

Price Dispersion Within and Across Stores in a Comparison site

Working Paper

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Abstract: The paper extends the information gatekeeper model of Baye and Morgan(2001) to allow for different levels of store service quality ratings, which are usually observable with the store product prices in the comparison site. The stores use mixed pricing strategies in equilibrium. In particular, higher service quality stores tend to price higher and lower service quality stores tend to price lower in the comparison site. In addition, the monopoly gatekeeper would either charge a click-through fee or a listing fee to maximize profits. In $N=2$ duopoly case, listing fee only is optimal when shoppers are few and click-through fee only is optimal when the size of shoppers gets larger.

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I. Introduction.

Clearinghouse models predict that stores set prices by drawing from a common mixed pricing strategy and hence, the equilibrium result is consistent with the price dispersion observed in reality. However, empirical research disputes that stores use a common mixed strategy to set their prices by examining the price rank in the price comparison sites. Baylis and Perloff(2002) find that the price rank of a store changes little over time in the market and the persistent pricing behavior of each store cannot be explained by a common mixed pricing strategy that is played by all the stores. At the same time, there is support for a mixed strategy of each store. Baye, Morgan and Scholten(2004a) study online monthly prices of 36 popular consumer electronics products listed at Shopper.com for 18 months. In the analytical part of the study, firm specific dummies and product specific dummies are added in the regression. They still find 28% of the unexplained price dispersion variation.

To explain the persistent pricing behavior of each store, we need asymmetric pricing strategies. And, the pricing strategies of each store should display dispersion. This paper provides a way to generate the above two results in equilibrium by including store service quality levels. Store service ratings are usually displayed together with the store prices in the comparison sites. Below is the screen snapshot of the search result of Canon SD600 on Mar.05.2007 in PriceGrabber.com, a leading price comparison site with a self-acclaimed 21 unique million visitors each month. The result is sorted by price. Due to space limit, the eight lowest priced stores out of 28 are displayed.








New (28 Sellers from \$199.99) Used & Refurbished (3 Sellers from \$189.95) View All Conditions						
	Seller	Price (USD)	Tax & Shipping	Availability	Seller Rating	
Shop →	 Featured Merchant Merchant Info	\$199.99	Enter Zip Code To View Shipping & Tax	In Stock	★★★★☆ 102 Reviews	
Shop →	 Merchant Info	\$211.00	Price + Shipping + Tax = BottomLinePrice	In Stock	★★★★★ 1243 Reviews	
Shop →	 Merchant Info	\$212.50		In Stock	See all-time ratings 239 Reviews	
Shop →	 Merchant Info	\$216.52		In Stock	★★★★☆ 1063 Reviews	
Shop →	 Merchant Info	\$217.65		In Stock	★★★★★ 505 Reviews	
Shop →	 Merchant Info	\$218.00		In Stock	★★★★☆ 1532 Reviews	
Shop →	ChiefValue.com Merchant Info	\$219.99		In Stock	★★★★☆ 573 Reviews	
Shop →	 Featured Merchant Merchant Info	\$221.79		See Site	★★★★☆ 4520 Reviews	

Table 1

The paper is built on the model of Baye and Morgan(2001). In their paper, there are N geographical separated towns where each is served by a local store and high transportation costs preclude local consumers to buy from out-of-town stores. The introduction of the internet takes away geographical boundaries. The monopoly comparison site provides a venue to allow the consumers in different towns to compare prices and to buy from the lowest priced store online. In response, the stores want to list prices in the comparison site to perk up sales. In a symmetric equilibrium, the price comparison site charges an optimal fixed advertising fee so that stores have a positive probability to list in the site. They employ a common mixed pricing strategy when listing in the site and they set the monopoly price when not listing in the site.

This paper includes two more recent practices of the comparison site. First, store service quality ratings are available with the store product prices. The asymmetric quality ratings generate price dispersion across stores while preserving the mixed pricing strategy played by each store. Second, the comparison site is allowed to use two pricing instruments: a listing fee and a click-through fee. In a special case of the model, the comparison site prefers a click-through fee to a listing fee or to some combination of the two instruments. In the general solution, the comparison site either charges a listing fee or a click-through fee but never the combination of the two instruments. A click-through fee is also known as a cost-per-click (CPC) fee in the comparison site's fee policy. When a viewer from the comparison site is directed to the merchant's own webpage, the fee is paid by the listing merchant to the site regardless of the purchase outcome. The sample CPC rates effective as of Mar.5th.2007 by PriceGrabber.com on popular categories are: \$0.35 for Books, \$0.75 for Digital Cameras, and \$1.00 for Plasma&LCD Televisions.

In the first point, the stores are endowed with fixed service quality levels in the study period. The symmetry of the model is restored by restraining the maximum profit margin of every store to be the same. That is, consumers understand that a high service quality store has a high cost for providing the service while a low service quality store has a low cost. The same technique to restore symmetry is used in Wildenbeest(2006). His interest is on estimating the distribution of search costs when the consumers use the fixed-sample search method.

The formulation of the same maximum profit margin across stores is most applicable to small businesses. For one reason, internet provides a detailed categorization of products, which allows the small stores to compete in a specific product category that may be covered by the big stores. Baye et al.(2006) find that consumers are more selective with the wealth of information on products, prices and stores from the internet and they search at the product level rather than at the store level. Furthermore, small stores have limited resources and hence, they are more or less on an equal footing in competition. For example, in store services such as the security of credit card information, the ease of checking out, finding technical support, delivery options and the refund policy, small stores are reasonably perceived by the consumers to have a higher cost if the product comes with a better service package. In addition, the stores do not command some preferential search order from the online consumers.

As for the second point, the above discussion on asymmetric mixed pricing strategies across stores is still valid in a general framework that accommodates the presence of loyal consumers who only consider buying from their local stores. In the theoretical part of Baye, Morgan and Scholten(2004a), classic price dispersion papers such as Baye and Morgan(2001), Varian(1980), Narasimhan(1988), Rosenthal(1980) and Shilony(1977) arise as special cases. Due to the broader applicability, I will borrow their model setup and notations to investigate the optimal pricing scheme of the comparison site. In their paper, the focus is on the pricing strategies of the stores and they do not consider the incentives of the comparison site.

The plan of the paper is as follows: the model is laid out in section II. Then, the optimal behaviors of the consumers, stores and comparison site are analyzed in each stage of the three-stage game in section III. Section IV shows Baye and Morgan(2001) is a special case where all firms offer the same service quality in the model. Section V concludes and provides future research directions.

II. The Model.

In a homogeneous product market, N geographically separated towns are each served by a local store. Local consumers do not travel to another town to make purchase due to the high transportation costs. In every town, there are shoppers who check out prices before purchase, and loyal consumers who do not compare prices before buying from the local stores. The total number of shoppers is denoted as S where each town has S/N shoppers and there are L the loyal consumers each town.

Before the internet, each local store is a monopoly and they charge the monopoly price. The shoppers would like to compare prices but the search technology is too costly. That is, the cost of traveling across town cannot offset the benefit of a low price. The internet breaks down the geographical boundary. Shoppers now can compare prices in a virtual market and buy from lowest priced store online without incurring the transportation costs.

Consumers care about both product prices and store service qualities. That is, they prefer the highest value of the service quality minus the product price. The product offered by a particular store is considered as a bundle of the service quality and the price. The utility of an individual consumer is $V_i - P_i$ where the subscript denotes the firm that offers the bundle.

Stores have the same marginal cost of production. Without loss of generality, the cost is normalized to be zero. Service quality level is endowed to firm i with cost r_i . The maximal margin is the same across all stores. It is denoted as X , a positive finite number. The setting has interpretation of the short run time frame when the firm is stuck with their service quality levels. Here, the interest is not the endogenous choice of quality service level by the firms.

The comparison site controls the online virtual market. The monopoly provides a platform for the consumers to access the price and quality service information. The site

charges 0 fee to the consumers while a listing fee(Φ) and a click-through fee(t) to the stores. A listing fee is paid for the right to be listed in the site. In addition to that, a click-through fee is charged for directing a consumer to the store webpage from the site.

As compared to the free comparison service online, consumers pay a transportation cost ε which is positive to travel to the local store. ε is assumed to be sufficiently small so that a consumer still obtains surplus at the monopoly price. That is, $S(P_m) > \varepsilon$ where $S(P_m)$ is

defined as $\int_{P_m}^{P_{max}} q(p) dp$. The assumption is to avoid the Diamond Paradox of an inactive

market. In the following example illustrated on the graph below, the store's commitment to linear pricing and a particular demand are required. The utility of a consumer $S(P_m)$ is the triangle area above the monopoly price. The condition ensures that the consumers still visit the local store even when they expect the monopoly price.

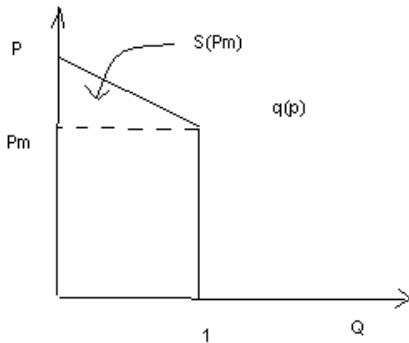


Figure 1

The stores choose to list or not in the comparison site, which publicizes both the price information and store service quality information to the consumers. Besides the advertising decision online, they also choose their pricing strategies when listing and when not listing.

In fact, in this formulation where price and service quality is offered as a bundle, the firms compete in setting the utility level offered to the consumers instead. Furthermore, the utility setting strategy is a one-to-one mapping of the price setting strategy. If the utility level offered by firm i is denoted as m_i , the profit margin for the firm is $X - (m_i - S(P_{mi}))$, the same expression as $P_i - r_i$. The highest utility level the firm can offer is $V_i - r_i + S(P_{mi})$ and the lowest is $S(P_{mi})$. The following two points should be noted.

First, $S(P_{mi})$ is some constant positive amount and it is assumed to be the same across all N firms. In the above example, all firms share the same shape of demand curve because of the restriction. Moreover, the higher the store service quality, the higher is the value to the consumers and the higher its monopoly price. The firm's demand shifts up and down vertically by the service cost depending on its service quality level. The maximal profit marginal X is constrained to be the same across all stores.

Second, for simplicity of notation, I will use m_i to denote $m_i - S(P_{mi})$. The derivation of the equilibrium will not be affected as $S(P_{mi})$ is just some constant positive number. Then, the profit margin is now $X - m_i$ where the largest is $V_i - r_i$ when $m_i=0$ the lowest is zero when $m_i=X$. In words, the lowest possible utility a store can offer is 0 to the consumers (essentially, $S(P_{mi})$) and obtain the monopoly price margin. The highest possible utility a store can offer is X to the consumers (essentially, $X + S(P_{mi})$) and obtain 0 marginal cost pricing margin. The two values of m_i define the set a store can choose to set utility. Before the internet, the stores obviously charge the monopoly price, that is, offer the lowest possible utility level.

If the firm makes the listing decision in the comparison site, the store product price can be updated with no extra cost. However, to prepare the updating documents to submit online, the firm is assumed to incur a fixed cost k , where k is nonnegative. The cost is specific when the firm lists online and does not exist in its off-line operation. Also, not-yet-changed store prices can be perceived unreliable by the consumers and negatively affects their purchase decision. Hence, the stores will put in extra efforts to update their prices online. Notice that the specification allows for k to be zero. Henceforth, the model can predict what happen if k decreases as the firms get used to the online advertising channel and become more efficient in preparing the files.

In reality, the store service quality ratings are evaluated either by the comparison site such as Shopper.com and MySimon.com or by the consumers who have purchase experience with the store such as PriceGrabber.com. The ratings can be biased in the sense that the comparison site may have incentive to disclose less information and hence, consumers click to check out more stores and the revenue of the gatekeeper increases. (although the exact equilibrium result of the practice should be formally analyzed in a full model, the above provides an explanation) Or, it is possible there are not enough consumer reviews that the law of large numbers does not take effect and the approximation of the true service quality level is bad. However, for the analysis of the current setting, let's assume away the two problems so that the listed service ratings reflect the truth.

The timing and nature of the decisions by consumers, firms and the gatekeeper are as follows: First, the gatekeeper announces the listing fee Φ and the referral fee t to the firms. Second, given Φ and t , firms decide whether or not to advertise on the site and what utility level to offer when they advertise and when they do not. Third, the consumers search and make the purchase decision.

In the next section, the three-stage game is analyzed using the subgame perfect equilibrium concept. Working backwards, the analysis starts with the optimal behavior of the consumers in the 3rd stage given the advertising and pricing strategies of the firms in part A. Then, the profit maximization problem of the firms is worked out given the optimal behavior of the consumer and the fees charged by the comparison site in part B. Finally, the monopoly site decides on the listing fee Φ and referral fee t in part C.

Section III. The Three-Stage Game.

A. Optimal Consumer Behaviors.

Recall that in each town, there are two types of consumers: S/N shoppers and L loyal consumers. Loyal consumers do not search online to compare prices while the shoppers take advantage of the virtual market provided by the comparison site. Henceforth, the stores advertise online to compete for the shoppers, whose total size is S, knowing the sales of the loyal consumers are secure.

To maximize their utilities, consumers buy from the store that offers the highest utilities: $V_i - P_i \equiv m_i + S(P_{mi}) \equiv m_i \quad \forall i \neq j$. m_i is defined in a way so as to be consistent with the simplified notation discussed in the previous section.

The optimal behaviors of the consumers are summarized in the following proposition.

Proposition 1: *A subgame perfect equilibrium is achieved when: the shoppers (a) first visit the comparison site and (b) buy from the firm that offers the highest utility level. (c) If no firm is listed in the site, consumers go to their local stores and buy the product. And, the loyal consumers buy from their local stores without comparing online the utility bundles of other firms.*

The optimal behaviors of consumers and the proof are an analogue of the proposition in Baye and Morgan(2001). They establish the theoretical ground while the current model is built upon the previous paper to provide a way that generates asymmetric pricing strategy from a symmetric setting. In their paper, the consumers compare prices and do not care store service qualities. In this paper, the consumers compare utility they receive because the commodity is offered as a bundle that consists of the product price and store service quality. The distinction of asymmetric pricing strategies will be apparent in the optimal behaviors of firms in part B.

The proof is straight forward in the following steps:

(a). Since the comparison site offers the service for free to the consumers, it is at least a weakly dominant strategy for the consumers to visit the site than to pay a positive transportation cost ε .

(b). Suppose firm i does not list in the site. It adopts a utility offering strategy $G_i(m)$ that is consistent with the belief of the consumers in the equilibrium path. Let m_{\max} be the highest observed utility offered in the site. A consumer is willing to pay ε to visit the off-line local store if and only if the expected gain from searching is greater than the cost.

That is, $\int_{m_{\max}}^X (G_i(m) - m_{\max}) dG_i(m) - \varepsilon \geq 0$. Hence, the lowest possible utility to entice

consumer search is defined as $m^* \geq m_{\max}$ such that $\int_{m^*}^X (G_i(m) - m^*) dG_i(m) \geq \varepsilon$.

Anticipating the search rule of the consumers, firm i will not offer a utility level above m^* , or, $\int_{m^*}^X (G_i(m) - m^*) dG_i(m) = 0$. The contradiction establishes the optimal behavior described in (b).

And, (c) follows from the specification of the model. The results are expected because price and utility are related one-to-one. The last part on the behavior of the loyal consumers is assumed.

B. Optimal Firm Behaviors

With the optimal consumer behavior in Section III, and the fee (Φ, t) charged by the comparison site, the firms make two decisions: first, whether or not to advertise; and second, their utility offering strategy when they advertise and when they do not advertise.

If a store chooses not to list, it is optimal to charge the monopoly price and to offer the lowest utility, i.e. $m_i=0$. If the store deviates to a negative utility, it will have no sales. On the other hand, if the store deviates to a positive utility, it will not have more sales but lose profit margin.

Throughout the paper, the focus is on a symmetric equilibrium where the stores adopts the same probability of listing in the comparison site, and use the same utility offering strategy $G(m)$, an atomless cumulative distribution function. The probability of listing is α . Then, $1 - \alpha$ is the probability of not listing. To ensure α is a probability measure, α is from $[0, 1]$. The utility offered by firms is m , from the set of $[0, X]$.

To derive α and $G(m)$, the profits of a firm that choose to list and not to list need to be calculated. Given the existence of the mixed strategy, α and $G(m)$ can be obtained by equating the two expressions of profits. Then, the posited equilibrium will be shown to be the true one.

If a firm does not list on the comparison site, it optimally offers the lowest possible utility 0, and charges the maximal margin X to the consumers. The sales to the loyal consumers, L are secure to a firm regardless of its decision of listing. Moreover, a firm will sell to its share of shoppers, S/N only when all the other firms do not list in the site.

$\pi^{NA} = L * \pi(0) + (1 - \alpha)^{N-1} * \frac{S}{N} + L * \pi(0)$ where $\pi(0) = X - 0 = X$. That is,

$$\pi^{NA} = L * X + (1 - \alpha)^{N-1} * \frac{S}{N} * X \quad (1)$$

If a firm lists to compete for the shoppers, S, its expected profit depends on the listing decisions of the remaining N-1 firms, given that all N firms are using G(m) as the utility offering strategy when listing. Recall that X-m_i is equivalent to P_i-r_i, the profit margin.

$$\pi^A = L * (X - m) + \sum_{j=0}^{N-1} \binom{N-1}{j} * \alpha^j * (1 - \alpha)^{(N-1)-j} * \pi(m) * G^j(m) - (\phi + k) \text{ where}$$

$$\pi(m) = S(X - m - t).$$

Again, the loyal consumers will buy from their local stores. Since the shoppers will buy from the firm that offers the highest utility, $G^j(m)$ captures the probability of a firm offering the higher utility m than any of the j competing firms on the comparison site and winning over all the sales of the shoppers. $\pi(m) * G^j(m)$ is the expected profit of the firm conditional on j listed firms out of the remaining N-1 firms. The shoppers are directed to the highest-utility store from the comparison site and the store pays a click-through fee for each online transaction. Hence, the profit margin of the online sales is X-m-t. Different combinations of the listing and not listing decisions are considered in the summation term. Two costs specific to online operation, listing fee Φ to the site and fixed cost k are subtracted.

Use the Binomial Theorem, the above equation can be simplified to

$$\begin{aligned} \pi^A &= L * (X - m) + (\alpha G(m) + (1 - \alpha))^{N-1} * S * \pi(m) - (\phi + k) \\ &= L * (X - m) + [1 - \alpha(1 - G(m))]^{N-1} * S * (X - m - t) - (\phi + k). \quad (2) \end{aligned}$$

Retrospectively, there are two cases regarding the advertising decision. First, when the fixed cost of the firm to advertise online k, is positive, there is no pure strategy in listing or no listing given a particular range of fixed fee Φ and referral fee t, which is discussed on the below paragraph. The existence of the mixed strategy in listing and no listing is shown after deriving the utility offering strategy of the firms in the posited equilibrium. Second, when the online fixed cost k incurred to a firm (not the fixed listing fee paid to the gatekeeper) is zero, pure strategy of advertising or not advertising exists, which is summarized in *Lemma 1*. When all firms advertise on the comparison site, the model is reduced to Varian(1980): the firm balances the tradeoff between securing the monopoly profits from its loyal consumers and competing for the shoppers with a higher utility but a lower profit from the loyal consumers. When all firms do not advertise, the monopoly price prevails in every town.

For the first case, we use that known fact that G(m) must be such that the profit level is constant regardless of the listing decisions. When the listed firm offers the lowest utility 0, the expected profit should be the same as any other utility offered over the support. $\pi(0) = L * X + (1 - \alpha)^{N-1} * (X - t) * S - (\phi + k)$. The firm is indifferent between listing and not listing because of the same expected payoff. Then, equating $\pi(0)$ with equation (1) yields

the firm's propensity to list: $\alpha = 1 - \left[\frac{\phi + k}{S[(1 - \frac{1}{N})X - t]} \right]^{\frac{1}{N-1}}$. For α to be a probability

measure, it should be between 0 and 1. The restrictions on Φ and t are such that (a) Φ is nonnegative. (b) $\Phi + k$ is no greater than $S[(1 - \frac{1}{N})X - t]$. Note that α is not properly defined when t equals $(1 - \frac{1}{N})X$.

Lemma1: When $t = (1 - \frac{1}{N})X$ and Φ is zero, pure strategies of advertising and pure strategies of not advertising are both symmetric equilibria. If all firms advertise, they offer utility according to $H(m) = \left[\frac{L * m}{S * (\frac{1}{N} * X - m)} \right]^{\frac{1}{N-1}}$ where $m \in [0, \frac{S}{S + L} * \frac{X}{N}]$. If all firms do not advertise, they offer 0 utility and charge monopoly prices.

The proof is as follows:

a). If all firms but i advertise on the comparison site. Suppose firm i lists and adopts a symmetric utility offering strategy $H(m)$. $\pi^A = S * \{(X - m) - t\} * H(m)^{N-1} + L * (X - m)$.

Plug in $t = (1 - \frac{1}{N})X$, $\pi^A = S * (\frac{X}{N} - m) * H(m)^{N-1} + L * (X - m)$. Especially when $m=0$,

$\pi^A = L * X - \phi = L * X$. The profit is constant for m over the support $[0, \bar{m}]$ and equating the above two equations, we can obtain $H(m)$.

$H(m) = \left[\frac{L * m}{S * (\frac{1}{N} * X - m)} \right]^{\frac{1}{N-1}}$ where $m \in [0, \frac{S}{S + L} * \frac{X}{N}]$. It is similar to the Varian(1980)

model. In the Varian model, the firms have zero listing fee, the same as this case when Φ is zero. The click-through fee serves as a marginal cost in the sales to the shoppers and it preserves the mixed equilibrium strategy. Whether or not it will be an equilibrium in the game depends on the fee decisions of the gatekeeper. The case will be revisited in part C. If firm i does not list, it will sell only to the loyal consumers. Optimally, the firm charges X , the maximal margin and earns $\pi^{NA} = L * X$. The firm cannot make a higher profit by deviation. Also, since the focus is on a symmetric equilibrium, all firms list on the comparison site and offer a utility according to $H(m)$ is the one we are looking for.

b). Suppose all firms but i do not advertise on the comparison site. If firm i does not advertise, its profit is $\pi^{NA} = L * X + \frac{S}{N}$. If firm i deviates and charges X on the

comparison site, $\pi^A = L * X + S * (X - t) - \phi = L * X + \frac{S}{N}$. Again, deviation is not a strictly dominant strategy. The symmetric equilibrium is that all firms do not list and offers 0 utility to the consumers.

Lemma 1 shows that pure strategies in advertising and not advertising exist for one combination of fees: $t = (1-1/N)X$ and Φ is zero in the feasibility set. The derivation of the equilibria is different from any other combination of fees, where the firms have a positive probability to advertise but not with certainty. Notice that when N goes to infinity, the upper bound of the mixed utility offering strategy when all firms advertise goes to 0. Intuitively, when there are a large number of firms in the industry, the incentive for the firms to compete for the highest utility on the site decreases exponentially. Instead, firms increasingly charge the monopoly prices to secure profits from the loyal consumers. Similar arguments are made in Varian(1980).

Now, we are in a position to discuss the first case of a general combination of Φ and t . When (a) Φ is nonnegative and (b) $\Phi+k$ is no greater than $S[(1-1/N)X-t]$, the optimal

probability of listing on the comparison site is mixed. $\alpha = 1 - \left[\frac{\phi + k}{S[(1-\frac{1}{N})X - t]} \right]^{\frac{1}{N-1}}$.

Since mixed strategy yields the same expected profit for every utility in its support, $G(m)$ is obtained by equating (1) and (2):

$$G(m) = 1 - \frac{1}{\alpha} \left(1 - \left[\frac{(\phi + k) + (1 - \alpha)^{N-1} * \frac{X}{N} + Lm}{S(X - m - t)} \right]^{\frac{1}{N-1}} \right) = 1 - \frac{1}{\alpha} \left(1 - \left[\frac{(\phi + k) * \frac{X - t}{(1 - \frac{1}{N})X - t} + Lm}{S(X - m - t)} \right]^{\frac{1}{N-1}} \right) \quad (3)$$

Setting $G(\bar{m})=1$ yields the upper bound \bar{m} .

$$\bar{m} = \frac{X - t}{S + L} * \left(S - \frac{\phi}{(1 - 1/N)X - t} \right) \quad \text{where } 0 < \bar{m} < X.$$

It is easy to verify that $G(m)$ is continuous and an increasing function of m over the support $[0, \bar{m}]$ where $G(0)=0$ and $G(\bar{m})=1$. It is indeed a properly defined c.d.f..

After deriving the probability of listing and the utility offering strategy of listing, we will show that the posited equilibrium is indeed the true one.

First, there is no pure strategy in listing or no listing in a symmetric equilibrium in this first case. Two conditions will be satisfied:

1. If all firms but i list in the comparison site, then, the expected profit to list as well is strictly less than the profit of not listing for firm i .

Suppose firm i decides to list, it uses $G(m)$ and the profit is the same over the support $[0, \bar{m}]$. In particular, when firm i offers the lower bound utility 0 , $\pi(0)^A = L * X - (\phi + k)$. If firm i decides not to list, firm i offers the lowest possible utility 0 , $\pi(0)^{NA} = L * X$. As k is positive in the first case, firm i is better off not listing.

2. If all firms but i choose not to list in the comparison site, then, the expected profit not to list as well is strictly less than the profit of listing for firm i .

Suppose firm i decides not to list, firm i offers the lowest possible utility 0 ,

$\pi(0)^{NA} = \frac{S}{N} * X + L * X$. If firm i decides to list, it chooses m from $G(m)$ over the

support $[0, \bar{m}]$. In particular, firm i offers the lower bound utility 0 since all other firms do not advertise. $\pi(0)^A = S * (X - t) - (\phi + k) + L * X$. By restriction (b) $\Phi + k$ is no greater than $S * [(1 - 1/N)X - t]$, the profit from listing on the site is no less than that from no listing. As a result, listing at least weakly dominates not listing.

The above shows that all firms advertising or all firms not advertising is not equilibrium. Hence, a symmetric equilibrium consists of a positive probability of listing but not with certainty.

If $N - 1$ firms list with the derived α probability, and offers utility according to $G(m)$, the best response for the remaining firm is to follow suit. The proof is straightforward from the derivation of α and $G(m)$. Q.E.D.

Proposition 2: *Given the fees (Φ, t) that the gatekeeper charges satisfy (a) Φ is positive. (b) $\Phi + k$ is no greater than $S * [(1 - 1/N)X - t]$, t is not equal $(1 - 1/N)X$ and k is positive, the optimal advertising and utility offering strategy of the firms in a symmetric equilibrium is described as follows:*

$$(i). \text{ Firms list with probability } \alpha = 1 - \left[\frac{\phi + k}{S[(1 - \frac{1}{N})X - t]} \right]^{\frac{1}{N-1}}.$$

(ii). *When a firm lists, it offers a utility level m according to the c.d.f. $G(m)$ over $[0, \bar{m}]$.*

$$G(m) = 1 - \frac{1}{\alpha} \left(1 - \left[\frac{(\phi + k) + (1 - \alpha)^{N-1} * \frac{X}{N} + Lm}{S(X - m - t)} \right]^{\frac{1}{N-1}} \right) = 1 - \frac{1}{\alpha} \left(1 - \left[\frac{(\phi + k) * \frac{X - t}{(1 - \frac{1}{N})X - t} + Lm}{S(X - m - t)} \right]^{\frac{1}{N-1}} \right) \quad (3)$$

$$\bar{m} = \frac{X-t}{S+L} * \left(S - \frac{\phi}{(1-1/N)X-t} \right) \text{ where } 0 < \bar{m} < X.$$

(iii). When a firm does not list in site, it offers the lowest possible utility level, 0. And, the

$$\text{expected profit for a firm is } \pi^{NA} = L * X + \frac{\phi + k}{(1-\frac{1}{N})X-t} * \frac{X}{N} \quad (1)$$

All the firms are symmetric in offering utilities to consumers in the subgame perfect equilibria discussed above. However, each firm with a different service quality level and the prices are different henceforth. There is a one-to-one mapping from utility m to price P. The price charged by every firm can be recovered from the utility it offers by the equation: $P_i = V_i - m_i$, where V_i is the consumer valuation of the service quality of firm i.

$$F_i(p) = \Pr(P_i \leq p) = \Pr(V_i - m_i \leq p) = \Pr(m_i \geq V_i - p) = 1 - G(V_i - p).$$

The support for firm i is between $[V_i - \bar{m}, V_i]$.

For stores with different service quality levels, the supports vary. The pricing distribution of every firm shares the same shape but over a different support. To ensure the lower bound of the price is non-negative, the value of the lowest service quality firm to the consumers should be greater than \bar{m} . The illustration shows the pricing distributions of five service quality levels that are recovered from a common utility offering strategy.

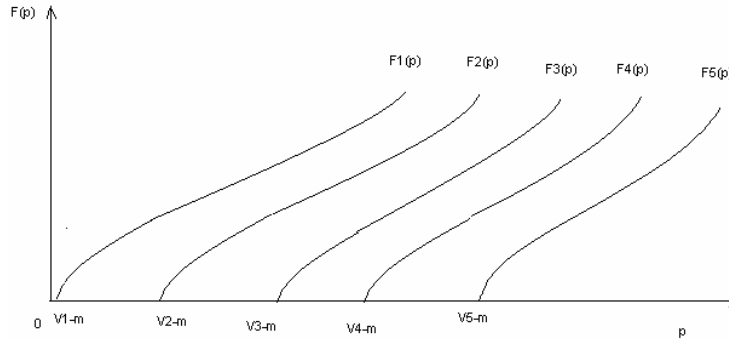


Figure 2

Store service quality levels are public knowledge and consumers value service quality in the same scale of price (the difference between the value of store service and the store product price). Firms compete in offering utilities to the shoppers rather than in the price levels only. As a result, firms with high service quality persistently charge high product prices while firms with low service quality persistently charge low product prices. The findings of Baylis and Perloff (2002) that the price rank of a store changes little over time in the market is consistent with the results. Moreover, each store uses its own mixed pricing strategy. Naturally, price dispersion is observed for a particular store listed on the comparison site. Baye, Morgan and Scholten (2004b) find that 28% of price dispersion is

unexplained even when product and firm dummies are added. The above two empirical findings are implied by the current model.

In empirical research involved data from the comparison site, the model includes more features and accommodates more general results. Two points are worth noting here.

First, when service quality ratings are available in the comparison site and they are not accounted for, the empirical result may be biased. The biased result can be a mixed distribution of all pricing strategies with different supports. A similar note of caution is called upon in Wildenbeest(2006) where he is interested in the estimation of the consumer search cost distribution. In addition, the service quality has the same unit of product price in the paper. That is, the dollar value of the service instead of the simple star ratings should be put in empirical estimation or testing. A possible way to convert between the two is to put a scale parameter for the star ratings where the scale parameter tells us how much dollar value it is worth of one star rating.

Second, service quality levels have discrete and continuous interpretations depending on the rating systems and the consumer perception. Both interpretations can be found in comparison sites. Commonly seen in the search result page, V_i in 0 to 5 star rating systems is discrete in 10 levels with an increment of a half star in the comparison site. The star ratings are displayed along with the product prices. Consumers do not need to search further for the information. On the other hand, the star ratings displayed is rounded from the continuous grades available in the comparison site from 0 to 5. The information is available with a few more clicks within the site. If consumers take the effort to search for the continuous grade, the pricing distribution will be different for every store. The key differences lie in the assumption of the consumer perception and the distinction only matters in empirical works.

C. Optimal Comparison Site Fee Scheme.

Bear in mind that the gatekeeper has two instruments to maximize its profit, one is the listing fee, Φ and the other is the click-through fee, t . If the site charges $t=0$, the comparison can charge a optimal Φ , which was the practice of the site at the beginning of the internet era. Baye and Morgan(2001) analyzes the optimal listing fee in the last stage of the game. If the site charges $\Phi=0$, the gatekeeper can still charge an optimal click-through fee to recoup profits for directing one shopper to the store webpage. Now, the leading comparison sites do charge a click-through fee and no listing fee in their stated pricing policies. In this part, the optimal fee combination will be studied in more details.

The conditions from the analysis of the firms to ensure a properly defined probability are Case1 when $k > 0$: (a) Φ is positive. (b) $\Phi+k$ is no greater than $S*[(1-1/N)X-t]$ and Case2 when $k=0$: t can equal $(1-1/N)X$ and $\Phi=0$ under the above (a) and (b).

In general, profits of the comparison site consist of two parts: one part is from the advertising fee Φ , $N*\alpha*\Phi$. The expected revenue is the probability of listing times the number of firms in the industry and the listing fee. The other part is from the click-

through fee t for directing every shopper, $(1-(1-\alpha)^N)*t*S$. The shoppers will visit the site and buy from a listed store that offers the highest utility. The probability that such an event occurs is $1-(1-\alpha)^N$ where $(1-\alpha)^N$ is the probability that no firm list and the shoppers go to their local stores. When the shoppers buy online, the comparison site collects $t*S$. Hence, the expected profits from t is the probability that at least one firm lists times $t*S$.

The objective function of the gatekeeper is:

$$N * \phi * \alpha + S * t * (1 - (1 - \alpha)^N) \quad \text{where } \alpha \text{ is specified in proposition 2. That is,}$$

$$N * \phi * \left(1 - \left(\frac{\phi + k}{S[(1-1/N)X - t]} \right)^{\frac{1}{N-1}} \right) + S * t * \left(1 - \left(\frac{\phi + k}{S[(1-1/N)X - t]} \right)^{\frac{N}{N-1}} \right).$$

Consider the natural range of the fee combination Φ and t , the graph below shows that the feasible values in a simplex. In Case1 when k is positive, the simplex is a compact set where t does not take $(1-1/N)X$. In Case2 when k is zero, $\max t$ is $(1-1/N)*X$ and we will resort to *Lemma1* if the point is called upon in the analysis. In sum, the feasible set of Φ and t is compact.

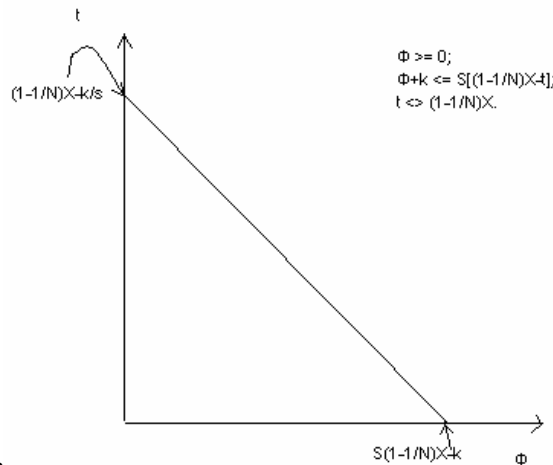


Figure 3

A more careful look at the objective function will verify that it is continuous in Φ and t in the above compact set. By the Weierstrass Theorem, a maximal solution exists. If it is an interior solution, the standard first order conditions with respect to Φ and t are the necessary conditions. Second order conditions should be negative semi-definite at the solution, which are defined among the critical points by the f.o.c.. If it is a boundary condition, it will exist on either three sides of the simplex. Along the longest side, the hypotenuse of the right triangle, profits are zero because the probability of listing is 0.

$$f.o.c. \text{ of } \phi: \left(\frac{\phi+k}{S[(1-1/N)X-t]} \right)^{\frac{1}{N-1}} \left(S^* t^* \frac{\phi+k}{(1-1/N)X-t} + (N-1)^* k + \phi^* N \right) = (N-1)^* (\phi+k)$$

$$f.o.c. \text{ of } t: \left(\frac{\phi+k}{S[(1-1/N)X-t]} \right)^{\frac{1}{N-1}} \left((N^2 * S^* X + S^* X + 1) * \left(\frac{\phi+k}{S[(1-1/N)X-t]} \right) + N^2 * \phi \right) = N^2 * S^* X + S^* X + 1 - N^2 * S^* t$$

The solutions defined by the two first orders conditions are hard to solve analytically.

Let's look at the Hessian matrix where the top left element of the 2*2 matrix is the second order conditions w.r.t. Φ , the bottom right element is the second order conditions w.r.t. t and the diagonal elements are the same, the cross partial of Φ and t .

$$\begin{aligned} \left(\frac{\partial^2 \pi}{\partial \phi^2} \right): & - \frac{N(S^* t^* \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{N}{N-1}} + (2k(N-1) + \phi^* N) \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{1}{N-1}}}{(\phi+k)^2 (N-1)^2} \\ \left(\frac{\partial^2 \pi}{\partial t^2} \right): & - \frac{(2S^* X^* (N-1)^2 + S^* t^* N) * \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{N}{N-1}} + \phi^* N^2 \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{1}{N-1}}}{(N-1)^2 ((1-1/N)^* X-t)^2} \\ \left(\frac{\partial^2 \pi}{\partial \phi \partial t} = \frac{\partial^2 \pi}{\partial t \partial \phi} \right): & - \frac{\left(N(k(N-1) + \phi^* N) * \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{1}{N-1}} + S^* (X^* (N-1)^2 + N^* t) * \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{N}{N-1}} \right)}{(\phi+k)(N-1)^2 ((1-1/N)^* X-t)} \\ \frac{\partial^2 \pi}{\partial \phi^2} * \frac{\partial^2 \pi}{\partial t^2} - \frac{\partial^2 \pi}{\partial \phi \partial t} * \frac{\partial^2 \pi}{\partial t \partial \phi}: & - \frac{\left(S^* X^* (N-1) * \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{N}{N-1}} - k^* N^* \left(\frac{\phi+k}{S^* ((1-1/N)^* X-t)} \right)^{\frac{1}{N-1}} \right)^2}{(\phi+k)^2 (N-1)^2 ((1-1/N)^* X-t)^2} \end{aligned}$$

The s.o.c. of Φ is negative for the values of Φ and t simplex except at the point where $\Phi=0$ and $t=(1-1/N)X-t$. The same is true with the s.o.c. of t . Suppose one of the instruments (either Φ or t) is fixed, there is a unique solution of the other instrument within the simplex.

In addition, the Hessian is indefinite in the simplex because the strictly negative determinant of the 2*2 matrix, which fail to meet the semi-negative definite matrix conditions. In other words, any Φ and t combination that is determined by the f.o.c. will not be the local maximum. Henceforth, the solution will be at the corner (either of the three sides of the simplex). The indefinite matrix holds for any possible parameter values of S , N , X and k .

Since for a given value of Φ , a unique t maximizes the profit function. At the corner where $\Phi=0$, let's denote t^* as the optimal solution. Again, for a given t , a unique Φ maximizes the gatekeeper's profit. Φ^* is the optimal advertising fee when $t=0$ at the corner. The profit along the hypotenuse is zero for all the combinations except for one point when $k=0$ ($\Phi=0$ & $t=(1-1/N)X$) which is discussed in *Lemma 1*. So long as $\pi(\Phi^*)$ or $\pi(t^*)$ is greater than 0, the hypotenuse corner will be ruled out. Intuitively, when the gatekeeper considers a fee structure, he will at least make sure the fee is not too high to shun away the firms to use the comparison site. Fees along the hypotenuse are such that

the firms will have zero possibility to list and hence, the site has zero economic profit. If it is not profitable to charge the fees along the hypotenuse, the solutions will either exist at Φ^* or t^* , at the other two corners. At the two possible corner solutions, the f.o.c. of t where $t=0$ (at Φ^*) is negative for any possible parameters (S, N, X, k) and the f.o.c. of Φ where $t=0$ (i.e. at t^*) is negative for any possible parameters (S, N, X, k). In other words, the candidates of the corner solution exist regardless of the exogenous parameters. In sum, the gatekeeper will either charge a listing fee Φ or charge a click-through fee t but not the combination of both instruments. Let's discuss the two above mentioned cases when $k>0$ and when $k=0$.

When the fixed cost of preparing the files to update prices on the comparison site is zero, or $k=0$, we can solve explicitly the two possible corner solutions when $\Phi=0$ and when $t=0$. Superimpose $t=0$ in the first order condition of Φ and the optimal Φ is derived as:

$$\phi^* = \left(1 - \frac{1}{N}\right)^{N-1} \left(1 - \frac{1}{N}\right) * X * S \quad \text{and} \quad \pi(\phi^*) = \left(1 - \frac{1}{N}\right)^{N-1} \left(1 - \frac{1}{N}\right) * X * S. \quad \text{When } \Phi=0$$

in Case2 where $k=0$, the gatekeeper charges no listing fee and the probability α is 1 when t does not take $(1-1/N)X$. The profits from the click-through fee t is $t*S$, which is linear in t . In order to maximize the profits, the firm should charge the t max value. *Lemma1* can be applied under this specific combination of fees. Always advertising on the comparison site is a symmetric equilibrium, where the firms offer utility according to $H(m)$. Using the change of variable technique demonstrated in the previous part B, firms with different service qualities will price according to their respective mixed pricing strategies. Then,

the optimal t is $(1-1/N)X$ and the profit of the gatekeeper is $\left(1 - \frac{1}{N}\right) * X * S$. Then,

$\left(1 - \frac{1}{N}\right)^{N-1}$ is the ratio of profits from a listing fee only and profits from a click-through fee only. The term is a decreasing function where $N=2$ yields the highest profits and $N=\infty$ yields the smallest profits from listing fee as compared to click-through fee. When $N=2$, the profits from charging listing fee only is a half of those from click-through fee only. When N goes to infinity, the ratio converges to $1/e$, which is approximately 0.37. That says, when there are numerous stores in the market, the profits from an optimal listing fee to the gatekeeper is about 37 percent of profits from an optimal click-through fee. In sum, a click-through fee strictly dominates a listing fee.

How do the stores arrive at the always advertising equilibrium? One possible way is that the gatekeeper charges a click-through fee only strictly less than the optimal t at the beginning. All firms will find the information platform valuable and all firms list on the comparison site as a result. Then, the gatekeeper increases the fee up to t^* . Given that all $N-1$ firms advertise, the remaining firm does not do better than switching its advertising strategy. This is a jump-start story of the gatekeeper and always advertising equilibrium is an equilibrium result. Supporting evidence of an increasing click-through fee can be found in PriceGrabber.com, where the rate increases by 5 cents in four of the major categories: Computers, Electronics, Office and Software from Mar. to Apr. in 2007.

Similar results are obtained in previous literature. For example, Varian(1980) considers a zero advertising fee, which is exogenously to the stores without the gatekeeper. *Lemma2* below summarizes the results of Case2 when $k=0$.

Lemma2: *When the opportunity cost of advertising online is zero to the stores, i.e. $k=0$, the monopoly gatekeeper always finds it optimal to charge an optimal click-through fee $t=(1-1/N)X$ under the always advertising online equilibrium(see discussion in Lemma1). In duopoly case when $N=2$, the profits from listing fee only are half as those from click-through fee. As N increases, the profits from listing fee only decreases. When $N=\infty$ in the extreme, the profits from listing fee is only 37 percent of click-through fee.*

In the more general case where $k>0$, the analysis still carries through but with one difference that t max in the illustrated graph above can be obtained without resorting to *Lemma1*. Although the gatekeeper can use two pricing instruments Φ and t , the optimal pricing is to use one instrument only. However, due to the non-linear nature of the maximization problem in N , general solutions of optimal advertising fee Φ and optimal click-through fee t are hard to solve analytically. The general results are presented in *Proposition3* below, which is followed by a more detailed discussion of the duopoly case when $N=2$.

Proposition3:

i). *The monopoly comparison site finds it optimal to use a unique click-through fee only, t or to use a unique fixed fee only, Φ to maximize profits although the gatekeeper can choose to use both pricing instruments.*

ii). *If click-through fee is optimal, the optimal t^* is between $[0, (1-1/N)*X-k/S]$ and*

$$\arg \max \left\{ \left(\frac{\phi}{S[(1-1/N)X - t]} \right)^{\frac{N}{N-1}} * (N^2 * S * X + S * X + 1) = N^2 * S * X + S * X + 1 - N^2 * S * t \right\}$$

iii). *If listing fee is optimal, the optimal Φ^* is between $[0, S*(1-1/N)*X-k]$ and*

$$\arg \max \left\{ \left(\frac{\phi}{S[(1-1/N)X]} \right)^{\frac{1}{N-1}} ((N-1)*k + N*\phi) = (N-1)*(\phi+k) \right\}$$

iv). *Depending on exogenous parameters(S, X, N, k), the monopoly gatekeeper charges click-through fee t^* when $\pi(t^*) > \pi(\Phi^*)$; and the gatekeeper charges listing fee Φ^* when $\pi(\Phi^*) > \pi(t^*)$.*

v). *As the size of shoppers S increases, t^* and Φ^* increases; As the maximal profit margin X increases either because consumers value the product/store service more highly or the firms lower costs of production/service, t^* and Φ^* increases; As the opportunity cost of listing online k increases for firms, t^* and Φ^* decreases.*

i) is a restatement of the existence of corner solution in the gatekeeper's profit maximization problem. The corner where profits are zero is ruled out because any arbitrary small t and/or Φ will yield positive profits. Hence, the solution exists at the two

corners of the t and Φ axis, which means either Φ is zero or t is zero in the maximal solution. Moreover, the second order conditions w.r.t. Φ and t are strictly negative. For $\Phi=0$, there is a unique solution of t^* and for $t=0$, there is a unique Φ^* . The solutions presented in ii) and iii) are from the first order conditions where $\Phi=0$ and $t=0$ are imposed respectively. iv) follows naturally from profit maximization assumption of the gatekeeper. The last point v) is the standard comparative statistics analysis. The signs of the analysis in N are not determined analytically.

When $N=2$, the nonlinearity in ii) and iii) of t^* and Φ^* goes away and analytical solutions can be obtained. Moreover, by comparing $\pi(t^*)$ and $\pi(\Phi^*)$, we can tell which pricing instrument is optimal for the gatekeeper. The explicit solutions are not presented because they are messy but available upon request.

The figure below shows the difference of $\pi(t^*) - \pi(\Phi^*)$ as a function of the size of shoppers S when k is 0.01 and X is 1. The difference function is monotone in S . That is, after passing the threshold of approximately 0.02, the profits from click-through fee are greater than those from listing fee.

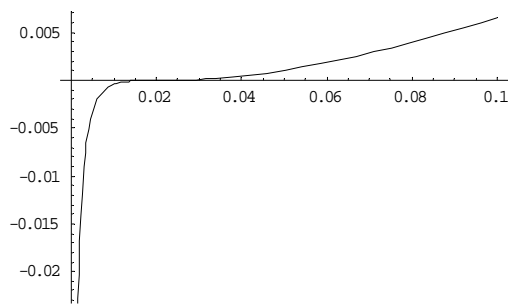


Figure 4

When S is less than 0.02, charging a listing fee yields higher profits while charging a click-through fee is optimal otherwise. In other words, when there are few shoppers, the gatekeeper optimally uses only an advertising fee. As more people use the internet, the gatekeeper switches to a click-through fee only. *Lemma3* summarizes the findings in duopoly case.

Lemma3: *When there are two firms in the market, the monopoly gatekeeper finds it optimal to charge a listing fee Φ^* when the size of shoppers is small and to charge a click-through fee t^* when the size of shoppers passes the threshold and gets bigger.*

Section IV. Discussion: Baye and Morgan(2001).

The model is built on Baye and Morgan(2001) and hence, it seems complete to show that the 2001 model is nested in the current model by specifying the parameters. Furthermore, the discussion also highlights the connection and the difference between the two models.

First, the current model includes store service qualities and the marginal costs for the services, which are different across stores. They consider store prices only and the marginal cost of the product is the same across all stores.

Second, the current model includes a click-through fee t as a pricing instrument at the gatekeeper's disposal. They consider a listing fee only situation. In *proposition 3*, the current model shows that either listing fee only or click-through fee only is optimal.

In addition, k is assumed to be zero.

Then, we take the valuation of store services to be constant across stores, which is the reservation price in the 2001 model and the marginal costs for the services are constant, which is called the marginal cost of production in their paper. Furthermore, we superimpose t to be zero in the model.

The pricing strategies of each store will be the same where the lower bound is

$V - \left(1 - \frac{\phi}{(1-1/N) * X}\right) * X$ and the upper bound is V . The maximal profit margin X is $V-r$,

the monopoly price minus the marginal cost of production. Hence, the lower bound is

$P_0 = \frac{N}{N-1} \phi + r$ and the upper bound is V , the monopoly price. The c.d.f. $F(p)$ is

recovered from $1-G(m)$, which is $\frac{1}{\alpha} \left(1 - \left[\phi * \frac{V-r}{(1-1/N)(V-r)} * \frac{1}{P-r}\right]^{\frac{1}{N-1}}\right)$. The solutions

are the same. In sum, the current model is indeed a general version from the one that is based upon.

Section V. Conclusion and Further Research.

The paper provides a way to generate price dispersion both within a store and across stores by introducing the store service quality levels. Price dispersion is shown to be robust whether the firms all advertise on the comparison site or choose a positive probability to do so. The contradiction in the previous theoretical work that firms use a common mixed pricing strategy and the empirical work that price ranks of firms change little is explained.

In addition, built on Baye and Morgan(2001), the model shows that the monopoly gatekeeper will use one pricing instrument rather than the combination of listing fee and click-through fee in equilibrium, which is consistent with our observations with the fee practices of comparison sites. In addition, in duopoly case, the gatekeeper optimally switches from a listing fee to a click-through fee as the size of shoppers gets larger.

One direction to go from here is to analyze the N firm case in general rather than the duopoly case. It may serve as a guide to the gatekeeper when it is optimal to use one pricing instrument and when to use the other in relation to the exogenous parameters.

Another possible further research is on the purchase decision of the consumers. In the current model, the consumers who click through the store webpage will buy from the store, which is far from the reality. The propensity to buy from a store depends on many other factors besides the information available from the comparison site. In turn, the probability of listing of stores will be affected if the gatekeeper charges a click-through fee.

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