Financing constraints and returns on Physical investment

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Abstract

Using an intertemporal model of investment decisions under financing constraints we show that the Euler equation approach can be used to identify the effect of financing constraints on investment returns even when the premium on external financing does not change over time. This result is made possible by the introduction of a tangibility constraint to a commonly used dynamic optimization model of investment. Nevertheless, we have to signal that the result is only true for firms that are severely constrained since the tangibility constraint has to be binding to make the identification of financing constraints possible under the above condition.

1 Introduction

In their model of perfect competition economists usually assume that firms choose the level of investment that maximizes their profit without having to take account of their ability to find the funds necessary to carry out their investment projects. In reality, many economists realize that it is not always possible for certain firms to finance the level of investment that would allow them to achieve the highest possible profit. The firms facing such situation are usually considered to be financially constrained. They are forced either to choose an investment level less than optimal or to pay a financing premium in order to find enough funds to carry out the optimal investment level. It is then logical to think that these firms may not be able to obtain the investment return that they would be able to achieve if they had easy access to the funds they need. Thus, financing constraints may be able to explain some aspects of a firm’s investment behavior. Some economists like Gertler and Gilchrist (1994), and Kashyap, Lamont, and Stein (1994) argue that models with financing frictions might provide a better understanding of the effect of monetary policy on firms’ activities over business cycles. Nevertheless, previous studies have shown that it is not easy to identify the effect of these constraints from available data. Eberly et al. (2006) show that it may be possible to find the presence of financing constraints from data simulated from a model with perfect capital markets using a linear regression of investment on the Tobin’s Q (average Q) and cash flows. The majority of the empirical studies found in the literature use linear panel models with investment as dependent variable. However, a dynamic optimization model of investment has also some implications for investment returns which will lead to a non-linear panel data model. The preceding observation differentiates the model presented in this paper from most of the models reported in the literature. In fact, the dynamic models used in this paper is closest to the model by Gomes et al. (2006). The main difference is that these authors tried to use investment returns to price stocks and bonds while in this paper, the objective is to determine whether or not financing constraints matter for investment returns. This paper is also different
from the paper by Gomes, Yaron, and Zhang (2006) because of the introduction of a tangibility constraint as in Almeida and Campello (2006). In fact, the amount of funds that a firm can borrow is usually limited by the amount of money that can be easily recovered from the firm’s assets in case they are liquidated, that is, by the tangibility of the firm’s assets. As a result, asset tangibility is supposed to affect firm’s investment returns. Almeida and Campello (2006) use a static model to derive the implications of asset tangibility for investment and test these implications through a reduced form linear econometric equation. However, using a similar tangibility constraint, this paper derives the implications of assets tangibility for investment returns from a theoretical dynamic optimization model of investment decision that provides directly the econometric equation to be estimated.

The paper is organized as follows. In section 2, the dynamic model used is presented and the hypotheses that will be tested are isolated. In section 3, the econometric methodology used to test the hypotheses is laid out. In section 4, the results obtained are presented and discussed. Finally, section 5 concludes the paper.

2 Model

The firm’s investment decision is considered when there exists some degree of information asymmetry between the managers of the firms and potential suppliers of funds. At first, a simple investment model is presented. The model is latter made more complex by the introduction of information asymmetry. In finance, the firm’s investment decision is usually considered as the problem of choosing the firm’s next period capital stock in order to maximize its expected value to the shareholders.

Let \( V(K_t, S_t) \) be the value of the firm at time \( t \). The firm’s value at time \( t \) depends on the amount of capital used at time \( t \) (\( K_t \)) and on the stochastic shock (\( S_t \)) faced by the firm. \( S_t \) includes not only a productivity shock, but also, the real prices of the variables inputs which are assumed to be chosen optimally. The Bellman’s equation for the firm’s problem can then be written as:

\[
V(K_t, S_t) = \max_{K_{t+1}} [\pi(K_t, S_t) - I_t - C(K_t, K_{t+1}) + E_t m_{t,t+1} V(K_{t+1}, S_{t+1})], \quad K_{t+1} \geq 0
\]

(1)

where \( \pi(K_t, S_t) \) is the firm’s profit when the variable factors of production are chosen optimally and when the capital stock at time \( t \) is known; \( I_t \) is the firm’s investment spending at time \( t \), \( C(K_t, K_{t+1}) \) is the cost of adjusting the firm’s capital stock from \( K_t \) to \( K_{t+1} \), and \( m_{t,t+1} \) is a stochastic discount factor, the value at time \( t \) of a dollar received at time \( t+1 \). Capital is accumulated according to the equation:

\[
I_t = K_{t+1} - K_t + \delta K_t
\]

(2)

Assume that \( V(K_t, S_t) \), \( \pi(K_t, S_t) \), and \( C(K_t, K_{t+1}) \) are differentiable with respect to the capital stock (\( K_t \)) and that the capital stock of an existing firm is strictly positive. Let \( f_i(\cdot) \) be the first derivative of the function \( f(\cdot) \) with respect to its \( i \)th argument. Then, the first order condition for the problem defined in equation (1) is given by:

\[
E_t [m_{t,t+1} V_1(K_{t+t}, S_{t+1})] = 1 + C_2(K_t, K_{t+1})
\]

(3)

We can rewrite the preceding equation as:
If \( V(K_t, S_t), \pi(K_t, S_t), \) and \( C(K_t, K_{t+1}) \) are twice differentiable and if the second order conditions are satisfied, using the envelope theorem the first derivative of the firm’s value is given by:

\[
V_1(K_{t+1}, S_{t+1}) = \pi_1(K_{t+1}, S_{t+1}) + 1 - \delta - C_1(K_{t+1}, K_{t+2}) \quad (envelope \ condition)
\]

Substituting equation 5 in equation 4, we obtain equation 6:

\[
E_t \left\{ m_{t,t+1} \left[ \pi_1(K_{t+1}, S_{t+1}) + 1 - \delta - C_1(K_{t+1}, K_{t+2}) \right] \right\} = 1 \quad (7)
\]

The left-hand side of the preceding equation (equation 7) is the expected change in the firm’s value induced by a one-unit increase in the firm’s capital stock divided by the cost of that unit of capital. This means that the left-hand side is the firm’s return on investment. Thus, equation 7 means that the optimal level of investment should be such that the expected gross return on investment is equal to 1. A gross return on investment higher than one would mean that the expected benefit of a one-unit increase in capital stock is higher than the expected cost. As a result, the corresponding choice of \( K_{t+1} \) would not be optimal; thus, if the firm’s value is concave, it will increase if the manager chooses a higher level of the capital stock. On the other hand, an expected gross return on investment less than one would not be optimal either since it would be possible to increase the firm’s value by choosing a lower level of investment. The latter case would be a situation of overinvestment.

### 2.1 Financing constraints

The optimal condition previously derived is true under the assumption that the firm faces no financing constraint. If there exists some degree of information asymmetry between the firm and its potential suppliers of funds, the firm may not be able to finance its investment to the level that is optimal in the absence of constraints. Without the ability to evaluate accurately the profitability of the firm’s projects, the suppliers of funds may not be willing to finance the firm’s investment or they may be willing to supply only a fraction of the funds needed by the firm. Information asymmetry may also lead funds providers to increase the financing cost the firm faces and lead to the choice of an investment level less than the optimal level that would be obtained in the absence of information asymmetry. As a result, returns on investment may be affected.

#### 2.1.1 The firm has no access to external financing

In the extreme case where the firm does not have access to external funds, its investment spending cannot exceed its cash flows which leads to the addition of a constraint to the problem previously defined. This constraint can be written as:

\[
\pi(K_t, S_t) - I_t - C(K_t, K_{t+1}) \geq 0 \quad (8)
\]
The first order condition for the problem becomes:

\[
E_t \left[ m_{t,t+1} V_1 (K_{t+1}, S_{t+1}) \right] \frac{(1 + \lambda_t)}{[1 + C_2 (K_t, K_{t+1})]} = 1
\]  

(9)

\[
\pi(K_t, S_t) - I_t - C(K_t, K_{t+1}) \geq 0, \lambda_t \geq 0 \text{ (with complementary slackness)}
\]  

(10)

\[
V_1 (K_{t+1}, S_{t+1}) = (1 + \lambda_{t+1}) [\pi_{t+1} (K_{t+1}, S_{t+1}) + 1 - \delta - C_1 (K_{t+1}, K_{t+2})] \text{ (envelope condition)}
\]  

(11)

The first order condition can then be written as:

\[
E_t m_{t,t+1} \left\{ \frac{(1 + \lambda_{t+1}) [\pi_t (K_{t+1}, S_{t+1}) + 1 - \delta - C_1 (K_{t+1}, K_{t+2})]}{(1 + \lambda_t) [1 + C_2 (K_t, K_{t+1})]} \right\} = 1
\]  

(12)

When the constraint is not binding, the complementary slackness condition implies \( \lambda_t = 0 \). In this case, the firm is unconstrained since the first order condition is exactly the same as in the unconstrained case.

When the constraint is binding, (\( \lambda_t \) is non-negative), the cost of increasing the firm’s capital stock has increased from \( 1 + C_2 (K_t, K_{t+1}) \) (unconstrained case) to \( (1 + \lambda_t) [1 + C_2 (K_t, K_{t+1})] \). So, \( \lambda_t \) can be considered as an additional opportunity cost that the firms has to bear each time the optimal investment level is higher than the level that can be financed by the firm’s internal funds. With the increase in cost, the value of \( V_k \) needed to satisfy equation 11 is higher than the value needed to satisfy the equation 12. When the value function is concave, this implies a lower value for \( K \). So, the optimal level of investment is lower under constraint.

When \( \frac{1 + \lambda_{t+1}}{1 + \lambda_t} = 1 \), equation 12 is the same as equation 11 and financial frictions would have no impact on investment returns. Then, if we can test whether or not the last equality is satisfied, we can detect the presence or the absence of financing constraints effects on investment returns. Note that the latter equation is satisfied when the financing premium is zero or constant over time. As a result, such a test would not allow the identification of financing frictions when they are associated to constant financing premia, which limits the implications of the test.

2.1.2 The firm faces borrowing constraint

Let us now consider the case where the amount of funds that the firm can borrow is restricted by the tangibility of its assets as in Almeida and Campello (2006). Let \( B_{t+1} \) be the amount of funds that the firm can borrow at time \( t \) and that will be repaid at time \( t+1 \) at a gross interest rate of \( R_{t+1} \), \( \tau_t \) be the fraction of the firm’s assets that can be recovered if these assets have to be liquidated at time \( t \) (which is what is called tangibility), and \( K_t \) be the firm’s capital stock at time \( t \). We can then write the following constraint \(^1\):

\[
B_{t+1} \leq \tau_t K_t, \tau_t \in [0, 1] \text{ (Borrowing constraint)}
\]

The firm’s problem can then be rewritten as:

\[
V(K_t, B_{t-1}, S_t) = \max_{K_{t+1}} \left[ \pi_t (K_t, S_t) + B_t - R_t B_{t-1} - I_t - C(K_t, K_{t+1}) + E_t m_{t,t+1} V(K_{t+1}, B_t, S_{t+1}) \right]
\]  

(13)

\(^1\)we consider here only the tangibility of the physical capital stock of the firm. It is very likely that a firms possesses other types of assets whose tangibility matters. In this case we assume that these assets are so tangible that the firm would prefer to liquidate before trying to get a debt.
\[ st : I_t + C(K_t, K_{t+1}) \leq \pi(K_t, S_t) + B_t - R_t B_{t-1}, B_t \leq \tau_t K_t \] (14)

Let \( \theta \) and \( \rho \) be the multipliers associated to the first and second constraints. The first order conditions are:

\[
K_{t+1} : \quad E_t \frac{m_{t,t+1} V_1(K_{t+1}, B_t, S_{t+1})}{(1 + \theta_t)[1 + C_2(K_t, K_{t+1})]} = 1
\] (15)

\[
B_t : 1 + E_t m_{t,t+1} V_2(K_{t+1}, B_t, S_{t+1}) + \theta_t - \rho_t = 0
\] (16)

\[
\theta_t : \pi(K_t, S_t) + B_t - R_t B_{t-1} - I_t - C(K_t, K_{t+1}) \geq 0, \theta_t \geq 0 \quad \text{(with complementary slackness)}
\] (17)

\[
\rho_t : \tau K_t - B_t \geq 0, \rho_t \geq 0 \quad \text{(with complementary slackness)}
\] (18)

\[
V_1(K_{t+1}, B_t, S_{t+1}) = (1 + \theta_{t+1})[\pi_1(K_{t+1}, S_{t+1}) + 1 - \delta - C_1(K_{t+1}, K_{t+2})] + \rho_{t+1} \tau_{t+1} \quad \text{(first envelope condition)}
\] (19)

\[
V_2(K_{t+1}, B_t, S_{t+1}) = -R_t(1 + \theta_{t+1}) \quad \text{(second envelope condition)}
\] (20)

Thus,

\[
E_t m_{t,t+1} \left\{ \frac{(1 + \theta_{t+1})[\pi_1(K_{t+1}, S_{t+1}) + 1 - \delta - C_1(K_{t+1}, K_{t+2})] + \rho_{t+1} \tau_{t+1}}{(1 + \theta_t)[1 + C_2(K_t, K_{t+1})]} \right\} = 1
\] (21)

\[-E_t m_{t,t+1} R_{t+1}(1 + \theta_{t+1}) = \rho_t - \theta_t - 1 \iff E_t \left[ m_{t,t+1} \left( R_t \frac{1 + \theta_{t+1}}{1 + \theta_t - \rho_t} \right) \right] = 1 \] (22)

Equation 21 and equation 22 are the most important to us. Equation 21 is similar to equation 14 with one term \( \rho_{t+1} \tau_{t+1} \) added in the numerator. So, asset tangibility matters for investment returns only when \( \rho_{t+1} \) is strictly positive. Equation 22, however, is a new one; it means that benefit of borrowing one dollar today (denominator) should be equal to the expected cost of that dollar (numerator). \( \theta_t \) can be interpreted as an additional marginal cost borne by the firm when it has to use external financing, that is, when the first constraint is binding. \( \rho_t \) is an additional marginal cost borne by the firm when its optimal level of debt is higher or equal to the amount of funds that can be generated by the liquidation of its assets. Thus, when the firm borrows an extra dollar, the denominator of equation 22 is equal to that dollar added to the extra benefit of reducing the possibility of facing a binding financing constraint minus the cost associated to the fact that the tangibility constraint is now more likely to be binding. The numerator \( R_{t+1}(1 + \theta_{t+1}) \) is the sum of the direct cost next period of a dollar borrowed today and the cost of increasing the possibility that the next period financing constraint binds.

From equation 21 and equation 22 it can be seen that financing frictions do not affect investment returns when \( \frac{1 + \theta_{t+1}}{1 + \theta_t - \rho_t} = 1 \) and \( \rho_t = 0 \). One main problem with the preceding condition is that for \( \{\theta_t\}_{t=1}^\infty = \{\theta\}_{t=1}^\infty \) with \( \theta > 0 \), the first condition is satisfied even though the firm is financially constrained which makes it impossible to identify financially constrained firms when the financing premium is constant.
over time if the tangibility constraint is not binding ($\rho_t = 0$). This is probably the main reason why some researchers argue that the Euler equation approach cannot be used to determine whether a firm is financially constrained or not if the firm is as constrained today as tomorrow. However, the addition of the tangibility constraint in this paper makes it possible to circumvent this problem when the firm is severely constrained. In this case $\rho_t > 0$ which is sufficient to claim that the firm is financially constrained. In fact, when the tangibility constraint is binding the firm has borrowed the maximum quantity of funds that can be supported by its assets and this will happen only when its expenses are at least equal to the sum of its cash flows and the amount of external funds it can get which is to say when the first constraint is binding ($\theta_t > 0$).

Let $\theta_t = \theta_0 + \theta h_t$ and $\rho_t \tau_t = a g_t$.

$h_t$ are the determinants of the external financing premium and $g_t$ are the determinant of the cost generated when the tangibility constraint is binding. some researchers have showed that the ratio of debt to capital $\left(\frac{B_t}{K_t}\right)$ is one of the most important determinants of financing premia and the second constraint in equation shows the same ratio will also determine whether the tangibility constraint binds or not. Based on the preceding observations $h_t$ and $g_t$ can be specified as:

$$h_t = \frac{B_t}{K_t}, g_t = \frac{\tau_t B_t}{K_t}$$

Many authors have also assumed that $\pi (K_t, S_t)$ is homogeneous of degree one which is consistent with the assumption of perfect competition. And using a quadratic adjustment cost, $C (K_t, K_{t+1}) = \frac{\theta}{2} \left(\frac{h_t}{K_t}\right)^2 K_t$, equation and equation become:

$$E_t m_{t, t+1} \left\{ \left(1 + \theta_0 + \theta \frac{B_{t+1}}{K_{t+1}} \right) \left[ \frac{\pi (K_{t+1}, S_{t+1})}{K_{t+1}} + (1 - \delta) \left(1 + c \frac{B_{t+1}}{K_{t+1}} \right) + \frac{a}{2} \left(\frac{B_{t+1}}{K_{t+1}}\right)^2 \right] + a \tau_{t+1} \frac{B_{t+1}}{K_{t+1}} \right\} = 1$$

(23)

(24)

To test whether or not financial constraints matter, we want to estimate $\theta_0, \theta, c$ and $a$ and test:

1) $H_0: a > 0, H_1: a = 0$
2) $H'_0: \theta > 0, a > 0, H'_1: \theta = 0, a = 0$

3 Econometric methodology

The parameters of the model will be estimated using the Generalized Method of Moments (GMM) with the moment conditions defined as in equation and equation Let:

$$f (X_{it+1}, \alpha) = \frac{\left(1 + \theta_0 + \theta \frac{B_{t+1}}{K_{t+1}} \right) \left[ \frac{\pi (K_{t+1}, S_{t+1})}{K_{t+1}} + (1 - \delta) \left(1 + c \frac{B_{t+1}}{K_{t+1}} \right) + \frac{a}{2} \left(\frac{B_{t+1}}{K_{t+1}}\right)^2 \right] + a \tau_{t+1} \frac{B_{t+1}}{K_{t+1}}}{\left(1 + \theta_0 + \theta \frac{B_t}{K_t}\right) \left[1 + c \frac{B_t}{K_t}\right]}$$

(25)
\[ g(L_{it+1}, \beta) = \left[ \left( R_{it} \frac{1 + \theta_0 + \theta \frac{B_{it+1}}{K_{it+1}}}{1 + \theta_0 + \theta \frac{B_{it}}{K_{it}} - a \tau} \right) \right] \]  

(27)

where

\[ X_{it+1} = \begin{bmatrix} B_{it}, I_{it}, B_{it+1}, I_{it+1}, \pi \left( K_{it+1}, S_{it+1} \right) \end{bmatrix} \]  

and \( \alpha = [\theta_0, \theta, c, a] \)  

(28)

\[ L_{it+1} = \begin{bmatrix} R_{it}, B_{it}, K_{it}, B_{it+1}, K_{it+1} \end{bmatrix} \]  

and \( \beta = [\theta_0, \theta, a] \)  

(29)

then, the moment conditions can be written as:

\[ E_t \{ m_{it,t+1} [ f(X_{it+1}, \alpha) ] - 1 \} = 0 \]  

(30)

\[ E_t \{ m_{it,t+1} [ g(L_{it+1}, \alpha) ] - 1 \} = 0 \]  

(31)

since any variable dated \( t \) is in the information set at time \( t \), lag variables should be orthogonal the previous moments which allows us to use them as instruments. Let

\[ m_{it,t+1} [ f(X_{it+1}, \alpha) ] - 1 = u_{it} \text{ where } u_{it} = \varphi_i + \eta_{it}, i = 1, ..., N; t = 1, ..., T_i \]  

(32)

\[ m_{it,t+1} g(L_{it+1}, \beta) - 1 = v_{it} \text{ where } v_{it} = \gamma_i + \epsilon_{it}, i = 1, ..., N; t = 1, ..., T_i \]  

(33)

\[ \eta_{it} \sim iid(0, \eta), E_t (\eta_{it}) = 0 \text{ where } E_t = E (\text{information set at time } t) \]  

(34)

\[ \epsilon_{it} \sim iid(0, \epsilon), E_t (\epsilon_{it}) = 0 \text{ where } E_t = E (\text{information set at time } t) \]  

(35)

\[ E_t (\varphi_i) \neq 0 \]  

(36)

\[ E_t (\gamma_i) \neq 0 \]  

(37)

\( \varphi_i \) and \( \gamma_i \) are used to capture the possibility of unobserved heterogeneity.

From the last equations we assume additive errors. However, since the dependent variable is here identically equal to one, it can be shown that assuming additive errors is equivalent to assuming multiplicative errors when we transform the model to remove the fixed effects.

At this stage to estimate the model parameters we need to use the sample moments:

\[ \frac{1}{T} \sum_{i=1}^{N} \sum_{t=2}^{T_i} (m_{it,t+1} f(X_{it+1}, \alpha) - 1) = 0, T = \sum_{i=1}^{N} T_i \]  

(38)

\[ \frac{1}{T} \sum_{i=1}^{N} \sum_{t=2}^{T_i} (m_{it,t+1} g(L_{it+1}, \beta) - 1) = 0 \]  

(39)

for \( N \) large
\[
\frac{1}{T} \sum_{i=1}^{N} \sum_{t=2}^{T_i} (m_{t,t+1} f(X_{it+1}, \alpha) - 1) \rightarrow E_t m_{t,t+1} f(X_{it+1}, \alpha) - 1 = E_t (\varphi_i + \eta_{it}) = E_t (\varphi_i) + 0 \neq 0
\] (40)

\[
\frac{1}{T} \sum_{i=1}^{N} \sum_{t=2}^{T_i} (m_{t,t+1} g(L_{it+1}, \beta) - 1) \rightarrow E_t m_{t,t+1} g(L_{it+1}, \beta) - 1 = E_t (\gamma_i + \epsilon_{it}) = E_t (\gamma_i) + 0 \neq 0
\] (41)

Thus, using equation 38 and equation 39 as our sample moments would lead to inconsistent estimators.

To get consistent estimators we will use the mean difference approach. Let:

\[
F(X_{it+1}, \alpha) = [m_{t,t+1} f(X_{it+1}, \alpha) - 1] - \left[ \frac{m_{t,t+1} f(X_{it+1}, \alpha)}{N} \right] = (\varphi_i + \eta_{it}) - (\bar{\varphi_i} + \bar{\eta_{it}}) = \eta_{it} - \bar{\eta_{it}}
\] (42)

\[
G(L_{it+1}, \beta) = [m_{t,t+1} g(L_{it+1}, \beta) - 1] - \left[ \frac{m_{t,t+1} g(L_{it+1}, \beta)}{N} \right] = (\gamma_i + \epsilon_{it}) - (\bar{\gamma_i} + \bar{\epsilon_{it}}) = \epsilon_{it} - \bar{\epsilon_{it}}
\] (43)

\[
\frac{m_{t,t+1} f(X_{it+1}, \alpha)}{N} = \frac{1}{N} \sum_{i=1}^{N} m_{t,t+1} f(X_{it+1}, \alpha)
\] (44)

\[
\frac{m_{t,t+1} g(L_{it+1}, \beta)}{N} = \frac{1}{N} \sum_{i=1}^{N} m_{t,t+1} g(L_{it+1}, \beta)
\] (45)

With the preceding transformations we have:

\[
\frac{1}{T} \sum_{i=1}^{N} \sum_{t=2}^{T_i} F(X_{it+1}, \alpha) \rightarrow E_t [F(X_{it+1}, \alpha)] = E_t (\eta_{it} - \bar{\eta_{it}}) = 0
\] (46)

\[
\frac{1}{T} \sum_{i=1}^{N} \sum_{t=2}^{T_i} G(L_{it+1}, \beta) \rightarrow E_t G(L_{it+1}, \beta) = E_t (\epsilon_{it} - \bar{\epsilon_{it}}) = 0
\] (47)

As signalled the orthogonality of the lag variables to the moment conditions give access to a large set of instruments. However, since we are estimating only 4 parameters, we only need at least 4 instruments. At first we have decided to choose as instruments the following variables: 1(vector whose elements are the number 1) \( \frac{B_{K_{it}}}{K_{it}} \) and \( \frac{\pi(K_{it}, S)}{K_{it}} \) for the first equation, and the vector \( 1_{R_{it-1}} \) and \( \frac{B_{K_{it}}}{K_{it}} \) for the second equation. We then have 7 moment conditions to estimate four parameters. The natural approach is then the Generalized Method of Moments. Suppose the vector \( M \) is the vector of moment conditions:

\[
M = \\
\begin{bmatrix}
E_t (F(X_{it+1}, \alpha)) \\
E_t \left( \frac{B_{K_{it}}}{K_{it}} F(X_{it+1}, \alpha) \right) \\
E_t \left( \frac{\pi(K_{it}, S)}{K_{it}} F(X_{it+1}, \alpha) \right) \\
E_t (G(L_{it+1}, \beta)) \\
E_t (R_{it-1} G(L_{it+1}, \beta)) \\
E_t \left( \frac{B_{K_{it}}}{K_{it}} R_{it} G(L_{it+1}, \beta) \right)
\end{bmatrix}
\]
We can obtain the parameters by solving:

\[
\text{Minimize} M' W M \quad (\theta_0, \theta, c, a)
\]

where \( W \) is a weighting matrix of dimension 7X7.

Looking at the preceding problem the first question that comes to mind is whether or not there exists a unique vector \((\theta_0, \theta, c, a)\) that solves the preceding problem which leads us to the issue of identification. If we could guarantee the strict convexity of the objective function, we would be able to guarantee the identification of the model parameters; however, this would involve messy algebra. Nevertheless, from equation 24 and equation 25 it can be seen that if \( \theta = 0 \) and \( a = 0 \), the model is not identified since the moment conditions would involve the term \( \frac{1+\theta_0}{1+\theta_0} \) which is equal to one for any real number different from minus one (-1). Which implies that we need to avoid starting values with \( \theta = 0 \) and \( a = 0 \). If \( a = 0 \) and the sequence \( \{B_t\} \) is constant, neither \( \theta \) nor \( \theta_0 \) is identified which implies that we cannot use a starting vector with \( a = 0 \).

4 Data

The data used in this study is obtained from COMPUSTAT (the industrial file) for the period going from 1980 to 2005. More specifically the dataset is made of annual observation for firms whose SIC code is between 1000 and 3999. Since most of the variables in this study are normalized by the firm’s stock of capital (data item 8), observations associated to a missing capital stock as well as observations with a reported capital stock of zero are deleted. Table I shows that such observations represented more than 50% of the observations.

<table>
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<th>Capital = 0</th>
</tr>
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</tr>
<tr>
<td>Percentage</td>
<td>55.1%</td>
</tr>
</tbody>
</table>

We have dropped 6,342 firm-year observations where the growth rate of the firm’s assets (data item 6) exceeds 100%. A big increase in a firm’s asset is in fact more likely to come the acquisition of another firm and is then incompatible with our model that is about incremental changes in a firm’s stock of capital. We also dropped 406 firms that have only 1 observation, and 734 firms with only 2 observations because it is not possible to use lag variables as instruments for the given firms. We also dropped 180 firms that show depreciation (data item 14 times 2 divided by the sum of data item 7 and data item 30) higher or equal to 1, a total of 1709 observations. It is in fact assumed that a firm’s stock of capital cannot become negative. The firms’ profits are obtained by adding income before extraordinary items (data item 18) and depreciation (data item 14). Investment is measured as capital expenditure (data item 30). Short term debt and long term debt correspond respectively to data item 34 and data item 9, while the variable interest is data item 15 divided by the sum of short term and long term debt. The final dataset is divided in four groups according to the variable tangibility defined as in Almeida and Campello (2006):

\[
\text{tangibility} = 0.715*\text{Receivables} + 0.547*\text{Inventory} + 0.535*\text{capital}
\]
Where Receivables is data item 2, Inventory, data item 3, and Capital, data item 8. The preceding measure is normalized by the book value of the firm’s total assets.

Even though the dropped observations consist mainly of outliers, the summary statistics presented in tables 2 shows that the standard deviation of most of the variables is very big with respect to their mean and their median suggesting that many outliers remain in the data set. It should be noted that the ratio of profit to capital shows a negative mean with a and a large standard deviation. However, if the observations located in the tails (approximately 1%) of the distribution, the ratio becomes positive with a much smaller variance.

Table 2: These statistics are obtained after applying the dropping rules provided in the text. Med = Median, Std = standard deviation.

<table>
<thead>
<tr>
<th>All firms</th>
<th>Mean</th>
<th>Med</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangibility</td>
<td>0.39</td>
<td>0.43</td>
<td>0.14</td>
</tr>
<tr>
<td>Short term debt to capital ratio</td>
<td>1.86</td>
<td>0.08</td>
<td>56.94</td>
</tr>
<tr>
<td>Long term debt to capital ratio</td>
<td>2.16</td>
<td>0.35</td>
<td>42.07</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.34</td>
<td>0.22</td>
<td>2.29</td>
</tr>
<tr>
<td>Profit to capital ratio</td>
<td>-2.91</td>
<td>0.18</td>
<td>105.26</td>
</tr>
<tr>
<td>Investment to capital ratio</td>
<td>0.75</td>
<td>0.19</td>
<td>18.05</td>
</tr>
<tr>
<td>Interest</td>
<td>0.10</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Depreciation</td>
<td>0.18</td>
<td>0.15</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 3: The groups are formed after ordering the firms according to asset tangibility and dividing them in four group of approximately equal size. The first group has the highest asset tangibility and the last group has the lowest asset tangibility. Med = Median, Std = Standard deviation.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Mean</td>
<td>Med</td>
<td>Std</td>
</tr>
<tr>
<td>Tangibility</td>
<td>0.42</td>
<td>0.29</td>
<td>1.82</td>
</tr>
<tr>
<td>Short term debt to capital ratio</td>
<td>1.65</td>
<td>0.11</td>
<td>31.67</td>
</tr>
<tr>
<td>Long term debt to capital ratio</td>
<td>3.31</td>
<td>0.74</td>
<td>66.04</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.42</td>
<td>0.29</td>
<td>1.82</td>
</tr>
<tr>
<td>Profit to capital ratio</td>
<td>-2.01</td>
<td>0.13</td>
<td>40.77</td>
</tr>
<tr>
<td>Investment to capital ratio</td>
<td>0.67</td>
<td>0.18</td>
<td>17.33</td>
</tr>
<tr>
<td>Interest</td>
<td>0.24</td>
<td>0.09</td>
<td>4.16</td>
</tr>
<tr>
<td>Depreciation</td>
<td>0.21</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>Group 1</td>
<td>Group 2</td>
<td>Group 3</td>
<td>Group 4</td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
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<tr>
<td>-1.0000</td>
<td>3.9477</td>
<td>12.6744</td>
<td>-1</td>
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<tr>
<td>0.0000</td>
<td>-0.0271</td>
<td>-0.0130</td>
<td>0.0000</td>
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<tr>
<td>0.0056</td>
<td>0.0009</td>
<td>-0.0044</td>
<td>0.0399</td>
</tr>
<tr>
<td>0.0000</td>
<td>10.4707</td>
<td>29.1047</td>
<td>0.0000</td>
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<td>Moments</td>
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<td></td>
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<tr>
<td>0.0000</td>
<td>-0.0000</td>
<td>-0.0000</td>
<td>-0.0000</td>
</tr>
<tr>
<td>-3081</td>
<td>-617.6578</td>
<td>7.6928</td>
<td>-824.9432</td>
</tr>
<tr>
<td>-260</td>
<td>0.0044</td>
<td>5.3396</td>
<td>4.9944</td>
</tr>
<tr>
<td>-10011</td>
<td>-256.9741</td>
<td>-263.3566</td>
<td>332.8353</td>
</tr>
<tr>
<td>0</td>
<td>-0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>592</td>
<td>-2.3952</td>
<td>-0.2327</td>
<td>0.6096</td>
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<tr>
<td>-1</td>
<td>0.4489</td>
<td>-0.0033</td>
<td>-0.9522</td>
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<td>Standard errors</td>
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<tr>
<td>0.0000</td>
<td>346.9728</td>
<td>29844</td>
<td>0.0000</td>
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<tr>
<td>0.0000</td>
<td>1.9018</td>
<td>28</td>
<td>0.0000</td>
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<tr>
<td>0.0013</td>
<td>0.0000</td>
<td>0</td>
<td>0.0118</td>
</tr>
<tr>
<td>0.0000</td>
<td>734.2871</td>
<td>63520</td>
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</tr>
<tr>
<td>Value of GMM objective function</td>
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</tr>
<tr>
<td>0.0078</td>
<td>0.01107</td>
<td>0.0064</td>
<td>0.0067</td>
</tr>
</tbody>
</table>

5 Results

Since the model is highly non-linear, the GMM estimation requires the use of iterative methods whose performance may be affected by the starting values. In general, the most frequent source of starting values is the linear version of the non-linear model considered. However, in our case there is no natural point around which to linearize the model. Moreover, the coefficients of the linear model would not be really useful since they are supposed to be complicated functions of the coefficients of the non-linear model. This observation makes it difficult to find a suitable starting vector of coefficients. Since we are trying to minimize the GMM, it appears to be necessary to choose starting vectors such that the Hessian matrix of the GMM objective function is positive definite. However, all the starting values that we were able to try produce Hessian Matrices with at least one negative eigenvalue. The results obtained do not appear to be interesting since the estimated standard deviations are very big. While errors coming from the use of numerical differentiation in the estimation of the gradient of the moment conditions may contribute to the big standard deviations, the problem of identification may also be responsible for the imprecision of the estimates.

6 Conclusion

In the absence of the identification of the model parameters, it is impossible to test the hypotheses specified in this paper and to assess how well the model fit to the given data set. The model is very much likely to be misspecified, but non-linearity plays a significant role in complicating its estimation.
References


Figure 1: Median of Short term Debt to Capital Ratio for all the four groups
Figure 2: Mean of Short term Debt to Capital Ratio for all the four groups

Figure 3: Median of Long term Debt to Capital Ratio for all the four groups
Figure 4: Mean of Long term Debt to Capital Ratio for all the four groups

Figure 5: Median interest rate for all the four groups
Figure 6: Mean interest rate for all the four groups

Figure 7: Median of Investment to Capital Ratio for all the four groups
Figure 8: Mean of Investment to Capital Ratio for all the four groups

Figure 9: Median of Profit to Capital Ratio for all the four groups
Figure 10: Median of Profit to Capital Ratio for all the four groups

Figure 11: Median of depreciation for all the four groups
Figure 12: Mean depreciation for all the four groups

Figure 13: Median tangibility for all the four groups
Figure 14: Mean tangibility for all the four groups