Captial-Skill Complementarity and Optimal Education Policy

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ABSTRACT

This paper analyses the general equilibrium implications on public education policies in an economy with heterogeneous agents, human capital investment and capital-skill complementarity. Human capital is accumulated through private and public education inputs. Workers choose between being goods workers or human capital workers and are taxed to support public education. The government hires human capital workers to maximize education quality given tax revenue. Due to collective bargaining in the human capital market, the government has a trade-off between the quantity and quality of human capital it can afford. We show that income inequality increases even when education quality increases and that as income inequality increases, it is optimal for the government to increase the number of human capital workers but with lower within short human capital levels.

1. Introduction

The supply and relative price of teachers has changed significantly over the last forty years. Many account this change to education policy which has increased the teacher-student ratio. At the same time, the market for skilled labor has also changed considerably. Krussell et al. (2000) document that there has been a large increase in the quantity of skilled labor relative to that of unskilled labor, and that the skill premium, defined as the wage of skilled labor relative to that of unskilled labor, has grown significantly since 1980. This paper is motivated by two questions: (1) How does government education policy change when the skill premium rises? and (2) Is there a relationship between the seemingly unrelated skill premium, teacher-student ratio, and teacher aptitude? The main conclusion of this paper

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is that as the skill premium rises, teachers with the same human capital level become more costly, and it is optimal for the government to employ more teachers with lower human capital levels.

This paper complements many existing papers concerning the effects of skill premium on economies. Galor and Maov (2000) find that as technical progress increases, the wage differential between and within groups of skilled/unskilled has widened. This is similar to the results of Acemoglu (1998) & (2000). Stokey (1996) finds that as physical capital increases, the skill premium increases and accelerates human capital accumulation. These effects will be the same effects that drive education policy towards hiring more teachers at a lower human capital level.

In the human capital literature, the main focus has been on human capital inequality, economic growth, and the effects on future distributions of human capital allowing human capital to be perfectly substitutable. The education production function frequently used is a measure of education quality per pupil as a function of output and/or the student-to-teacher ratio (See Glomm & Ravikumar 1992, Blanenau & Simpson, 2004, Benabou 1996, Gradstein & Justman 1996). This functional form’s main features are rising education quality with rising output and rising education quality with rising teacher-student ratio. A natural next step is to improve on the definition of educational quality per pupil. This can be done by separating the quantity-quality aspects of human capital and to learn more on how the government’s optimal policy reacts to skill premium. Many empirical studies separate education quality on quantity and quality dimensions (see Hanushek & Leque, 2003; Hanushek et al., 2005; Ingersoll, 1997). However, the implications of skill premium on the market for teachers still have not been investigated.

The main conclusion is that the rise in the teacher-student ratio has accompanied lower human capital levels of teachers due to rising skill premium. The skill premium in this paper is assumed to come from capital-skill complementarity. This causes the skilled laborer’s income up and due to a union wage structure in the teacher market, all teacher must be paid more. In respecting the government’s budget constraint, this causes optimal educational policy to change to hiring not as high of human capital but more of them to compensate. The mechanism that produce the results is as follows: when the physical capital stock is low, then higher ability individuals choose to teach as they can gain a higher wage in teaching then in the private sector. As the physical capital stock grows causing the marginal wage in the goods sector to rise for skilled individuals, this entices individuals at the higher end of human capital amongst teachers to switch from being teachers to private sector workers as they can now gain a higher income. This will cause the human capital of teachers to fall over time as more higher human capital teachers leave for the goods production sector. In
terms of the intertemporal human capital level, the human capital of teachers is increasing over time. At the same time, the relative human capital level of teachers to other skilled workers decreases over time.

The paper is organized as follows. In Section II, the relevant data and literature is explored. In Section III, a closed economy is developed. The main features of this economy are capital-skill complementarity and collective bargaining in the teacher’s market. Government policy considered are the aptitude or ability requirement for entering into teaching and the salary offered to all teachers. Individuals must decide in this economy which occupation to work in: a goods worker or a teacher. Households with children can also choose to supplement child’s education. Section IV talks about the parameter values chosen for simulation and the results. Section V finish with the main conclusions.

2. The Data

There are six data facts for the last forty years that we would like to highlight. They are, (1) teachers have preferred to bargain collectively for wages, (2) the teacher-student ratio has increased, (3) teacher salary has declined relative to pay of other college educated peers, (4) the percent of GDP towards education has increased by 1%, (5) teacher aptitude, relative to their peers, has declined, and (6) skill premium has risen.

(1) Teachers have preferred to bargain collectively for wages then individually for the last 40 years. The first reason is how would teachers be paid based on marginal productivity. It is not clear how one measures these gains with different human capital levels as each level has different marginal productivities. If marginal productivity is based on the marginal increase in human capital of the students, then tracking of students performance is the first step in measuring teachers productivity. Hanushek et al (2005) address this issue when estimating the value-add of teachers when students are tracked over their academic career. They find little relationship between teacher characteristics and gains on test score except with drops in scores from first year teachers and matching of teacher/student ethnicities. Epple, Newlon, and Romano (2002) studied effects of tracking students ability in a theoretical model when students were assigned to a classroom based on test performance. They find that tracking has small aggregate effects on achievement and welfare. From the empirical literature, it is not clear how much and to whom the gains can be attributed.

Another difficulty in attributing increases or decreases in student scores to teachers if students receive shocks throughout the year that may not be accounted for or no exact way of measuring how they will affect students. If a student has an undiagnosed head injury
that lowers their performance, then his teacher will receive a deduction in pay due to a non-
performance related issue. Even if the shock was known, each student may react differently
in terms of test performance. Shocks such as the Columbine massacre and Hurriance Katrina
are known but the actual effect on each students’ performance seems impossible to track.

A third difficulty is the tracking of additional private education supplements by parents
and organizations. A student’s marginal gain in a test score may be due to a combination of
their teacher and the private education supplements. This means that the tracking process
for students would also have to track private supplements for all students.

For these reason, it is extremely implausible that teachers will prefer individual bar-
gaining over collective bargaining over based on some value-added calculation done by the
government. It also should be noted that private teachers’ pay roughly follows the pay scale
of public school teachers in the same school district, and in some cases they receive lower
monetary benefits than the public school teachers.

The unionization of teachers does not come without a cost. First, there is an automatic
wage inflation for lesser ability teachers that could have been paid less. Second, wages have
been linked to tenure for teachers by the union. If the government knew they had a poor
teacher, they couldn’t remove the teacher from the human capital production.

(2) Over the last forty years, the teacher-student ratio has risen. Glomm and Ravikumar
(2001) display data on teachers per 1,000 students for Canada and the United States from
1900 to 1990. This figure is replicated in the appendix. The data indicates that the number
of teachers increased steadily from 1900 to 1940 and increased more rapid during 1960 to
1990 with Canada outpacing the U.S from 1985 onward.

(3) Over the last forty years, teacher have been losing ground on their salaries compared
to their peers with a college degree. Up to around 1955, 50% of the female workforce earned
less then female teachers. From 1960 to present, this has fallen to 37%. Male teachers earned
significantly less in 1955 with only 38% of college educated men earning less then them. This
also has declined to roughly 29% (see figure 1 of Hanushek and Revkin, 2003).

(4) Over the last forty years, the percent of GNP allocated towards education has
increased by 1%. Rangazas (2002) estimates the share of GDP in 1960 to be 3.2% and 4.1%
in 1980. Glomm & Ravikumar (2001) estimate the share of GNP in 1960 to be 4% and 5.5%
in 1990. Either estimate reveals that spending has increased by roughly 1%.

(5) Teacher aptitude has declined over the last forty years relative to peers. Hoxby and
in 1963 41% of all teachers were of middle aptitude relative to their peers with 17% above
average and 42% below average. This has declined significantly after forty years with 67% of the all teachers being in the lowest aptitude and 28% being middle aptitude relative to their peers. Only 5% were above average aptitude (see Table 1).

(6) The skill premium has risen over the last thirty years. Krusell et al (2000) estimate the skill premium for the U.S from 1963 to 1990. Normalizing the skill premium to 1 in 1963, the skill premium has risen to 1.16. This however came after a sharp decline in skill premium then a rapid rise from 1980 onward.

3. The Model

We develop a simple overlapping generations model of growth. The economy is populated by a sequence of three-period lived within generation heterogenous individuals, a representative firm producing a single consumable good, a technology sector producing human capital, and a government that finances and administers public education. The government is able to set a human capital entry requirement for new teachers in a human capital sector and set a salary of all teachers that is irrespective of ability. It is assumed that there is no population growth and a continuum of individuals are born in each generation $G_t$ on the interval $[0, 1]$. We refer to an individual in the first period of life as a learner, in the second period as young, and the final period as old.

In the first period of life, learners depend on their parents and only gain education through public schooling and private educational supplements. At the beginning of the second period, the young have a child, choose to work in either the private goods sector or the human capital technology sector if they qualify, and save for retirement. The choice of occupation is based solely on maximization of income. If a young individual chooses to work in the goods sector, he will earn a competitive market income, $I^g_{it}$, based on his human capital level. If a young individual has the required level of human capital to qualify him to work in the human capital sector and choose to work in this sector, he will earn an income, $I^h_t$. This will be at least what they would have earned in the goods sector and does not differ across human capital sector workers. In the last period of life, individuals are old, retire from work, and consume off of their savings from young.

All individuals born at $t = 0, 1, 2, 3, ...$ have identical preferences over consumption and children's human capital. Formally, the preferences of an individual $i$ born at time $t - 1$ are represented by

$$U(c_{it}, c_{it+1}, h_{it+1}) = \ln c_{it} + \beta \ln c_{it+1} + \gamma \ln h_{it+1},$$

where $c_{it}$ is consumption when young at time $t$, $c_{it+1}$ is consumption when old at time $t + 1$.
and \( h_{it+1} \) is his child’s human capital.

The goods production uses three factors of production - physical capital \( K_t \), unskilled labor \( H^u_t \), and skilled labor \( H^s_t \) to produce a homogenous output. The production technology has constant returns to scale and the output can be used for consumption, private education supplements, or physical capital. The aggregate production function is

\[
Y_t = A \left[ aK_t^\theta + (1-a) \left[H^u_t\right]^\theta \right] \left[ H^u_t \right]^{1-\alpha},
\]

where \( \theta, a, \alpha \in [0,1] \) and \( A > 0 \). Physical capital and unskilled workers are combined together into an aggregate using a CES technology with an elasticity of substitution of \( 1/(1-\theta) > 1 \). If \( \theta = 1 \), they are perfect substitutes. This aggregate is then combined with human capital in a Cobb-Douglas technology with parameter \( \alpha \).

This technology allows physical capital and unskilled labor to be perfect substitutes and skilled labor is complementarity to both unskilled and physical capital. The parameter \( e \) adjusts the skill premium. If \( e = 0 \), then skilled labor’s share of output is fixed at \( 1 - \alpha \). \( e > 0 \) decreases the skilled labor’s share of output and allows unskilled wages to increases when the capital stock increases. If \( e < 1 \), then wages for unskilled will not rise as rapidly for unskilled labor as skilled labor.

The human capital \( h_{it+1} \) of a young individual is produced by combining children’s ability, \( a_{it} \), with public education, \( E_t \), and private parental education supplements, \( \Theta_{it} \), according to the law of motion

\[
h_{it+1} = B \left( E_t + b \Theta_{it} \right)^\nu h_{it}^{1-\nu}, \quad \nu \in [0,1] \text{ and } B > 0.
\]

Public education is assumed to be financed through a proportional income tax, \( \tau_t \), on all workers.

Heterogeneity of individuals arises from the initial young’s human capital endowment. The initial young are endowed with \( h_{i0} \) units of human capital distributed lognormally and the initial old individuals are endowed with \( s_0 \) units of physical capital. The lognormal distribution has parameters \( m_0 > 0 \) and \( s_0 < 1 \) with density \( f(a_{i0}; m_0, s_0) = \frac{1}{a_{i0}s_0 \sqrt{2\pi}} e^{-\left(\ln a_{i0} - m_0\right)/\left(2s_0^2\right)} \). Unskilled laborers have human capital levels below the 30th percentile. This threshold is labeled \( h^* \).

3.1. Households

The first choice young skilled individuals make is what sector to work in. If individual \( i \) chooses to work in the goods production, he supplies \( h_{it} \) units of human capital inelastically
to the goods sector. Given the wage rate at time \( t \), \( w^s_t \), the individual’s income is \( I^s_{it} = w^s_t h_{it} \).

If individual \( i \) has a human capital level of \( h_{it} > h_t \) and chooses to work in the human capital sector, then he supplies \( h_{it} \) units of human capital inelastically to the human capital sector and receives income \( I^h_t = w^h_t h_{it} \), where \( h_t \) is the highest human capital level in the human capital sector. The occupational choice for skilled labor is defined by the binary variable \( \phi_{it} \) such that

\[
\phi_{it} = \begin{cases} 
1 & \text{if } i \text{ works in the human capital production sector} \\
0 & \text{if } i \text{ works in the goods production sector}
\end{cases}.
\]

Thus income of skilled labor is defined as

\[
I_{it} = (1 - \phi_{it}) I^s_{it} + \phi_{it} I^h_t.
\]

Unskilled individuals have no choice of occupation and work solely in the goods sector. These individuals provide \( h_{it} \) units of human capital inelastically to the goods sector. Given the wage rate at time \( t \), \( w^u_t \), unskilled individual’s income is \( I^u_{it} = w^u_t h_{it} \).

We can now define a skilled individual’s maximization problem. Individual \( i \) chooses his occupation, consumption when young and old, private educational supplements, and savings decisions to solve

\[
\max_{\phi_{it}, s_{it}, \Theta_{it}} U(c_{it}, c_{it+1}, h_{it+1})
\]

subject to the following constraints

\[
c_{it} = (1 - \tau_t) I_{it} - s_{it} - \Theta_{it} \\
c_{it+1} = R_{t+1} s_{it} \\
h_{it+1} = B (E_t + b \Theta_{it})^{\nu} h^\nu_{it} \\
I_{it} = (1 - \phi_{it}) I^s_{it} + \phi_{it} I^h_t
\]

given

\[
(I^s_{it}, I^h_t, R_{t+1}, E_t, \tau_t)
\]

where \( s_{it} \) is savings from period \( t \) and \( R_{t+1} \) is the return on savings at time \( t + 1 \).

An unskilled individual \( i \)‘s maximization problem is choosing consumption when young and old, private educational supplements, and savings decisions to solve

\[
\max_{s_{it}, \Theta_{it}} U(c_{it}, c_{it+1}, h_{it+1})
\]
subject to the following constraints

\begin{align*}
    c_{it} &= (1 - \tau_t) I_{it}^u - s_{it} - \Theta_{it} \\
    c_{it+1} &= R_{t+1} s_{it} \\
    h_{it+1} &= B (E_t + b \Theta_{it})^\nu h_{it}^\sigma \\
    I_{it}^u &= w_{it}^a h_{it}
\end{align*}

given

\[(w_{it}^u, R_{t+1}, E_t, \tau_t).\]

If we assume each individual has measure zero effect on the human capital production process, the household’s maximization problem is solved using a two step process. Individuals first choose their occupation, \( \phi_{it} \), then consumption, savings, and educational supplement decisions. This technique is permitted because the utility function only reallocates earnings saved when young to old and individuals choose \( \phi_{it} \) based on the maximization of income. This assumption rules out the possibly of individuals increasing their child’s human capital by working in the human capital sector for less income then maximizing income and increasing child’s human capital through private supplements.

### 3.2. Production

There are two sectors in this economy. The goods sector is private and produces a consumable output using labor and physical capital as inputs. The human capital sector is run by the government and produces human capital using only labor as an input. I will address the human capital sector first as the goods sector takes government policy as given while maximizing profit.

#### 3.2.1. The Human Capital Sector

The government’s purpose in this economy is the provide public education. It is not assumed the government knows how human capital is completely formed for an individual while they maximize education quality. The public education produced by the government augments childrens’ abilities and parents’ private education supplements. Public education, \( E_t \), is solely a function of the quality and quantity of human capital workers. We assume
that public education quality per pupil takes the form

\[ E_t = \int^{\bar{h}_t}_{\underline{h}_t} h_{it}dG_t(h_t). \]

The total number of teachers each period is defined as

\[ \chi_t = \int^{\bar{h}_t}_{\underline{h}_t} \phi_{it}dG_t \text{ where } \chi_t \in [0, 1]. \] (6)

Since each generation has been normalized to 1, it should be noted that aggregate values and per capita values are the same. This means that \( E_t \) is total education quality and per pupil education quality and \( \chi_t \) is total number of teachers and the teacher-student ratio. \( E_t \) can be interpreted as the total stock of human capital in the human capital sector.

### 3.2.2. The Goods Sector

The goods sector consists of a firm that represents a large number of firms in a perfectly competitive market with identical technology that produce final goods. The final good can be used for consumption or physical capital investment. Goods are produced using physical capital and human capital of workers. In our model we assume that laborers with different human capital levels are not perfectly substitutable and are aggregated by unskilled and skilled human capital types.

The level of human capital of an individual determines where they are best suited in the production of output. If an individual has human capital level of \( h < h^* \) then they are assigned to unskilled labor, \( H^u_t \). If they have \( h \geq h^* \), then they are assigned to skilled labor, \( H^s_t \).

This yields the following two human capital stocks

\[ H^u_t = \int_0^{h^*} h_{it}dG_t(h_t) \] (7)

\[ H^s_t = \int_{h^*}^{1} h_{it}dG_t(h_t) + \int_{\underline{h}_t}^{h^*} h_{it}dG_t(h_t) \] (8)

Thus, the firm’s problem is

\[ \max_{\kappa_t, H^u_t, H^s_t} Y_t - q_t K_t - w^u_t H^u_t - w^s_t H^s_t \]

given

\( (w^u_t, w^s_t, q_t) \) and \( Y_t \).
In the production function, the Allen-Uzawa partial elasticity of substitution between capital and skilled labor $\sigma_{KH} = 1$. Under the condition that $\theta$ is strictly greater than zero, $\sigma_{KH} > \sigma_{KH}$ and the production function exhibits capital-skill complementarity.

Given $H_t^s$, $H_t^u$, and $K_t$, the interest rate is

$$r_t = A\alpha \left[ aK_t^\theta + (1 - a) [H_t^u]^\theta \right]^{-1} [H_t^s]^1-\alpha K_t^\theta-1$$

and competitive skilled and unskilled wages are

$$w_t^s = A(1 - \alpha) \left[ aK_t^\theta + (1 - a) [H_t^u]^\theta \right]^{-\alpha} [H_t^s]^{-\alpha}$$

$$w_t^u = A\alpha (1 - a) \left[ aK_t^\theta + (1 - a) [H_t^u]^\theta \right]^{-1} [H_t^s]^{1-\alpha} [H_t^u]^{\theta-1}.$$ 

Using the wages, the skill premium is the ratio of skilled to unskilled workers’ wages

$$z_t = \frac{(1 - \alpha)}{\alpha (1 - a)} \left[ aK_t^\theta + (1 - a) [H_t^u]^\theta \right] [H_t^s]^{-1} [H_t^u]^{-1-\theta}.$$ 

The skill premium comes directly through capital-skill complementarity. The capital-skill complementarity can be seen by differentiating $z_t$ with respect to $K_t$. Since the derivative is greater than zero, the relative productivity of skilled labor is increasing in the amount of capital as is across group inequality.

### 3.3. Government

The government’s purpose is to finance and administer public education, $E_t$. The government has two policies to accomplish this: income of human capital sector workers and the minimum requirement of human capital, $h_t$, in the human capital sector. By the functional form of $E_t$, human capital is perfectly substitutable in this sector. This implies the government does not impose a strict structure on quantity or quality of human capital during the maximization of $E_t$.

The government’s first policy is to administer a costless screening of all potential human capital workers to ensure a minimum human capital level. Potential human capital workers must have a minimum human capital capability, $h_t$, in period $t$ (See Angrist and Guryan, 2004). The government’s second policy is to set the salary in the human capital sector. Since they only offer one income in the human capital sector, this forces it to be equal to what the highest human capital worker, $\bar{h}_t$, in the human capital sector would have received in the
goods sector; that is, they set all human capital worker’s income equal to what his income in the goods sector would have been if he had chosen to be a goods sector worker. This can be thought of as a wage premium due to collective bargaining.

The government may not borrow or lend and must balance its budget each period. Total revenue is equal to taxes on income and total government expenditures is total teacher salary. This can be written as the government budget constraint as

\[
\chi_t I_t^h = \tau_t \int_0^{h^*} I_h^y dG_t(h_t) + \tau_t \int_{h^*}^{\bar{h}_t} I_h^y dG_t(h_t) + \tau_t \int_{\bar{h}_t}^{1} I_h^y dG_t(h_t) + \tau_t \chi_t I_t^h.
\] (11)

The government’s objective is to maximize education quality, \( E_t \), using \( h_t \) and \( I_t^h \) as policies while satisfying its budget constraint and takings as given the private sector’s response to changing government policy. Since \( I_t^h = w_t^y h_t \), choosing \( I_t^h \) or \( \bar{h}_t \) is identical in this model. \( h_t \) can be set directly as it is government policy. \( \bar{h}_t \) is not government policy but the government sets \( I_t^h \) to assure that the workers with human capital between \( \bar{h}_t \) and \( h_t \) choose to work in the human capital sector as they earn higher wages then in the goods sector.

We can now state the government’s maximization problem

\[
\max_{h_t, I_t^h} E_t
\]

subject to

\[
(1 - \tau_t) \chi_t I_t^h = \tau_t \int_0^{h^*} I_h^y dG_t(h_t) + \tau_t \int_{h^*}^{\bar{h}_t} I_h^y dG_t(h_t) + \tau_t \int_{\bar{h}_t}^{1} I_h^y dG_t(h_t)
\]

\[
H_t^s = \int_{h^*}^{\bar{h}_t} h_t dG_t(h_t) + \int_{\bar{h}_t}^{1} h_t dG_t(h_t)
\]

It is important to note that the private sector will react to government policy. The human capital stock in the goods sector is directly effected by the individuals the government allows to work in the goods sector. This in turn effects the wage rates on unskilled and skilled human capital and the rental rate on physical capital.

4. Definition of Equilibrium

A competitive equilibrium is a collection of sequences of individuals’ occupation choice \( \{\phi_{it}\}_{i=0}^{\infty} \) for all \( i \) and decisions of individual consumption, savings, and private education
supplements \( \{c_{it}, c_{it+1}, \Theta_{it}, s_{it}\}_{i=0}^{\infty} \) for all \( i \), a sequence of outputs and aggregate inputs of physical and human capital \( \{K_t, H_t^y\}_{i=0}^{\infty} \) chosen by the firm each period, government policies \( \{\tau_t, h_t, \widetilde{h}_t\}_{i=0}^{\infty} \), prices \( \{w_t, R_{t+1}\}_{i=0}^{\infty} \), and the initial distribution of human capital, \( h_{i0} \) and savings \( s_{i0} \) such that

(i) \( \{c_{it}, c_{it+1}, \Theta_{it}, s_{it}\}_{i=0}^{\infty} \) solves the household’s maximization problem for all \( i \) given government policies and prices,

(ii) \( \{K_t, H_t^y\}_{i=0}^{\infty} \) solves the firm’s maximization problem given prices,

(iii) \( \{h_t, \widetilde{h}_t\}_{i=0}^{\infty} \) solves the government’s maximization problem

(iv) the stock of human capital follows the law of motion

\[
h_{it+1} = B (E_t + b \Theta_{it})^\nu \frac{h_{it}^{1-v}}{v} ,
\]

(v) Factor prices are determined competitively

(vi) the capital market clears: \( K_{t+1} = (1 - \delta) K_t + S_t \), where \( S_t = \int s_{it} dG_t(h_t) \)

(vi) the goods market clears: \( Y_t = C_{it} + C_{it-1} + S_t \),

(vii) the labor market clears:

\[
\chi_t = \int \varphi_{it} dG_t(h_t)
\]

\[
1 - \chi_t = \int \varphi_{it} dG_t(h_t)
\]

(viii) \( \{\tau_t\}_{i=0}^{\infty} \) is set exogenously from the model.

5. Inspecting the Mechanism

Before exploring the full model, we first use a simple model to demonstrate the mechanism behind changing optimal education policy. In this section, we omit physical capital and allow all workers to be skilled working in either the human capital sector or home production with technology \( y_{it} = h_{it} \). It should be noted that the household decisions rules will remain as the fully specified model. These rules for private savings and education supplements are respectively

\[
s_{it} = \begin{cases} 
\frac{(\beta + \beta^2 + \gamma v)(1-\tau_t) I_{it}}{(1+\beta)(1+\gamma v+\beta)} - \frac{\beta E_t}{b(1+\gamma v+\beta)} & \text{if } \frac{b(\beta + \beta^2 + \gamma v)(1-\tau_t) I_{it}}{(1+\beta)(1+\gamma v+\beta)} > E_t \\
0 & \text{otherwise}
\end{cases}
\]

\[
\Theta_{it} = \begin{cases} 
\frac{\gamma v(1-\tau_t) I_{it}}{(1+\gamma v+\beta)} - \frac{(1+\beta) E_t}{b(1+\gamma v+\beta)} & \text{if } \frac{b\gamma v(1-\tau_t) I_{it}}{(1+\beta)} > E_t \\
0 & \text{otherwise}
\end{cases}
\]

As stated previously, the policy variables the government controls are the human capital requirement in the human capital sector, $h$, and the salary of teachers, $I^h$. The government’s problem is illustrated in Figure 2 for a log normal human capital distribution with parameters $\mu = 3.73$ and $\sigma = .53$, a 5% tax rate, and $h^* = 0$. In this example, human capital is the same as income. Figure 2a graphs the human capital levels of the current young entering into workforce from lowest human capital level to highest. This figure highlights three policy sets, $(h, I^h)$, the government can choose while satisfying the government’s budget constraint. Due to the rigid wage structure in the human capital sector, workers receive a premium above their human capital levels which produces a dead weight loss. As $h$ is lowered, the dead weight loss increases as new workers are not as capable as the existing workers. As $h$ is raised, this also increases the dead weight loss as the government now pays all workers a higher salary. This stops the government from hiring the lowest (highest) human capital individuals as it requires a large (small) number of them but is more costly then the interior of the distribution where the dead weight loss is minimized.

Figure 2b & 2c graphs $E_t$ and $\chi_t$ as a function of $h$ that satisfy the government’s budget constraint. At the low end of the distribution, it requires a large percent of the workers and has a large dead weight lose. As $h$ increases, the government can not afford as many workers, but is better off by hiring fewer higher human capital individuals. Increasing $h$ away from the low tail of the distribution shrinks the dead weight loss as workers are more similar in human capital levels. As $h$ approaches the highest human capital individuals, the dead weight loss again increases as the difference in human capital levels becomes larger and only a few human capital workers are affordable. Figure 2d displays the dead weight loss as a function of $h_t$ for different percentile levels of $h$. Optimal government policy is to hire individuals who are very similar in human capital levels and not of the tail ends of the distribution of human capital. This forms a U-shaped cost curve and a monotonic concave maximization of $E_t$.

Optimal policy will yield maximum public education quality per pupil, $E_t$, of 2.32 units attained with policies $h_t = 51.82$ and the salary of the human capital individual with human capital $I^h_t = 55.29$. The human capital sector uses approximately 4.34% of the economies labor force or has a student-teacher ratio of 23 to 1. While the distribution of human capital remains stationary, this will remain optimal.

If the distribution of human capital changes, then the government must reoptimize their policies. During the last forty years there has been a large increase in the skill premium and the income of skilled workers has become more dispersed. This has caused the dead weight loss to increase for the same percentile of human capital that was optimal under the stationary equilibrium. The government will react to these changes in the human capital
distribution by adjusting the entry requirement and income of teachers. For simplicity, we only consider two periods. Output increases only as human capital increases and this is contributed to the human capital parameter, $B$, and the public and private educational inputs. In the full model we account for the increase in human capital and rising dispersion of income that is generated by increasing skill premium over the past three decades. For now the desire is to understand the mechanism driving government policy before complicating the model.

Figure 3 demonstrates how optimal policy changes as the human capital distribution changes. The first period, marked as the blue line, is the same as figure 2a and the second period, marked as the green line, is simulated using the optimal decision rules. During this transition, the human capital stock doubles and the coefficient of variation goes from 1.76 to 1.98. This implies that human capital is become more unequal. Note that this increase inequality does not come at the expense of the below-mean human capital individuals (see Figure 4 for a mean preserving spread). Thus it is not a general statement that inequality induces the changes in optimal policy that we propose of an increase in the number of teachers with lower teacher aptitude. Indeed, a mean preserving spread will give exact opposite results.

The results of this example are that human capital can rise for all individuals and that due to the greater dispersion of human capital (income), the government will maximize education quality per pupil by hire more lower human capital workers in percentile. In terms of comparing one generation to another, the second period teachers have higher human capital then the first but relative to their peers, human capital of teachers has fallen. Education quality per pupil has also risen. The number of teachers also has risen from 4.3% to 5.0%. This is equivalent of going from a student-teacher ratio of 23:1 to 20:1.

6. Rising Skill Premium and Optimal Policy

We numerically simulate individuals over time to find how the distribution of income changes and the resulting optimal public policies. The initial distribution of human capital is calibrated to match the gini coefficient of income of .43 in 1960 taken from Atkinson, 2003 and mean and median income. The TFP on goods and human capital production are then matched to Diaz-Gimenez et al, 1997 gini coefficient estimates of .73 for 1990. The model coefficients are:
As a first pass, I draw from the log normal distribution with parameters, the mean equal to 3.73 and the standard deviation equal to .53. This yields a mean income of 48 and median of 41. This can be consider a close approximation to the United States income in 1000s. I then back out the human capital distribution which would generate this income distribution. This give an initial human capital distribution to commence simulating with.

The initial value distribution of human capital back out from the income distribution has a mean value of 5.431 and median of 4.7123. After one iteration, the human capital changes to a mean value of 15.498 and median of 13.472. This implies that over time, all individuals increasing in human capital. The education quality also rises over time. Education quality was maximized at .26 initially and rises to .75. However, due to the income inequality that widens over time, this pushes the percent of workers in the human capital sector to rise from 4.6% to 5.0%. This equates to a 22:1 student-to-teacher ratio to 20:1 student-to-teacher ratio. Private supplements over this time period also decreased over time from 99.86% to 80.62%.

The human capital of teachers also changed over time. In absolute terms, the entry requirement in human capital rises over time from 5.48 value of human capital to 14.46 value of human capital. However, in terms of percentile of human capital, the entry require fell from 61.2 percentile to 55.3 percentile. These results indicate that it is best to hire more, but at a lesser percentile of human capital as income inequality rises.

It should be clear from this analysis that an arbitrary decreases in the student-teacher ratio caused by lowering the human capital requirement and adjusting teacher’s pay to clear the government’s budget constraint is not optimal. This will only increase the dead weight loss and decrease education quality per pupil.
7. Conclusion

In the data we find that the student-teacher ratio has risen along with the skill premium over the last forty years. We also find that the percent of GDP allocated towards education has risen by 1%. This paper goals was to explain the mechanism behind optimal public education policy when the skill premium rises. The means to produce skill premium in this paper has been a result of capital-skill complementarity. As the skill premium rises, this causes the economy to move along a transition path to a new steady state in balanced growth. Along the transition path, the rising skill premium causes the government to reoptimize its policy towards education. This is due to the unionized wage structure of teachers. Because all teachers earn the same income, this implies that all teachers must now be paid more. This increases the dead weight loss and the government reoptimizes by lowering the human capital requirement of teachers relative to the peers and lower teacher salary relative to their peers. However, at this new location, the government can afford to hire more teachers then previous.

8. References


9. Appendix

Source: Hanushek & Revkin, 2003

Source: Hoxby & Leigh, 2004
FIGURE 3  Quality of education: teachers per 1,000 students

Source: Glomm & Ravikumar, 2001

FIGURE 2  Percentage of GNP allocated to public expenditures on education

Source: Glomm & Ravikumar, 2001
Figure 3. The skill premium: Skilled vs. unskilled wages per hour (normalized with 1983=1).

Source: Krusell et al, 2000
Figure 2b: Public Education, $E_t$, as a function of $h_t$

$E_t$ in Percentile of Human Capital

Figure 2c: Percent of Labor Force in Human Capital Sector as a function of $h_t$

Percent of Labor Force in Human Capital Sector

$h_t$ in Percentile of Human Capital