Monetary Policy Under Switching Fiscal Regime

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April 4th, 2008

Abstract
The purpose of this research is to investigate, using a New Keynesian forward looking sticky price model, what are the consequences of ignoring or bad integration of fiscal policy features for the conduct of monetary policy? The fiscal policy feature I will model in this exercise is fluctuations between high and low taxation periods, as mostly happen in the real world when for example two different political parties of different orientation succeed one after the other in running a country. I do the analysis under the optimal policy framework. My findings suggest several facts: 1. the observed switching nature of the monetary policy might be a result of the economy state dependency induced through the regime switching fiscal policy; 2. ignoring tax policy fluctuations causes inflation and output be more volatile than necessary with considerable welfare losses on the private agents side; 3. microfounded loss function based on linearization or perturbation methods might not be appropriate for monetary policy analysis if regime switching is introduced through disturbances;

1 Introduction

Post World War II analysis of political actions regarding US fiscal policy unveil a bouncing up and down policy with switching roles and rationales. The chronology of major tax events, as surveyed by Yang(2007) display a fluctuating fiscal policy between tax cuts and increases, government spending increases and decreases, actions operated under different motivations and which do not seem to be correlated.

1 This is a very preliminary and incomplete draft. All errors are my own. I thank my advisor and chair of the committee, Professor Eric Leeper, for stimulating my interest in this topic and guidance. I, also, thank Professor Kim Huyhn, Professor Todd Walker, Professor Brian Peterson, Professor Bruce McGough, Bing Li and Spring 2008 Micro-workshop participants for helpful discussions. Any comments and suggestions are highly appreciated.

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Inflation today is regarded as the main’s monetary authority responsibility, tough fiscal policy actions are ones of the price level main determinants in the economy. Therefore, by ignoring this tax policy fluctuations, central bankers might be missing their inflation targets and cause the inflation and output be more volatile than is necessary with considerable welfare losses on the private agents side.

The purpose of this research is to investigate an issue unexplored so far in the literature which is how switching fiscal policy will impact monetary policy conduct? Could switching fiscal policy induce monetary policy switching? Given that we observe a monetary policy that is state dependent, could switching fiscal policy be one of the economic feature that might cause monetary policy to switch? What will be an optimal monetary policy response when it has to deal with switching fiscal policy? How much of the ”big spike” in after World War II US inflation is due to ignoring/bad integration of fiscal policy particularities? My claim is that there could be potential suboptimal/wrong monetary policy responses if fiscal policy features are ignored.

Empirical work and theoretical models showed that interest rates affect inflation and output with a lag. Therefore policy makers set them making a forecast, given their information set, about how the economy might change by the time interest rates will impact the inflation and output. Usual practice to model the economy, as pointed out by Moessner(2006) is assuming that the economy does not change over time and the only uncertainty faced by the policymaker is about the type and duration of the shocks that hit the economy. Little attention was payed, tough, to what will be a proper objective function for monetary policy given that she has to face regime switching shocks as for example fiscal policy shocks. Choosing a proper objective function is an important issue that any modern central banker should be concerned with as pointed out by Alan Blinder(2006).

So far, in the literature, several directions were undertaken with respect to the central bank criterion: 1. specify a monetary policy rule, 2. posit an ad-hoc loss function, 3. derive a microfounded loss function.

Under the regime switching framework, for the first case, work by Chung, Davig and Leeper(2004), Davig and Leeper(2007), Farmer, Waggoner and Zha(2006, 2007) is referential. Chung, Davig and Leeper(2004) explores an environment in which both, monetary and fiscal policy, evolve according to a Markov process, and how this environment can change the impact of policy shocks. They show that when fiscal policy is unresponsive to debt, active monetary policy (Taylor rule) in one regime is not sufficient to insulate the economy against tax shocks in that regime and it can have unintended consequence of amplifying and propagating the aggregate demand effects of tax shock.

Davig and Leeper (2005) estimate regime-switching rules for the monetary and tax policy over the post-war period in United States and finds that U.S. monetary and fiscal policies have fluctuated
among passive and active rules, where passive and active terms should be understood as in Leeper (1991).
Farmer, Waggoner and Zha (2006) provides a modification of Chris Sims’ Gensys algorithm in order to compute the minimum state variable solution for a regime switching model and show how to test a given solution for uniqueness and boundedness. They apply their method to a monetary policy model when monetary policy follows a rule and provide three potential explanations for the persistence and volatility in interest rate: i. policy change, ii. shock variances change, iii. private sector parameters change.
Davig and Leeper (2007) investigates an environment in which reaction coefficients in the monetary policy rule evolve according to a Markov process, and find that regime change alters the quantitative and qualitative predictions of a conventional, forward looking New Keynesian model.

For case two, work by Swensson and Williams (2007) and Alexandre and Baçao and Driffill (2007) make the first steps. Swensson and Williams(2007) is a technical paper that examines optimal and other monetary policy in a markov-jump-linear quadratic system extended to include forward-looking variables. The authors posit an ad-hoc switching loss function, and uses Marcet and Marimon (1999) recursive saddle point method to find optimal policy as well as the policy functions. Swensson and Williams does not tell us why the weighting matrix in the loss function should be random or coefficients should be switching. They do not provide us, also, with a criterion how to pick which coefficients should switch and which ones not. Moreover Farmer, Waggoner and Zha (2006) point out that Swensson and Williams method lack a diagnostic to inform the experimenter when solution is unique and that their algorithm may converge to a non-existent solution or to a set of indeterminate equilibria.
Alexandre, Baçao and Driffill(2007) apply both methods: Farmer, Waggoner and Zha (2006), and Swensson and Williams(2007) to a open economy model with exchange rate uncertainty. They apply Farmer, Waggoner and Zha method to select monetary policy rules that provide uniqueness and boundedness then use Swensson and Williams method to evaluate losses using an ad-hoc loss function with switching weight matrix versus one without switching weight matrix in order to compare the performance of time invariant rules to regime switching rules tough they, also, doubt about the solution uniqueness in the case of Swensson and Williams’ method. After evaluating losses they conclude that taking into account the switching nature of the economy in the case of the exchange rate uncertainty is important only in extreme cases: shocks are very persistent with an autocorrelation coefficient of 1.1, and bubble regime, which means that the probability is greater than 0.75.
Regarding case three, work by Davig (2007) is first undertaken in this direction. He mentions evidence of flattening Phillips curve in recent periods: inflation became less responsive in movements to measures of aggregate economic activity as for example the output gap, and he advances a Micro argument to explain this phenomenon. He assumes that firms face a changing cost of price adjustment, derives a switching-Phillips curve, and assess the implications for the optimal discretionary policy of such Phillips curve. Under his assumption, the microfounded based loss function is able to capture the economy state dependency as introduced by switching cost of price adjustment.

In this project, the switching source is introduced in a DSGE-forward looking sticky price model through the fiscal side, therefore trying to relax two key assumptions necessary for a unique and stable equilibrium (=good policy) at the same time. The two assumptions are: fiscal policy is passive and policy rule is permanent.

The benchmark model is based on Woodford & Benigno (2003, 2005, and 2006), but unlike part of the Woodford work, I will not allow fiscal policy to adjust endogenously. Taxes will follow an exogenous process with switching autoregressive coefficient. Government spending will follow an exogenous nonswitching AR1 process. Taxes and government spending are assumed to be uncorrelated. In the background, transfers are assumed to adjust endogenously.

Initially I will assume the monetary policy follows a Taylor rule in order to derive basic intuition about the model workings then I will derive a loss function based on the second order approximation of the utility function following Woodford, and Woodford and Benigno work, and analyze how switching fiscal policy will impact the monetary policy when it is set optimally. I will look at several cases for the Taylor rule and loss function.

In the case of Taylor rule, I will consider 4 cases: i. switching tax process and invariant Taylor rule, i. switching tax process and Taylor rule with exogenous switching coefficients, iii. switching tax process, regime switching Phillips curve and invariant Taylor rule, iv. switching tax process, regime switching Phillips curve, and Taylor rule with exogenous switching coefficients. For the loss function I will consider, also, two cases: i. switching tax process and microfounded loss function when Calvo parameter is constant, switching tax process and microfounded loss function when Calvo parameter is regime switching.

The reasons why I considered the cases with the state dependent Calvo parameter is to try to move away from the Lucas critique. If only the autoregressive coefficient in tax rule and parameters in the monetary policy rule vary with state, but the parameters that reflect behavioral rules as the ones in wage and price setting, do not adjust to changes in the state of the economy or to changes in policy rule, the analysis is subject to Lucas critique. I wanted to keep constant only the deep structural parameters of the model, meaning the ones that reflect preferences and technology.
The method I used for the optimal policy part of the analysis follows very closely Woofford and Benigno (2005, 2007) linear quadratic approach, which means that central bank will minimize a loss function derived using a second order approximation of the utility function, subject to the linearized constraints.

My findings, after investigating these cases, are quite interesting.

Firstly, I found that the microfounded analysis will deliver loss function with constant weights on inflation and output gap when the behavioral parameter are not allowed to switch (constant Calvo parameter). This is due to the fact that perturbation method used in obtaining the loss function is made around the constant, unique steady state, assuming the deviations from the steady state are small. The weights in the loss function will be functions only of the deep parameters of the model, Calvo parameter and steady state values. In other words monetary policy is instructed, if guided by the microfounded analysis, to follow time-invariant policy rules as being welfare loss minimizing.

Secondly, from Monte Carlo experiment I found that, certain combination of switching weights in the loss function can be considerable welfare improving, given assumed calibration across different types of tax rule: switching versus non-switching. This findings needs to be investigated further analytically. The basic intuition lies in the solutions for interest rate, inflation and output gap. Switching tax process will induce switching in output gap and inflation, and consequently in the interest rate, increasing their volatility. If we reweigh inflation and output gap in such a way that we put more aggressive weights toward the more volatile ones, then we will get considerable welfare loss reductions. This potentially can explain the observed switching nature of the monetary policy. My hypothesis is the following: fiscal policy switches exogenously, this translates in regime switching output and prices, which will imply regime switching monetary policy actions as reactions to observed switching output gap and inflation.

The mention I want to make in this case is that, unlike Alexandre, Bacão and Driffill(2007), I made the comparison across the same welfare criterion but in an economy facing different type of shock: switching versus non-switching tax process. The interesting result is that welfare loss ranking for .

Thirdly, with regime switching tax process and Calvo state dependent parameter, we get a switching intercept and slope Phillips curve, but a microfounded loss function with switching weight only on inflation variable, the one on output gap remaining fix.

This suggests that linear quadratic approximation used for obtaining the microfounded loss function might not be a good indicator for what should be the weights in the central bank objective function, no matter what order of linearization or perturbation used. It also tell us that it matters a lot which coefficients in the monetary policy rules or weighting matrix in the loss function gets switching and which not, we just cannot posit ad-hoc that all or part of them switches randomly and get the
minimum welfare losses.

2 Simple model dynamics - a DSGE model with Taylor rule and switching tax rule

Before jumping to more complicated variants, I will work through this simple enough model so that I will be able to derive analytical solutions and develop intuition about its working mechanism.

In this section I investigate, using a New Keynesian forward looking model, the solutions under the exogenous switching type tax rule. The linearized equations describing private sector behavior are the consumption-Euler equation and the aggregate supply relation. Exogenous disturbances are autoregressive and mutually uncorrelated. In all cases fiscal policy is the only source of switching, and I assume that monetary policy follow a Taylor rule responsive to inflation and output gap. In the background, I assumed that lump sum taxes and transfers adjust passively to ensure fiscal solvency.

2.1 Model A

The equations for the model A are:

\[ x_{it} = E_{it}x_{t+1} - \sigma^{-1}(i_t - E_{it}\pi_{t+1}) + u^D_t \]
\[ \pi_{it} = \beta E_{it}\pi_{t+1} + \kappa(x_{it} + \psi\tau_t) + u^S_t \]
\[ i_{it} = \alpha\pi_{it} + \gamma x_{it} \]
\[ u^D_t = \rho^D u^D_{t-1} + \epsilon^D_t \]
\[ u^S_t = \rho^S u^S_{t-1} + \epsilon^S_t \]
\[ \tau_t = \rho^\tau(S_t)\tau_{t-1} + \epsilon^\tau_t \]

where \( x_t \) is the output gap, \( \pi_t \) is the inflation, \( i_t \) is the nominal interest rate, \( \tau_t \) is the tax gap, \( u^D_t \) is the aggregate demand shock, \( u^S_t \) is the aggregate supply shock with \( |\rho^l| < 1 \) for \( l = D, S, \tau \). \( \epsilon^l \) is mean zero, uncorrelated random variables with bounded supports for \( l = D, S, \tau \). \( s_t \) is assumed to follow a finite two state Markov chain with transition probability matrix:

\[ P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \]
\[ E_{tt} \pi_{tt+1} = E[\pi|s_t = i, \Omega_t^{-s}] = p_{i1} E[\pi_{tt+1}|s_t = i, \Omega_t^{-s}] + p_{i2} E[\pi_{tt+1}|s_t = i, \Omega_t^{-s}] = \]

\[ p_{i1} E[\pi_{tt+1}|\Omega_t^{-s}] + p_{i2} E[\pi_{tt+1}|\Omega_t^{-s}] \]

\[ E_{tt} x_{tt+1} = E[x|s_t = i, \Omega_t^{-s}] = p_{i1} E[x_{tt+1}|s_t = i, \Omega_t^{-s}] + p_{i2} E[x_{tt+1}|s_t = i, \Omega_t^{-s}] = \]

\[ p_{i1} E[x_{tt+1}|\Omega_t^{-s}] + p_{i2} E[x_{tt+1}|\Omega_t^{-s}] \]

for \( i = 1, 2 \). \( E_{tt} \) is the expectation operator at time \( t \) for \( s_t = i \).

The agents information set at time \( t \) is \( \Omega_t = \Omega_t^{-s} \cup s_t \), where \( \Omega_t^{-s} = \{ u_{i,t}, u_{i-1,t}, ..., u_t^S, u_{i-1,t}^S, ... \} \).

Using the method of undetermined coefficients we posit solutions for inflation and output gap of the following form:

\[ \pi_{it} = a_{i}^D u_{it}^D + a_{i}^S u_{it}^S + \alpha_i \tau_t \]

\[ x_{it} = b_{i}^D u_{it}^D + b_{i}^S u_{it}^S + \beta_i \tau_t \]

Since we assume that there are only two states:

\[ (\rho^r(S_t)) = \begin{cases} \rho_1^r; S_t = 1 \\ \rho_2^r; S_t = 2, \end{cases} \]

(12)

where:

\( S_t = i \) with \( i = 1, 2 \) identifies the two regimes.

In order to get the MSV (minimum state variable) solution, we need to solve for the regime-contingent coefficients \( \{ a_i^D, a_i^S, a_i^r, b_i^D, b_i^S, b_i^r \} \). The solutions for the total of 12 coefficients we will get by solving the following system of equations:

a. for \( u_{i}^D \)

\[
\begin{pmatrix}
\beta_{p11} \rho_{p} - 1 \\
\beta_{p21} \rho_{p} \\
\sigma^{-1} p_{11} \rho_{p} - \sigma^{-1} p_{21} \rho_{p}
\end{pmatrix}
\begin{pmatrix}
\rho_1^r \\
\rho_2^r \\
0
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
-1
\end{pmatrix}.
\]

b. for \( u_{i}^S \)

\[
\begin{pmatrix}
\beta_{p11} \rho_{p} - 1 \\
\beta_{p21} \rho_{p} \\
\sigma^{-1} p_{11} \rho_{p} - \sigma^{-1} p_{21} \rho_{p}
\end{pmatrix}
\begin{pmatrix}
\rho_1^r \\
\rho_2^r \\
0
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
-1
\end{pmatrix}.
\]
c. for $\tau_t$

\[
\begin{pmatrix}
\beta p_{11}\rho_1^2 - 1 & \beta p_{12}\rho_2^2 \\
\beta p_{21}\rho_1^2 & \beta p_{22}\rho_2^2 - 1 \\
\sigma^{-1} p_{11}\rho_1^2 - \alpha \sigma^{-1} & \sigma^{-1} p_{12}\rho_2^2 \\
\sigma^{-1} p_{21}\rho_1^2 & \sigma^{-1} p_{22}\rho_2^2
\end{pmatrix}
\begin{pmatrix}
\kappa \\
0 \\
\kappa \\
0
\end{pmatrix}
\begin{pmatrix}
\alpha_1^t \\
0 \\
\alpha_2^t \\
0
\end{pmatrix}
= \begin{pmatrix}
\kappa - \kappa \psi \\
0 \\
\kappa - \kappa \psi \\
0
\end{pmatrix}. \tag{15}
\]

Solutions for the coefficients can be found in the Appendix.

One important feature of Model A is that we get a switching Phillips curve, inflation, output gap and interest rate rule, but the determinacy conditions of the system remain the same as for the fixed regime model counterpart. Model A’s eigenvalue are the same as the ones for the fixed model counterpart.

Full DSGE specification will deliver a model of the following type:

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta \beta)(1 - \theta)}{\theta} \left[ (\frac{\sigma}{\alpha + \varphi}) \hat{y}_t + \frac{\bar{r}}{1 - \bar{r}} \tilde{r}_t - \frac{\sigma(1 - \alpha)}{\alpha} \hat{y}_t - \frac{1}{\varepsilon - 1} \hat{\varepsilon}_t \right] \tag{16}
\]

\[
\hat{y}_t = E_t \hat{y}_{t+1} - \frac{\alpha}{\sigma} (\hat{r}_t - E_t \pi_{t+1}) + (1 - \alpha)(1 - \rho^g) \hat{y}_t \tag{17}
\]

\[
\hat{r}_t = \alpha \pi \hat{r}_t + \gamma \hat{x}_t \tag{18}
\]

\[
\hat{\varepsilon}_t = \rho^\varepsilon \hat{\varepsilon}_{t-1} + \varepsilon_t^\varepsilon \tag{19}
\]

where the first equation is the Phillips curve derived from the firms profit maximization problem, the second is the forward-looking IS curve, derived from the representative household’s optimization problem, the third is the monetary policy rule which is responsive to output gap and inflation, and the rest of the equations are the exogenous processes for tax, government spending, and mark up shock.

As for the rest of the parameters: $\theta$ is the degree of price stickiness (Calvo parameter), $\sigma$ is the risk aversion parameter, $\alpha = \frac{C}{\bar{Y}}$ where $\bar{C}$ is the steady state output and $\bar{Y}$ is the steady state output, $\bar{r}$ is the steady state rate, and $\mu$ is the steady state mark up.

As notation, throughout the paper, inflation, output gap, interest rate, and taxes should be understood as representing 2X1 vectors as following: $x_t = vec([x_{it}]_{i=1}^2)$, $\pi_t = vec([\pi_{it}]_{i=1}^2)$, $r_t = vec([r_{it}]_{i=1}^2)$ and $\tau_t = vec([\tau_{it}]_{i=1}^2)$, where $i = 1, 2$ indicates the state.

In the above derivation and throughout this paper the assumed household utility function is of the following form:

\[
U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} \tag{22}
\]

where $C_t$ denotes the level of the composite consumption good and $N_t$ is the composite of the labor services.
In case of the state dependent Calvo parameter, \( \tilde{\theta} \), the derived Phillips curve has the following form:

\[
\pi_t = \tilde{\theta}^{-1}(I - \tilde{\theta})(I - P\tilde{\theta}\beta)\left[\left(\frac{\sigma}{\alpha} + \varphi\right)\hat{y}_t + \frac{\tau}{1 - \tau}\hat{r}_t - \frac{\sigma(1 - \alpha)}{\alpha}\hat{\theta}_t - \frac{1}{\varepsilon - 1}\hat{\epsilon}_t\right] + P\tilde{\theta}\beta(I - P\tilde{\theta})\beta(I - P\tilde{\theta})^{-1}P\tilde{\theta} + I|PE_t\pi_{t+1}
\]

where \( P \) is the above mentioned matrix probability, \( \tilde{\theta} \) follows the same Markov process as the tax process, \( E_t\theta(S_{t+j}) = P^j\tilde{\theta} \), and:

\[
\tilde{\theta} = \begin{pmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{pmatrix}
\]

with \( \theta_1 \) is the fraction of firms that changes prices if state 1, and \( \theta_2 \) is the fraction of firms that changes prices if state 2.

### 3 Optimal monetary policy with discretion under switching fiscal policy

This section explores one type of a Central Bank’ systematic response. The type of response is targeting rule that results from setting monetary policy with discretion: central bank choices at \( t \) does not bind in the future, so the central bank cannot affect private sector expectations.

In what follows I will look at analytically results from two cases: 1. loss function is derived following Woodford and Beningno method in the case of constant Calvo parameter, 2. loss function is derived using the same method but with regimes-switching Calvo parameter.

#### 3.1 Discussion about using a microfounded central bank objective function when fiscal policy follows a regime switching process

An important issue that a central banker should be concerned with, as pointed out by Binder(2006), is to find the proper, model-consistent objective function for the monetary policy. This means, in particular, to find the right loss functional form and coefficients in front of the gaps such that the loss minimization problem will deliver a monetary policy rule that allows a containment of inflation with lower welfare costs for the agents in the economy. It is believed so far, that a welfare criterion which is based on the parameters that describe the structure of the model is able to deliver such a policy.

Woodford and Benigno(2003, 2005 and 2007) provide a linear quadratic approach to derive the loss function for the case of the distorted steady state which for discretion case look as following:

\[
L = \frac{q_1}{2}\pi_t^2 + \frac{q_2}{2}(\hat{Y}_t - \hat{Y}_t^*)^2
\]
I will provide here the expressions for $q\pi$, $qy$ and $\hat{Y}_t^*$, and I will relegate the full derivation to the appendix. Given that I use a slightly different utility function than Woodford and Benigno I got the following results for the above mentioned terms:

$$q\pi = \frac{\varepsilon (\varphi + \sigma)(1 - \Phi)}{k} + \Phi \frac{\varepsilon}{k}$$  \hspace{1cm} (26)

where

$$k = \frac{(1 - \theta)(1 - \theta \beta)(\sigma + \varphi)}{\theta \alpha}$$  \hspace{1cm} (27)

and

$$\Phi = 1 - \frac{(\varepsilon - 1)(1 - \bar{\tau})}{\varepsilon}$$  \hspace{1cm} (28)

$$qy = (\sigma + \varphi) - \Phi (1 + \varphi) + \Phi (2 + \varphi - \frac{\sigma}{\alpha})$$  \hspace{1cm} (29)

$$\hat{Y}_t^* = \frac{1}{q_y} \{ (\sigma - \Phi) \frac{2\sigma (\alpha - \sigma)(1 - \alpha)}{\alpha (\sigma + \alpha \varphi)} \} \hat{g}_t + \frac{1}{q_y} \{ \Phi \frac{\varphi}{1 - \bar{\tau}} \} \hat{\tau}_t + \frac{1}{q_y} \{ \Phi \frac{2(\varepsilon \alpha + \varepsilon \varphi \alpha - \alpha \varphi + \alpha)}{(\varepsilon - 1)(\sigma + \alpha \varphi)} \} \hat{\varepsilon}_t$$  \hspace{1cm} (30)

As one can notice, the weights are functions of the deep parameters of the model, Calvo parameter and steady state values. Given that these parameters are assumed to be nonswitching, the micro-founded criterion instructs central banker to follow time-invariant policy rules. Since central banker is concerned, besides inflation containment, with minimizing household expected welfare losses, then with constants weights in the loss function:

$$E[L] = E[q\pi^2 + qy(\hat{Y}_t - \hat{Y}_t^*)^2] = \frac{q\pi}{2} E[\pi_t^2] + \frac{qy}{2} E[(\hat{Y}_t - \hat{Y}_t^*)^2] = \frac{q\pi}{2} var(\pi_t^2) + \frac{qy}{2} var(\hat{Y}_t - \hat{Y}_t^*)^2$$  \hspace{1cm} (31)

given that $E[\pi_t]$ and $E[(\hat{Y}_t - \hat{Y}_t^*)]$ are assumed to be zero.

On the other hand with state dependent weights, we have:

$$E[L] = E[q\pi(S_t)\pi_t^2 + qy(S_t)(\hat{Y}_t - \hat{Y}_t^*)^2] = E[q\pi(S_t)\pi_t^2] + E[qy(S_t)(\hat{Y}_t - \hat{Y}_t^*)^2]$$  \hspace{1cm} (32)

which is not be equal to $\frac{q\pi}{2} var(\pi_t^2) + \frac{qy}{2} var(\hat{Y}_t - \hat{Y}_t^*)^2$.

We will get two different random variables, with different variances than the previous case of constant weight. Therefore, welfare loss comparison between a loss function with constant coefficients and one with regime switching coefficients is not an appropriate approach. Alternatively we can look for the same criterion at different type of tax policies: switching versus non switching and see how different criterions do the loss ranking, or compare losses from the approximated function with the ones from the non approximated criterion to check if results in important differences.
So far, given that Calvo parameter is a behavioural parameter, the analysis may fall under the Lucas critique. Therefore I will allow it to become regime switching with the same probability matrix as in the case of the tax process. State dependent modeling of the Calvo parameter is based on the empirical findings by Midrigan (2007), that firms are more likely to change prices whenever the aggregate and idiosyncratic shocks reinforce each other and trigger desired price changes in the same direction. He finds that in times of monetary expansions the fraction of adjusting firms that have negative idiosyncratic technology shocks increases. In my case the aggregate shock is the tax shock.

With state dependent Calvo parameter, Phillips curve will have a state dependent slope and intercept from the tax process. In the loss function, only the weight $\tilde{q}_\pi(S_t)$ will become state dependent through $\tilde{k} = \tilde{\theta}^{-1}(1 - \tilde{\theta})(1 - \tilde{P}\tilde{\theta}\beta)^{\tilde{a}+\tilde{\sigma}}$. Given that the weight on the output gap $q_y$ will remain constant, while the Phillips curve become more instable (now slope and intercept switches instead of only intercept), intuitively we will expect worse results. This is the intuition why not any kind regime switching monetary policy rule or random weights in the loss function will deliver better results then their constant counterparts, and why Alexandre, Baçao, and Driffill might have get the results that taking into consideration the switching nature of the exchange rate will not bring significant improvements from the welfare losses point of view.

### 3.2 Optimal policy with discretion

Under discretion, the policymaker minimizes the loss function subject to the Phillips curve. In this case, the central banker choices at date $t$ do not bind in the future therefore he cannot affect the private sector expectations. For this part, I will solve the central banker problem using two alternative combinations of loss functions and Phillips curve constraints: 1. the microfounded one derived under the assumption of constant Calvo parameter and exogenous switching process for taxes, 2. the microfounded loss function obtained under the state dependent Calvo parameter (which means regime switching weight on inflation but fix weight on the output gap) and regime-switching slope and intercept Phillips curve.

#### 3.2.1 Case 1-Discretion: microfounded loss function, with switching tax process, without state dependent Calvo parameter

The central bank faces a single-period problem which is to minimize:

$$L = \frac{q_\pi}{2} \pi_t \pi_t' + \frac{q_y}{2} (\hat{Y}_t - \hat{Y}_t^*)(\hat{Y}_t - \hat{Y}_t^*)'$$

subject to:

$$\pi_t = \beta E_t \pi_{t+1} + k(\hat{Y}_t - \hat{Y}_t^*)$$  \hspace{1cm} (34)
where
\[ \hat{Y}_t^n = \frac{\alpha}{\alpha \bar{\phi} + \sigma (1 - \alpha)} \hat{\tau}_t - \frac{\sigma (1 - \alpha)}{(\sigma + \alpha \bar{\phi})} \hat{g}_t - \frac{\alpha}{(\sigma + \alpha \bar{\phi})(\varepsilon - 1)} \hat{\epsilon}_t \] (35)

The first order conditions are:
\[ \pi_t: q_{\pi} \pi_t + \omega_t = 0 \] (36)
\[ y_t: q_{y}(\hat{Y}_t - \hat{Y}_t^*) - k\omega_t = 0 \] (37)
where \( \omega_t \) is the lagrange multiplier on the Phillips curve constraint.

The first order condition will imply a monetary policy rule of the form:
\[ \pi_t + \frac{q_{y}}{kq_{\pi}} (\hat{Y}_t - \hat{Y}_t^*) = 0 \] (38)

In order to get numerical solutions: impulse response functions, the system to be fed in the Chris Sims’ Gensys algorithm is formed from the previous equation, the Phillips curve, and the processes for tax, government spending and mark up shocks.

In what is following I will obtain and discuss the analytical solution of the system.
Substituting the monetary policy rule we got from the loss minimization problem into the Phillips curve, we get the following first order difference equation in output gap \( \hat{Y}_t \):
\[ E_t \hat{Y}_{t+1} - \frac{1}{\beta} (1 + \frac{k^2 q_{\pi}}{q_y}) \hat{Y}_t = E_t \hat{Y}_t^* - \frac{1}{\beta} \hat{Y}_t^* - \frac{k^2 q_{\pi}}{\beta q_y} \hat{Y}_t^n \] (39)

If we define \( L^{-j} E_t \hat{Y}_t = E_t \hat{Y}_{t+j} \), where \( j \) is an integer, then the previous equation becomes:
\[ (L^{-1} - \frac{1}{\beta} (1 + \frac{k^2 q_{\pi}}{q_y})) \hat{Y}_t = E_t \hat{Y}_t^* - \frac{1}{\beta} \hat{Y}_t^* - \frac{k^2 q_{\pi}}{\beta q_y} \hat{Y}_t^n \] (40)

In this case we have an unique, determinate equilibrium if \( |\frac{1}{\beta} (1 + \frac{k^2 q_{\pi}}{q_y})| > 1 \). Given that from calibration we have \( 0 < \beta < 1 \), \( k > 0 \), and \( q_{\pi} > 0 \) and \( q_y > 0 \), we will get an unique determinate equilibrium.

Let
\[ a = \frac{1}{q_y} \left\{ \sigma - \Phi \left[ \frac{2(\alpha - \sigma) \sigma (1 - \alpha)}{\alpha} \right] \right\} \] (41)
\[ b = \frac{1}{q_y} \Phi \frac{2\alpha (\sigma + 1)}{1 - \Phi \sigma + \alpha \bar{\phi}} \] (42)
\[ c = \frac{1}{q_y} \Phi \frac{2(\varepsilon \sigma + \varepsilon \bar{\phi} \alpha - \alpha \bar{\phi} + \alpha)}{(\varepsilon - 1)(\sigma + \alpha \bar{\phi})} \] (43)
\[ d = -\frac{\sigma (1 - \alpha)}{\sigma + \alpha \bar{\phi}} \] (44)
\[
e = \frac{\alpha}{\alpha \phi + \sigma (1 - \bar{\tau})}
\]
\[
f = \frac{-\alpha}{(\sigma + \alpha \phi)(\varepsilon - 1)}
\]

The solutions for output gap, inflation, and interest rate are as following:

output gap
\[
\hat{Y}_t = [I - \frac{1}{\beta}(1 + \frac{k^2q\pi}{q_y})\rho^\beta]^{-1}[\rho^\beta a - \frac{a}{\beta} - \frac{k^2q\pi}{\beta q_y}d]\hat{y}_t + [I - \frac{1}{\beta}(1 + \frac{k^2q\pi}{q_y})P\rho^\beta]^{-1}[bP\rho^\beta - \frac{b}{\beta} - \frac{k^2q\pi}{\beta q_y}e]\hat{\tau}_t + [I - \frac{1}{\beta}(1 + \frac{k^2q\pi}{q_y})\rho^\beta]^{-1}[cP\rho^\beta - \frac{c}{\beta} - \frac{k^2q\pi}{\beta q_y}f]\hat{\xi}_t
\]

inflation
\[
\pi_t = \frac{-q_y}{kq_{\pi}}([I - \frac{1}{\beta}(1 + \frac{k^2q\pi}{q_y})\rho^\beta]^{-1}\{[\rho^\beta a - \frac{a}{\beta} - \frac{k^2q\pi}{\beta q_y}d] - a\})\hat{y}_t + [I - \frac{1}{\beta}(1 + \frac{k^2q\pi}{q_y})P\rho^\beta]^{-1}\{[bP\rho^\beta - \frac{b}{\beta} - \frac{k^2q\pi}{\beta q_y}e] - b\}\hat{\tau}_t + [I - \frac{1}{\beta}(1 + \frac{k^2q\pi}{q_y})\rho^\beta]^{-1}\{[cP\rho^\beta - \frac{c}{\beta} - \frac{k^2q\pi}{\beta q_y}f] - c\}\hat{\xi}_t
\]

interest rate
\[
r_t = (\frac{\sigma}{\alpha}(\rho^\beta[I - (1 - \frac{1}{\beta} + \frac{k^2q\pi}{\beta q_y})\rho^\beta]^{-1}[a\rho^\beta - \frac{a}{\beta} - \frac{k^2q\pi}{\beta q_y}d] + \frac{q_y}{kq_{\pi}}[I - (1 - \frac{1}{\beta} + \frac{k^2q\pi}{\beta q_y})\rho^\beta]^{-1}[a\rho^\beta - \frac{a}{\beta} - \frac{k^2q\pi}{\beta q_y}d] - 1) + \frac{q_y}{kq_{\pi}}(I - (1 - \frac{1}{\beta} + \frac{k^2q\pi}{\beta q_y})\rho^\beta]^{-1}[bP\rho^\beta - \frac{b}{\beta} - \frac{k^2q\pi}{\beta q_y}e] - b)\hat{\tau}_t + \{[I - (1 - \frac{1}{\beta} + \frac{k^2q\pi}{\beta q_y})\rho^\beta]^{-1}\{[cP\rho^\beta - \frac{c}{\beta} - \frac{k^2q\pi}{\beta q_y}f] - c\}\hat{\xi}_t
\]

where \(P\) is the probability matrix, and \(\rho^\beta\) is
\[
\rho^\beta = \begin{pmatrix}
\rho_1^\beta & 0 \\
0 & \rho_2^\beta
\end{pmatrix}
\]

Note that in this case the eigenvalues of the system is the same as in the fixed regime counterpart model.

Solutions inherit from the tax process the regime switching characteristic.
3.2.2 Case 2-Discretion: microfounded loss function, with switching tax process, with state dependent Calvo parameter

From the loss minimization problem we will get a monetary policy rule of the form:

\[ \pi_t + k(S_t)^{-1} q_\pi(S_t)^{-1} q_y(\hat{Y}_t - \hat{Y}_t^*) = 0 \] (51)

where in this case \( k(S_t) \) and \( q_\pi(S_t) \) are 2x2 matrices.

As in the previous case, the system of equations that can be solved numerically are formed from: the monetary policy rule (previous equation), Phillips curve, the processes for taxes, government spending and mark up shocks.

Supposing that \( \theta(S_t) \) follows the same Markov process as the tax process, we have that \( E_t \theta(S_{t+j}) = P^j \tilde{\theta} \), where:

\[ \tilde{\theta} = \begin{pmatrix} \theta_1 \\ 0 \\ \theta_2 \end{pmatrix} \] (52)

Following the same steps as in the previous case, we will get a first difference system of equations in output gap:

\[ E_t \hat{Y}_{t+1} - \{(P\gamma(Pk)^{-1}(Pq_\pi)^{-1})^{-1}[k^{-1}q_\pi^{-1} + k]\} \hat{Y}_t = E_t \hat{Y}_{t+1}^* - [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}k^{-1}q_\pi^{-1}\hat{Y}_t^* \]

\[ -(P\gamma(Pk)^{-1}(Pq_\pi)^{-1})^{-1}k\hat{Y}_n^t \] (53)

where I defined the matrix \( \gamma = \beta P\theta([I - P\theta]^{-1}P\theta + I) \), and \( P \) stands for the probability matrix as defined before.

Using lag operators, the equation becomes:

\[ \{L^{-1} - [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}[k^{-1}q_\pi^{-1} + k]\} \hat{Y}_t = E_t \hat{Y}_{t+1}^* - [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}k^{-1}q_\pi^{-1}\hat{Y}_t^* \]

\[ -(P\gamma(Pk)^{-1}(Pq_\pi)^{-1})^{-1}k\hat{Y}_n^t \] (54)

Assuming that \( Pk \), \( Pq_\pi \), \( k \) and \( [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}[k^{-1}q_\pi^{-1} + k] \) are invertible matrices, the determinacy of the system depends on the last mentioned matrix. In this case we need both of the respective matrices’ eigenvalues to be greater than one in absolute value.

Let matrix

\[ A = [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}[k^{-1}q_\pi^{-1} + k] \] (55)

\[ B = [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}k^{-1}q_\pi^{-1} \] (56)

\[ D = [P\gamma(Pk)^{-1}(Pq_\pi)^{-1}]^{-1}k \] (57)

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Solving for output gap, inflation and interest rate we get:

output gap

\[ \hat{Y}_t = [I - A \rho^g]^{-1}[a \rho^g - aB - dD]\hat{g}_t + [I - AP \rho^\tau]^{-1}[bP \rho^\tau - bB - cD]\hat{\tau}_t + [I - A \rho^\varepsilon]^{-1}[c \rho^\varepsilon - cB - fD]\hat{\varepsilon}_t \] (58)

inflation

\[ \pi_t = -k^{-1}q_\pi^{-1}q_y\{([I - A \rho^g]^{-1}[a \rho^g - aB - dD] - a)\hat{g}_t + ([I - AP \rho^\tau]^{-1}[bP \rho^\tau - bB - cD] - b)\hat{\tau}_t + ([I - A \rho^\varepsilon]^{-1}[c \rho^\varepsilon - cB - fD] - c)\hat{\varepsilon}_t \} \] (59)

interest rate

\[ \hat{r}_t = \left\{ \frac{\sigma}{\alpha}(\rho^g[I - A \rho^g]^{-1}[a \rho^g - aB - dD] - 1) + \rho^g[-k^{-1}q_\pi^{-1}q_y([I - A \rho^g]^{-1}[a \rho^g - aB - dD] - a)] + \frac{(1 - \alpha)(1 - \rho^g)}{\alpha}\right\}\hat{g}_t \\
+ \left\{ \frac{\sigma}{\alpha}(P \rho^\tau[I - AP \rho^\tau]^{-1}[bP \rho^\tau - bB - cD] - 1) + P \rho^\tau[-k^{-1}q_\pi^{-1}q_y([I - AP \rho^\tau]^{-1}[bP \rho^\tau - bB - cD] - b)]\right\}\hat{\tau}_t \\
+ \left\{ \rho^\varepsilon([I - A \rho^\varepsilon]^{-1}[c \rho^\varepsilon - cB - fD] - 1) + \rho^\varepsilon[-k^{-1}q_\pi^{-1}q_y([I - A \rho^\varepsilon]^{-1}[c \rho^\varepsilon - cB - fD] - c)]\right\}\hat{\varepsilon}_t \] (60)

4 Welfare costs of deviations from the inflation and output targets

Using the second order approximation to the utility, it is possible to derive the losses of the representative consumer, following Woodford (2003). The losses, expressed as a fraction of steady state consumption, can be written as:

\[ V = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}\left\{ \frac{q_\pi}{2} \pi_t^2 + \frac{q_y}{2} (\hat{Y}_t - \hat{Y}_t^*)^2 \right\} \] (61)

As the two aspects of policy can be independently specified, the expected welfare losses of any policy that deviates from the strict targeting are:

\[ E[V] = -E\left[ \frac{q_\pi}{2} \pi_t^2 + \frac{q_y}{2} (\hat{Y}_t - \hat{Y}_t^*)^2 \right] \] (62)

where we took unconditional variance of loss function and let \( \beta \to 1 \).

Table 2 captures the variation in the inflation and output gap for four cases: case 1. without Calvo state dependent parameter, without switching Taylor coefficients; case 2. without Calvo state dependent parameter, with switching Taylor rule coefficients; case 3. with state dependent Calvo, without switching Taylor coefficients; case 4. with state dependent Calvo, with switching Taylor rule.
coefficients.

Table 3 makes use of the above equation to assess welfare implications of monetary policy rules under discretion and rank those rules on the welfare grounds for five cases: case 1. without Calvo state dependent parameter, without switching weights in loss, case 2. without Calvo state dependent parameter, with switching weights in loss, case 3. without Calvo state dependent parameter, with switching \( q_\pi \) only, case 4. without Calvo state dependent parameter, with switching \( q_y \) only, case 5. with Calvo state dependent parameter. Table 4 displays simulation results for the discretionary cases across different type of tax policy: 1. low tax, 2. regime switching tax, 3. high tax.

5 Calibration

I calibrate the model for an initial data set following Woodford(2003, 2006).

I set the value for \( \beta \) to 0.99 implying a 4% annual interest rate. For \( \theta \), the fraction of firms that do not change price, I chose 0.75 implying a value consistent with an average period of one year between the price adjustments. Then in the case of of state dependent Calvo parameter I allowed it to switch between \( \theta_1 = 0.4 \) and \( \theta_2 = 0.75 \). I calibrated for a steady state mark-up of 1.2 which implied a value for \( \varepsilon \) of 6. I chose \( \varphi \) equal to 3, which implies a labor supply elasticity of 0.33. The value for \( \psi \) is 0.25 implying a steady state tax rate of 20%. The elasticity of intertemporal substitution I chose to be 2. For the transition probabilities,I set \( p_{11} = p_{22} = 0.55 \). For the rest of the parameters I chose: \( \rho^\pi = 0.4 \), \( \rho^\varepsilon = 0.4 \), \( \rho^\tau_1 = 0.9 \), and \( \rho^\tau_2 = 0.2 \).

Tax, supply and demand shocks are assumed to be uncorrelated, and the variance-covariance matrix of \( \varepsilon^d_t, \varepsilon^\pi_t \), and \( \varepsilon^\tau_t \) I assumed to be a normal distribution with mean 0 and variance 0.1 * \( I_3 \).

6 Some dynamics and results interpretations

Figures 1, 2, 3 and 4 present impulse response functions to a tax, government and mark up shocks for four different cases, DSGE model with Taylor rule type monetary policy: 1. without Calvo state dependent parameter, without switching Taylor coefficients: black line, 2. without Calvo state dependent parameter, with switching Taylor rule coefficients: red line, 3. with state dependent Calvo, without switching Taylor coefficients: blue line, 4. with state dependent Calvo, with switching Taylor rule coefficients: green.

In the case of constant regime the dynamics can be explained as following. A temporary tax increase results in an increase in inflation and decrease in output. This is because a higher tax rate implies a lower real wage, and thus a lower labor supply. Due to the fact that the downward adjustment of real wage is sluggish (prices adjust sluggishly), even though the labor costs fall, they rise relative to the marginal revenue. This cost push induces firms to increase their prices, resulting in a rise in inflation. The response of the monetary policy to the positive tax shock is an increase in the interest rate.
As regarding the government and mark up shocks, these have a stabilizing effect on inflation and output gap.

In regime switching case the qualitative explanations hold as for the first two cases without switching Calvo parameter. When regime switching Calvo parameter is considered, dynamics imply more aggressive actions with inflation even going negative and higher stabilizing effect from the government spending and mark up shocks part. Figures 5, 6, 7, 8 present impulse response functions when tax rule follows regime switching process for the following five cases: case 1. without Calvo state dependent parameter, without switching weights in loss: black line; case 2. without Calvo state dependent parameter, with switching weights in loss: red line; case 3. without Calvo state dependent parameter, with switching $q_{\pi}$ only: blue line; case 4. without Calvo state dependent parameter, with switching $q_{y}$ only: green line; case 5. with Calvo state dependent parameter: magenta line. The qualitative explanations hold as in the previous figures.

Tables 2, 3 and 4 reports the welfare losses associated with each policy as mentioned in section 4. All entries are percentage units of steady state consumption. Using the numerical solution from Chris Sim’s algorithm, Gensys, I simulated the model economies from the deterministic steady state for 10000 periods with the first 10% as the burn-in phase. The data simulated are assumed to be quarterly data. Given the calibrated transition probabilities, I simulated a Markov chain. After discarding the 10% burn-in phase of the Markov chain, I determined the realization of output gap, inflation gap, and $\lambda(S_t)$ according to the realized Markov chain. The calculations in Tables 2 and 3, are based on the simulated data for 9000 periods.

As showed in section 3.1, we cannot compare welfare across different types of loss functions, as for example between a loss function without switching weights and one with switching weights. But we can do the comparison, using the same loss function across different types of fiscal policy: low tax ($\rho_{\tau} = 2$), high tax ($\rho_{\tau} = 9$), and regime switching between high and low taxation.

The interesting simulation result is the difference in welfare loss ranking made by different type of loss functions across different type of fiscal policies. Intuition for why I consider this results interesting and as invalidating the constant weights monetary policy functions is the following: given that taxes are regime switching, prices and output become regime switching (households and firms’ optimization problem), therefore output gap and inflation will become regime switching. A constant weights monetary policy rule or criterion, responsive to output gap and inflation, will instruct the monetary policy to react in the same way (constant) no matter if monetary policy has to face higher inflation than in previous periods as for example due to a tax cut, or lower inflation and output due to a tax increase. Being less cautious(non-switching weights case) will make the economy more volatile.

Given that the monetary policy authority’s main roles of containing inflation with lowest welfare costs, and encourage economic activity during downturns (which imply variable weights on gaps), constant weights criterion seems to be an awkward central bank objective function.

Since what we observe in the data is a regime switching monetary policy, with possible induced
switching from an exogenous switching fiscal policy (switching type shocks), then microfounded based
criterion obtained by using linearization or perturbation methods which deliver constant weights type
objective functions, with weights depending only on deep parameters and steady state values might
not be appropriate to be used as guidance by policy makers. It, also, matters a lot what weights one
choose as regime switching in the loss function, as simulation results suggested that if we action on
the wrong weight we might get worse results.

7 Conclusion

As future work, I plan to move toward nonlinear model, and compare results from the nonlinear model
with the ones from the linear-quadratic problem under the tax switching framework.
References


Walsh, Carl (2003), Monetary Theory and Policy, The MIT Press


APPENDIX

MSV Solution for Model A

In obtaining the solutions, the demand, supply, and tax shocks are assumed to be uncorrelated, so the coefficients on the demand, those of the supply, and those on the tax shocks can be solved separately. The coefficients are the impact elasticities of demand, supply and tax shocks on output and inflation. The matrix equations are of the form $A \ast \vec{a} = \vec{b}$. The solutions are given by $\vec{a} = A^{-1} \ast \vec{b}$, where $A^{-1} = \frac{1}{\Delta} \ast C$, $\Delta$ is the determinant of matrix $A$, and $C$ is the adjoint matrix.

$$\begin{pmatrix} a_1^h \\ a_2^h \\ a_3^h \\ a_4^h \end{pmatrix} = \frac{1}{\Delta A} \begin{pmatrix} C_{11}^{th} & C_{12}^{th} & C_{13}^{th} & C_{14}^{th} \\ C_{21}^{th} & C_{22}^{th} & C_{23}^{th} & C_{24}^{th} \\ C_{31}^{th} & C_{32}^{th} & C_{33}^{th} & C_{34}^{th} \\ C_{41}^{th} & C_{42}^{th} & C_{43}^{th} & C_{44}^{th} \end{pmatrix} \begin{pmatrix} b_1^h \\ b_2^h \\ b_3^h \\ b_4^h \end{pmatrix}$$  \hspace{1cm} (A-1)

for $h = A, B$ models and $l = D, S, \tau$ for the demand, supply and tax shocks.

In the case of model A, for the demand shock, the solutions are:

$$\begin{pmatrix} a_1^{DA} \\ a_2^{DA} \\ b_1^{DA} \\ b_2^{DA} \end{pmatrix} = \frac{-1}{\Delta A} \begin{pmatrix} C_{11}^{DA} + C_{12}^{DA} \\ C_{13}^{DA} + C_{14}^{DA} \\ C_{23}^{DA} + C_{24}^{DA} \\ C_{33}^{DA} + C_{34}^{DA} \end{pmatrix}$$  \hspace{1cm} (A-2)

Let $a_i^{DF} = \beta p_i \rho^D - 1$ and $b_i^{DF} = (p_i \rho^D + \sigma^{-1} \gamma - 1)$ be the "fixed" regime coefficients, then

$$\Delta^{DA} = a_1^{DF} (a_2^{DF} b_1^{DF} b_2^{DF} - k b_1^{DF} \sigma^{-1} p_{12} \rho^D - p_{12}(\rho^D)^2 p_{21} a_2^{DF} + k p_{21} \sigma^{-1} p_{12}(\rho^D)^2 - \beta p_{12} \rho^D [\beta p_{21} \rho^D b_1^{DF} b_2^{DF} + k p_{21} \rho^D (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) - k b_1^{DF} \sigma^{-1} p_{21} \rho^D - p_{21}(\rho^D)^2 (p_{21})^2 \beta] + k [\beta p_{21} \sigma^{-1} p_{12}(\rho^D)^2 b_2^{DF} + k \sigma^{-1} p_{22} \rho^D (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) + p_{21}(\rho^D)^2 a_2^{DF} \sigma^{-1} p_{21} - k (\sigma^{-1})^2 p_{12} p_{21}(\rho^D)^2] - p_{12} p_{22} p_{21} \beta (\rho^D)^3 - (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) a_2^{DF} b_2^{DF}]$$  \hspace{1cm} (A-3)

$$C_{13}^{DA} = \beta p_{21} (\rho^D)^2 \sigma^{-1} p_{12} a_2^{DF} + (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) \sigma^{-1} p_{22} \rho^D k + \sigma^{-1} p_{21} (\rho^D)^2 b_2^{DF} p_{12} - k (\sigma^{-1})^2 (\rho^D)^2 p_{12} p_{21} - (\rho^D)^3 \sigma^{-1} p_{12} p_{22} p_{21} - b_2^{DF} a_2^{DF} (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1})$$  \hspace{1cm} (A-4)

$$C_{14}^{DA} = -(p_{21})^2 p_{12} \beta \sigma^{-1} (\rho^D)^3 + b_1^{DF} a_2^{DF} \sigma^{-1} p_{21} \rho^D - \beta p_{21} p_{22} \sigma^{-1} (\rho^D)^2 b_1^{DF} - p_{21} \rho^D a_2^{DF} (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1})$$  \hspace{1cm} (A-5)
\[ C_{23}^{DA} = -[\sigma^{-1}p_{12}\rho^D b_1^{DF} a_1^{DF} + (p_{12})^2 p_{21} \beta (\rho^D)^3 - \beta p_{12} (\rho^D)^2 \sigma^{-1} p_{22} a_1^{DF} - \beta p_{12} \rho^D (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) b_2^{DF}] \] 

\[ C_{24}^{DA} = p_{21} \sigma^{-1} p_{12} (\rho^D)^2 a_1^{DF} + k \sigma^{-1} p_{22} \rho^D (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) + \sigma^{-1} p_{21} (\rho^D)^2 \beta p_{12} b_1^{DF} - k (\sigma^{-1})^2 p_{12} p_{21} (\rho^D)^2 a_1^{DF} \sigma^{-1} p_{22} \rho^D b_1^{DF} - p_{21} \beta p_{12} (\rho^D)^2 (\sigma^{-1} p_{11} \rho^D - \alpha \sigma^{-1}) \] 

\[ C_{33}^{DA} = a_1^{DF} a_2^{DF} b_1^{DF} + k \beta p_{12} (\rho^D)^2 \sigma^{-1} p_{21} - k \sigma^{-1} p_{22} \rho^D a_1^{DF} - (\beta)^2 p_{21} (\rho^D)^2 p_{12} b_2^{DF} \] 

\[ C_{34}^{DA} = -[p_{21} \rho^D a_1^{DF} a_2^{DF} + k \beta p_{21} (\rho^D)^2 \sigma^{-1} p_{22} - k \sigma^{-1} p_{21} \rho^D a_2^{DF} - (p_{21})^2 (\rho^D)^3 (\beta)^2 p_{12}] \] 

\[ C_{43}^{DA} = -[p_{12} \rho^D a_2^{DF} a_1^{DF} + k \beta p_{12} \rho^D (\sigma^{-1} p_{11} - \alpha \sigma^{-1}) - k \sigma^{-1} p_{12} \rho^D a_1^{DF} - (p_{12})^2 (\rho^D)^3 (\beta)^2 p_{21}] \] 

\[ C_{44}^{DA} = a_1^{DF} a_2^{DF} b_1^{DF} + k \beta p_{21} (\rho^D)^2 \sigma^{-1} p_{12} - k a_2^{DF} (\sigma^{-1} p_{11} - \alpha \sigma^{-1}) - b_1^{DF} (\beta)^2 (\rho^D)^2 p_{12} p_{21} \] 

For the supply shock let \( a_i^{SF} = \beta p_{i1} \rho^S - 1 \) and \( b_i^{SF} = (p_{i1} \rho^S + \sigma^{-1} \gamma - 1) \) be the "fixed" regime coefficients, then the solutions are of the following form:

\[
\begin{pmatrix}
  a_1^{SA} \\
  a_2^{SA} \\
  b_1^{SA} \\
  b_2^{SA}
\end{pmatrix} = \frac{-1}{\Delta^{SA}}
\begin{pmatrix}
  C_{11}^{SA} + C_{12}^{SA} \\
  C_{21}^{SA} + C_{22}^{SA} \\
  C_{31}^{SA} + C_{32}^{SA} \\
  C_{41}^{SA} + C_{42}^{SA}
\end{pmatrix}
\] 

\[ \Delta^{SA} = a_1^{SF}[a_2^{SF} b_1^{SF} b_2^{SF} - k b_1^{SF} \sigma^{-1} p_{22} \rho^S - p_{12} (\rho^S)^2 p_{21} a_2^{SF} + k p_{21} \sigma^{-1} p_{12} (\rho^S)^2] - \beta p_{12} \rho^S[\beta p_{21} \rho^S b_1^{SF} b_2^{SF} + k p_{21} \rho^S (\sigma^{-1} p_{11} \rho^S - \alpha \sigma^{-1}) - k b_1^{SF} \sigma^{-1} p_{21} \rho^S - p_{12} (\rho^S)^3 (p_{21})^2 \beta] + k [\beta p_{21} \sigma^{-1} p_{12} (\rho^S)^2 b_2^{SF} + k \sigma^{-1} p_{22} \rho^S (\sigma^{-1} p_{11} \rho^S - \alpha \sigma^{-1}) + p_{12} (\rho^S)^2 a_2^{SF} \sigma^{-1} p_{21} - k (\sigma^{-1})^2 p_{12} p_{21} (\rho^S)^2 ] - p_{12} p_{22} p_{21} (\rho^S)^3 (\sigma^{-1} p_{11} \rho^S - \alpha \sigma^{-1}) a_2^{SF} b_2^{SF}
\] 

\[ C_{11}^{SA} = a_2^{SF} b_1^{SF} b_2^{SF} - k \sigma^{-1} p_{22} \rho^S b_1^{SF} + k p_{21} (\rho^S)^2 \sigma^{-1} p_{12} - a_2^{SF} (\rho^S)^2 p_{12} p_{21} \]
\[ C_{12}^{SA} = -[\beta p_{21} \rho^S b_1^{SF} b_2^{SF} + k p_{21} \rho^S (\sigma^{-1} p_{11} \rho^S - \alpha \sigma^{-1}) - k b_1^{SF} \sigma^{-1} p_{21} \rho^S - \beta (p_{21})^2 p_{12} (\rho^S)^3] \quad (A-15) \]

\[ C_{21}^{SA} = -[\beta p_{12} \rho^S b_1^{SF} b_2^{SF} + k p_{12} (\rho^S)^2 \sigma^{-1} p_{22} - (p_{12})^2 (\rho^S)^3 p_{21} \beta - k \sigma^{-1} p_{12} \rho^S b_2^{SF}] \quad (A-16) \]

\[ C_{22}^{SA} = a_1^{SF} b_1^{SF} b_2^{SF} + k p_{12} (\rho^S)^2 \sigma^{-1} p_{22} - p_{21} p_{12} \sigma^{-1} (\rho^S)^2 a_1^{SF} - k b_2^{SF} (\sigma^{-1} p_{11} \rho^S - \alpha \sigma^{-1}) \quad (A-17) \]

\[ C_{31}^{SA} = k^2 \sigma^{-1} p_{22} \rho^S - k p_{21} (\rho^S)^2 \beta p_{12} - k a_2^{SF} b_2^{SF} \quad (A-18) \]

\[ C_{32}^{SA} = -[k^2 \sigma^{-1} p_{21} \rho^S - k a_1^{SF} p_{21} \rho^S - k b_2^{SF} \beta p_{21} \rho^S] \quad (A-19) \]

\[ C_{41}^{SA} = -[k^2 \sigma^{-1} p_{12} \rho^S - k b_1^{SF} \beta p_{11} \rho^S - k p_{12} \rho^S a_2^{SF}] \quad (A-20) \]

\[ C_{42}^{SA} = k^2 (\sigma^{-1} p_{11} \rho^S - \alpha \sigma^{-1}) - k a_1^{SF} b_1^{SF} - \beta k p_{21} p_{12} (\rho^S)^2 \quad (A-21) \]

For the tax shock let \( a_i^{TF} = \beta p_{ii} \tau_i - 1 \) and \( b_i^{TF} = (p_{ii} \rho_i^T) + \sigma^{-1} \gamma - 1 \) be the fixed regime coefficients, then the solutions are of the following form:

\[
\begin{bmatrix}
  a_1^{TA} \\
  a_2^{TA} \\
  b_1^{TA} \\
  b_2^{TA}
\end{bmatrix} = \frac{-k \psi}{\Delta^{TA}} \begin{bmatrix}
  C_{11}^{TA} + C_{12}^{TA} \\
  C_{21}^{TA} + C_{22}^{TA} \\
  C_{31}^{TA} + C_{32}^{TA} \\
  C_{41}^{TA} + C_{42}^{TA}
\end{bmatrix}
\quad (A-22)
\]

\[
\Delta^{TA} = a_1^{TF} \sigma^{-1} p_{22} \rho^T - p_{12} p_{21} \rho_1^{T} \rho_2^{T} a_2^{TF} + k p_{21} \sigma^{-1} p_{12} \rho_1^{T} \rho_2^{T} b_2^{TF} + k p_{21} \rho_1^{T} (\sigma^{-1} p_{11} \rho_1^T - \alpha \sigma^{-1}) - k b_1^{TF} \sigma^{-1} p_{21} \rho_1^T - p_{12} \rho_2^{T} (\rho_1^{T})^2 \beta^{T} + k [ \beta p_{21} p_{12} \rho_1^{T} \rho_2^{T} \sigma^{-1} b_2^{TF} \\
+(\sigma^{-1} p_{11} \rho_1^T - \alpha \sigma^{-1}) \sigma^{-1} p_{22} \rho_2^{T} k + p_{12} \rho_2^{T} a_2^{TF} \sigma^{-1} p_{21} \rho_1^T - k (\sigma^{-1})^2 p_{12} \rho_2^{T} p_{21} \rho_1^T - p_{12} (\rho_2^{T})^2 \rho_1^{T} / \beta \\
\sigma^{-1} p_{21} p_{22} - (\sigma^{-1} p_{11} \rho_1^T - \alpha \sigma^{-1}) a_2^{TF} b_2^{TF}]
\quad (A-23)
\]

\[ C_{11}^{TA} = a_1^{TF} b_1^{TF} b_2^{TF} + k p_{21} p_{12} \sigma^{-1} \rho_1^{T} \rho_2^{T} - k \sigma^{-1} p_{22} \rho_2^{T} b_1^{TF} - p_{12} \rho_2^{T} p_{21} \rho_1^{T} \sigma^{-1} a_2^{TF} \quad (A-24) \]

\[ C_{12}^{TA} = -[\beta p_{21} \rho_1^{T} b_1^{TF} b_2^{TF} + k p_{21} \rho_1^{T} (\sigma^{-1} p_{11} \rho_1^T - \alpha \sigma^{-1}) - k b_1^{TF} \sigma^{-1} p_{21} \rho_1^T - p_{12} \rho_2^{T} (\rho_1^{T})^2 \beta] \quad (A-25) \]
\[ C_{21}^{\tau A} = -[\beta p_{12} \rho_2^2 b_1^\tau F b_2^\tau F + k p_{12} (\rho_2^\tau)^2 \sigma^{-1} p_{22} - (p_{12})^2 p_{21} \beta \rho_1^\tau (\rho_2^\tau)^2 - k \sigma^{-1} p_{12} \rho_2^2 b_1^\tau] \] (A-26)

\[ C_{22}^{\tau A} = a_1^\tau F b_1^\tau F b_2^\tau F + k p_{12} \rho_2^2 \sigma^{-1} p_{21} \rho_1^\tau - p_{12} \rho_2^\tau p_{21} \rho_1^\tau a_1^\tau F - k b_2^\tau F (\sigma^{-1} p_{11} \rho_1^\tau - \alpha \sigma^{-1}) \] (A-27)

\[ C_{31}^{\tau A} = k^2 \sigma^{-1} p_{22} \rho_2^\tau - k p_{21} \rho_1^\tau \beta p_{12} \rho_2^\tau - a_2^\tau F b_2^\tau F k \] (A-28)

\[ C_{32}^{\tau A} = -[k^2 \sigma^{-1} p_{21} \rho_1^\tau - k p_{21} \rho_1^\tau a_1^\tau F - k \beta p_{21} \rho_1^\tau b_2^\tau F] \] (A-29)

\[ C_{41}^{\tau A} = -[k^2 \sigma^{-1} p_{12} \rho_2^\tau - k p_{12} \rho_2^\tau a_2^\tau F - k \beta p_{12} \rho_2^\tau b_1^\tau F] \] (A-30)

\[ C_{42}^{\tau A} = k^2 (\sigma^{-1} p_{11} \rho_1^\tau - \alpha \sigma^{-1}) - k b_1^\tau F a_1^\tau F - k \beta p_{21} p_{12} \rho_1^\tau \rho_2^\tau \] (A-31)
Linear quadratic approximation of the utility function to be added

Derivation of the Phillips curve with state dependent Calvo parameter to be added
Calibrated parameters

Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>3</td>
<td>$\varepsilon$</td>
<td>6</td>
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<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>$\psi$</td>
<td>0.25</td>
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<tr>
<td>$\theta_1$</td>
<td>0.4</td>
<td>$\theta_2$</td>
<td>0.75</td>
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<tr>
<td>$p_{11}$</td>
<td>0.55</td>
<td>$p_{22}$</td>
<td>0.55</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>0.4</td>
<td>$\lambda_2$</td>
<td>1.9</td>
</tr>
<tr>
<td>$\rho^d$</td>
<td>0.7</td>
<td>$\rho^s$</td>
<td>0.7</td>
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<tr>
<td>$\rho^r_1$</td>
<td>0.9</td>
<td>$\rho^r_2$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>1.5</td>
<td>$\gamma_y$</td>
<td>1.9</td>
</tr>
<tr>
<td>$\alpha_{x1}$</td>
<td>1.5</td>
<td>$\gamma_{y1}$</td>
<td>1.1</td>
</tr>
<tr>
<td>$\alpha_{x2}$</td>
<td>0.9</td>
<td>$\gamma_{y2}$</td>
<td>1.9</td>
</tr>
<tr>
<td>$q_{r1}$</td>
<td>192</td>
<td>$q_{r1}$</td>
<td>2</td>
</tr>
<tr>
<td>$q_{r2}$</td>
<td>150</td>
<td>$q_{r2}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
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</tbody>
</table>
### Welfare Calculations

**Table 2: Contribution to welfare losses-Model with Taylor rule**

<table>
<thead>
<tr>
<th></th>
<th>Taylor1</th>
<th>Taylor2</th>
<th>Taylor3</th>
<th>Taylor4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{var}(\pi_t)$</td>
<td>0.6543</td>
<td>0.6686</td>
<td>0.0023</td>
<td>0.0038</td>
</tr>
<tr>
<td>$\text{var}(y_t)$</td>
<td>0.0745</td>
<td>0.0826</td>
<td>0.0035</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

**Table 3: Contribution to welfare losses-Optimal policy under discretion I**

<table>
<thead>
<tr>
<th></th>
<th>Discretion1</th>
<th>Discretion2</th>
<th>Discretion3</th>
<th>Discretion4</th>
<th>Discretion5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[q_\pi \pi_t^2]$</td>
<td>0.0656</td>
<td>0.0051</td>
<td>0.0678</td>
<td>0.0301</td>
<td>0.3114</td>
</tr>
<tr>
<td>$E[q_y y_t^2]$</td>
<td>0.0433</td>
<td>0.0093</td>
<td>0.0449</td>
<td>0.0695</td>
<td>0.2463</td>
</tr>
<tr>
<td>Total $= \frac{1}{2} {E[q_\pi \pi_t^2] + E[q_y y_t^2]}$</td>
<td>0.0544</td>
<td>0.0072</td>
<td>0.0563</td>
<td>0.0498</td>
<td>0.2789</td>
</tr>
</tbody>
</table>

**Table 4: Contribution to welfare losses-Optimal policy under discretion II**

<table>
<thead>
<tr>
<th></th>
<th>$\rho' = 2$</th>
<th>$\rho' = 2 \mapsto \rho' = 9$</th>
<th>$\rho' = 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretion1</td>
<td><strong>0.0544</strong></td>
<td>$= 0.0544&lt; 0.0549$</td>
<td>0.0549</td>
</tr>
<tr>
<td>Discretion2</td>
<td>0.0561</td>
<td>$&gt; 0.0072&lt; 0.0565$</td>
<td></td>
</tr>
<tr>
<td>Discretion3</td>
<td><strong>0.0545</strong></td>
<td>$&lt; 0.0563 &gt; 0.0550$</td>
<td></td>
</tr>
<tr>
<td>Discretion4</td>
<td>0.0560</td>
<td>$&gt; 0.0498&lt; 0.0564$</td>
<td></td>
</tr>
<tr>
<td>Discretion5</td>
<td><strong>0.1761</strong></td>
<td>$&lt; 0.2789 &gt; 0.1772$</td>
<td></td>
</tr>
</tbody>
</table>

Note: entries in Table 2 and 3 are percentage units of steady-state consumption.
• Taylor rule
  – 1. without Calvo state dependent parameter, without switching Taylor coefficients: black line
  – 2. without Calvo state dependent parameter, with switching Taylor coefficients: red line
  – 3. with state dependent Calvo, without switching Taylor coefficients: blue line
  – 4. with state dependent Calvo, with switching Taylor coefficients: green

Figure 1: Impulse responses to tax shock
Figure 2: Impulse responses to tax shock

- Taylor rule
  - 1. without Calvo state dependent parameter, without switching Taylor coefficients: black line
  - 2. without Calvo state dependent parameter, with switching Taylor coefficients: red line
  - 3. with state dependent Calvo, without switching Taylor coefficients: blue line
  - 4. with state dependent Calvo, with switching Taylor coefficients: green
Figure 3: Impulse responses to government spending shock

- Taylor rule
  - 1. without Calvo state dependent parameter, without switching Taylor coefficients: black line
  - 2. without Calvo state dependent parameter, with switching Taylor coefficients: red line
  - 3. with state dependent Calvo, without switching Taylor coefficients: blue line
  - 4. with state dependent Calvo, with switching Taylor coefficients: green
Figure 4: Impulse responses to mark up shock

- Taylor rule
  - 1. without Calvo state dependent parameter, without switching Taylor coefficients: black line
  - 2. without Calvo state dependent parameter, with switching Taylor rule coefficients: red line
  - 3. with state dependent Calvo, without switching Taylor coefficients: blue line
  - 4. with state dependent Calvo, with switching Taylor rule coefficients: green
Figure 5: Impulse responses to tax shock

- Optimal policy with discretion, switching tax rule and
  - 1. without Calvo state dependent parameter, without switching weights in loss: black line
  - 2. without Calvo state dependent parameter, with switching weights in loss: red line
  - 3. without Calvo state dependent parameter, with switching $q_{\pi}$ only: blue line
  - 4. without Calvo state dependent parameter, with switching $q_{\pi}$ only: green line
  - 5. with Calvo state dependent parameter: magenta line
Figure 6: Impulse responses to tax shock

- Optimal policy with discretion, switching tax rule and
  - 1. without Calvo state dependent parameter, without switching weights in loss: black line
  - 2. without Calvo state dependent parameter, with switching weights in loss: red line
  - 3. without Calvo state dependent parameter, with switching $q_x$ only: blue line
  - 4. without Calvo state dependent parameter, with switching $q_y$ only: green line
  - 5. with Calvo state dependent parameter: magenta line
Figure 7: Impulse responses to government spending shock

- Optimal policy with discretion, switching tax rule and
  - 1. without Calvo state dependent parameter, without switching weights in loss: black line
  - 2. without Calvo state dependent parameter, with switching weights in loss: red line
  - 3. without Calvo state dependent parameter, with switching $q_y$ only: blue line
  - 4. without Calvo state dependent parameter, with switching $q_y$ only: green line
  - 5. with Calvo state dependent parameter: magenta line
Figure 8: Impulse responses to mark up shock

- Optimal policy with discretion, switching tax rule and
  - 1. without Calvo state dependent parameter, without switching weights in loss: black line
  - 2. without Calvo state dependent parameter, with switching weights in loss: red line
  - 3. without Calvo state dependent parameter, with switching $q_y$ only: blue line
  - 4. without Calvo state dependent parameter, with switching $q_y$ only: green line
  - 5. with Calvo state dependent parameter: magenta line