepreneur disappears, the cross point of returning workers and permanent migrants

\[
\left( \frac{w_{r,t+1} + J}{w_{u,t+1}} \right) (1 + r) + \frac{w_{r,t+1} + J}{w_{u,t+1}} + w_{r,t+1}
\]

must be on the line of entrepreneurs. This will give the result of above equation.

Because at stationary equilibrium, the rural and urban wages are both rising at the same rate \( \rho \), I can simplify equation (38) as following

\[
C\left( \frac{w_{u,0}}{w_{r,0}} \right)^{\frac{\alpha_1}{\alpha_1 + \alpha_2}} \rho^{1 - \frac{\alpha_2}{\alpha_1 + \alpha_2}} \left( \frac{w_{r,0}}{w_{u,0}} + \frac{J}{w_{u,0}\rho^{t+1}} \right) - D = (w_{r,0}\rho^{t+1} + J)(1 + r)
\]

Apparently, there is a unique solution \( t \) to the above equation. So the time at which the last returning entrepreneur disappears is determined. Notice that there are several factors determines the time of entrepreneurs’ disappearance. First, the initial rural-urban wage difference \( w_{r,0}/w_{u,0} \). The larger is the initial wage differential, the longer the entrepreneurs’ group lasts. This is partly because the large wage differential provides returning entrepreneurs a good opportunity to make profit. The cheap labor in the rural area offers low-cost complement with the capital brought back by returnees, which improves the rural industrialization. The fixed cost \( D \) has a negative effect on the length of entrepreneur’s existence, whereas a higher living cost in cities prolongs the entrepreneurship in rural area. The constant growth rate of wages \( \rho \) goes in the opposite direction with the length of entrepreneurship. This is because, the higher the growth rate \( \rho \), to permanently migrate become attractive to more people at an increasing speed.

**Claim 3:** The rural population is shrinking at a rate that is less than geometric decreasing rate, but asymptotically the rural population decreases at a geometric rate.

**Proof:** From Claim 1 and Claim 2, we know that the permanent migrants’ group will replace the return entrepreneurs in finite time and replace the return workers when \( t \to \infty \). However, the mass of non-migrants will never be zero, i.e., even if we give it infinite time, the non-migrant group will never be completely replaced by the permanent migrants. By the definition of stationary equilibrium, \( h_{1,t} = h_1, \forall t \). As in figure 1, the threshold \( h_{3,t} \) approaches \( h_1 \) from the right overtime, and after \( h_{3,t} \) reaches \( h_1 \), it will not go further to the left of \( h_1 \) anymore. Now, let us focus on the shrinking rate of rural population.

At time 0, the population of the rural young \( N_{y,0} \). The thresholds for occupational choices are \( h_1 < h_{3,0} \). The proportion of young people staying in cities is given by \( 1 - \Phi(h_{3,0}) \), who are going to have their children in urban area. Thus, the population of rural born young people at time 1 is given by:

\[
N_{y,1} = N_{y,0} \Phi(h_{3,0})
\]

(45)
At time 1, $h_1 < h_{3,1} < h_{3,0}$, so the rural young population at time 2 is given by:

$$N^y_2 = N^y_1 \Phi(h_{3,1}) = N^y_0 \Phi(h_{3,0}) \Phi(h_{3,1})$$

(46)

In the infinite time horizon, $h_{3,t} \rightarrow h_1$, thus the rural population is evolving in the following motion:

$$N^y_t = N^y_{t-1} \Phi(h_{3,t-1}) = N^y_0 \Phi(h_{3,0}) \Phi(h_{3,1}) \cdots \Phi(h_{3,t-1})$$

(47)

Because $h_{3,t} \rightarrow h_1$ as $t \rightarrow \infty$, asymptotically, the shrinking rate of rural born population is $\Phi(h_1)$. The rural population is depleted to zero in the infinite horizon.
References


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<tr>
<td><strong>Technology Parameters</strong></td>
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<td><strong>Urban cost</strong></td>
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Figure 2: A simulation of migration and occupation evolution