I. Introduction: Motivation and Questions of Interest:

One of the largest and fastest growing sectors in the U.S. and many industrialized nations is the medical or health care sector. Further complicating matters is the simultaneous aging in these nations as individuals continue to live longer and have fewer children. Medical care, retirement and nursing home expenses, and prescription drug use necessarily increase as individuals age, and these increased costs can be difficult to bear for the elderly and also difficult to pay for through government intervention. This growth in longevity has led many countries to question the extent to which these expenses should be subsidized through programs such as Medicare or a universal health care system.

What pressures will this aging put on the economy, and what effects will there be on economic growth? On the surface, it would seem that increasing funding of health care for seniors to prolong life in retirement may be a “waste” from the perspective of growth, when compared to funding of programs such as education which would increase the productivity of future workers. In addition, subsidizing expenditures in retirement reduces the need to save, and could result in lower levels of capital accumulation.

However, in a model in which human capital is produced with both private and public inputs, reduced private expenditure on health in retirement frees up resources for altruistic parents to invest in their children’s human capital and might therefore have positive economic effects. This mechanism is directly related to that explored by Kaganovich and Zilcha (1999) who examined the relationship between Social Security funding and human capital investment,
and found conditions under which the presence of a social security system can increase parental investment in education, and thus enhance growth.

The main question this paper seeks to answer is: When longevity increases, are there conditions under which the presence of a health care subsidy such as Medicare has a positive effect on the growth rate? We will explore those conditions in this paper and then simulate an economy to estimate the size of the effects.

The preliminary answer to this question seems to be that such conditions may exist, but under some plausible conditions, the presence of a Medicare system has no positive effect on growth.

II. Basic Model:

Individuals in this economy each live for a maximum of three periods. Each period, a population of $N$ individuals, normalized to one enters the economy. In the first period, individuals receive education inelastically and make no decisions. This education is provided through a combination of public and private inputs.

Individuals work in the second period. While working, individuals choose their level of consumption, savings, and private educational investment for their children, while earning income for time spent in the labor force. Utility is gained from consumption and from the human capital of the offspring.

At the beginning of the last period, agents retire and choose their level of consumption and health care, which are purchased with the assets carried over from the previous period. The government provides additional funding to retirees in the form of Medicare which acts as unit subsidy for health expenditures for the individual. Utility in retirement is gained from
consumption and health care. Once the health care and consumption decisions are made for the retirement period, households then enter a lottery and receive a draw to see if they will survive into the third period. Saving is done in the form of perfect annuities so that the assets of those individuals who do not survive to retirement are split among the survivors.

A proportional income tax rate \( \tau \) exists and the proceeds of this tax fund public education to produce human capital. The Medicare subsidy is financed by a separate proportional tax rate \( \theta \). Medical care is produced using a combination of human capital and physical capital, and purchased with government and private inputs.

**Preferences:**

The household problem in this economy is as follows:

Individuals of generation \( j \) in period \( t \) maximize:

\[
E[U] = \ln c_{j,t}^y + \gamma_2 \ln h_{j+1,t} + \rho \gamma_3 \ln c_{j,t+1}^o + \rho \gamma_4 \ln d_{j,t+1}^o
\]

s.t.:
\[
c_{j,t}^y + s_{j,t} + e_{j,t} = (1 - \tau_t - \theta_t)w_t h_{j,t}
\]
\[
c_{j,t+1}^o + (1 - z_{t+1})p_{t+1}^m d_{j,t+1}^o = \frac{1+r_{t+1}}{\rho} s_{j,t}
\]
\[
h_{j+1,t} = Bx_t^{1-\eta} e_{j,t}^\eta
\]

where \( c_{j,t} \) represents the consumption expenditures of the individual of generation \( j \) in period \( t \), \( s_{j,t} \) is the savings in period \( t \), \( e_{j,t} \) is the level of private parental investment in their children’s education, \( x_t = \frac{x_t}{N} \) is the uniform per student level of public investment in education, \( d_{j,t} \) is the level of private medical expenditures purchased. The return to savings is \( 1+r_t \), which is split among the \( \rho*N \) survivors each period. After-tax labor income is \( (1-\tau_t-\theta_t)w_t h_{j,t} \), where \( h_{j,t} \) is the individual’s level of human capital, \( w_t \) is the
wage, and \( \tau_t \) and \( \theta_t \) are the proportional income tax rates devoted to education and health care funding, respectively.

The price of medical care is given by \( p^m_t \) and the government Medicare subsidy rate is given by \( z_t \). Individuals therefore chose consumption, medical expenditures, private education supplements, and savings taking prices and government policies as given. The parameter \( \gamma_i \) corresponds to the relative weight of medical care, consumption, and children’s human capital in the individual’s utility function, and the parameter \( \rho \) is the exogenous survival parameter, with the condition \( 0 < \rho < 1 \).

**Technology:**

There are two different types of firms in this economy: factories and hospitals. We will assume that each are competitive profit maximizers and hire workers and capital accordingly. Workers do not care which sector they work in, and supply their human capital wherever they receive the highest wage. Production in each sector takes place according to:

- **Consumption/Capital Goods sector:**
  
  \[
  Y_t = A(K_{yt})^\alpha (H_{yt})^{1-\alpha}
  \]

  where \( K_{yt} \) is the amount of physical capital used in the goods sector, \( H_{yt} \) is the amount of human capital used in the goods sector.

- **Medical/Health Care sector:**
  
  \[
  M_t = D(K_{mt})^\psi (H_{mt})^{1-\psi}
  \]

  where \( K_{mt} \) is the amount of physical capital used in the health sector, \( H_{mt} \) is the amount of human capital used in the goods sector.
The Government budget constraints:

The government taxes individuals’ labor income while working with two separate tax rates, \( \tau \) and \( \theta \) which fund public education spending and the Medicare subsidy, respectively. The two relevant constraints are therefore:

- **Government education funding**

  \[
  X_t = \tau_t w_t \sum_i^n h_{it}
  \]

  where \( X_t \) is the total amount of government funding devoted to education in period \( t \).

- **Government health care funding**

  \[
  Z_t = \theta_t w_t \sum_i^n h_{it} = p_{mt} z_t \sum_i^n \rho d_{it}
  \]

  where \( Z_t \) is the total amount of government funding devoted to health care subsidies in period \( t \).

III. Definition of a Competitive Equilibrium in this Economy:

A competitive equilibrium in this model will consist of a collection of sequences of household decisions \( \{c^y_t, c^e_t, s_t, e_t, d_t\}_{t=0}^{\infty} \), sequences of aggregate capital stocks \( \{K_t, H_t\}_{t=0}^{\infty} \) and their distribution between sectors \( \{K_{yt}, H_{yt}, K_{mt}, H_{mt}\}_{t=0}^{\infty} \), sequences of prices \( \{w_t, r_t, p^m_t\}_{t=0}^{\infty} \), and sequences of government policies \( \{z_t, \tau_t, \theta_t\}_{t=0}^{\infty} \) such that:

1. given prices and government policies, the household sequences solve the individual’s maximization problem

2. given prices, the firms in each sector are profit maximizing (no-arbitrage conditions hold)
3. capital and labor markets clear:

\[ H_{yt} + H_{mt} = H_t = \sum_{i=1}^{N} h_{it}, \]
\[ K_{yt} + K_{mt} = K_t = \sum_{i=1}^{N} s_{it-1} \]

4. The goods market clears: \( Y_t = \sum_{l=1}^{N} (c_{ij,t}^y + \rho c_{i-1,t}^y + s_{ij,t}) \)

5. The medical care market clears: \( M_t = \sum_{i=1}^{N} \rho d_{ij-1,t}^p \)

6. The government budgets are balanced:

\[ X_t = \tau_t w_t \sum_{i=1}^{N} h_{it} \]
\[ Z_t = \theta_t w_t \sum_{i=1}^{N} h_{it} = p_{mt} z_t \sum_{i=1}^{N} \rho d_{ij-1,t}^p \]

IV. Solving the model and finding the balanced growth path:

The individual household problem yields the following first order conditions:

\[ \left[ c_{j,t}^y \right] : \quad \frac{1}{c_{j,t}^y} = \lambda \]
\[ \left[ e_{j,t} \right] : \quad \frac{\eta y_2}{e_{j,t}} = \lambda \]
\[ \left[ c_{j,t+1}^p \right] : \quad \frac{\rho y_3}{c_{j,t+1}^p} = \lambda \left( \frac{\rho}{1 + r_{t+1}} \right) \]
\[ \left[ d_{j,t+1}^p \right] : \quad \frac{\rho y_4}{d_{j,t+1}^p} = \lambda \left( \frac{\rho (1 - z_{t+1}) p_{mt+1}}{1 + r_{t+1}} \right) \]

Which can be re-written as:

\[ c_{j,t+1}^p = \gamma_3 (1 + r_{t+1}) c_{j,t}^y \]
\[ e_{j,t} = \eta y_2 c_{j,t}^y \]
\[ d_{j,t+1}^p = \gamma_4 \left( \frac{1 + r_{t+1}}{1 - z_{t+1} p_{mt+1}} \right) c_{j,t}^y \]
These conditions together with the budget constraint

\[ c_{j,t}^y + \frac{\rho c_{j,t+1}^o}{1 + r_{t+1}} + \frac{\rho d_{j,t+1}^o}{1 + r_{t+1}} (1 - z_{t+1}) p_{mt+1} + e_{j,t} = (1 - \tau_t - \theta_t) w_t h_{j,t} \]

yield the following household decision rules:

\[ c_{j,t}^y = \left[ \frac{1}{(1+\eta_t + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_t - \theta_t) w_t h_{j,t} \]

\[ s_{1,t} = \left[ \frac{\rho \gamma_3 + \rho \gamma_4}{(1+\eta_t + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_t - \theta_t) w_t h_{j,t} \]

\[ e_{j,t} = \left[ \frac{\eta_t}{(1+\eta_t + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_t - \theta_t) w_t h_{j,t} \]

\[ d_{j,t+1}^o = \left[ \frac{\gamma_4}{(1+\eta_t + \rho \gamma_3 + \rho \gamma_4)} \right] \frac{(1+r_{t+1})}{(1-z_{t+1}) p_{mt+1}} (1 - \tau_t - \theta_t) w_t h_{j,t} \]

\[ c_{j,t+1}^o = \left[ \frac{\gamma_3}{(1+\eta_t + \rho \gamma_3 + \rho \gamma_4)} \right] (1 + r_{t+1}) (1 - \tau_t - \theta_t) w_t h_{j,t} \]

**Firm’s profit maximization problems:**

We will assume that both types of firms are competitive profit maximizers, and hire workers and capital accordingly. This yields the following conditions for prices:

1) **Goods sector:**

\[ 1 + r_{yt} = \alpha \frac{Y_t}{K_{yt}} \]

\[ w_{yt} = (1 - \alpha) \frac{Y_t}{H_{yt}} \]

2) **Medical sector:**

\[ 1 + r_{mt} = p_{mt} \psi \frac{M_t}{K_{mt}} \]

\[ w_{mt} = p_{mt} (1 - \psi) \frac{M_t}{H_{mt}} \]
The no-arbitrage conditions for the rate of return on human capital and physical capital across sectors yield the following conditions:

\[ \frac{1-\alpha}{\alpha} \left( \frac{K_{yt}}{H_{yt}} \right) = \frac{1-\psi}{\psi} \left( \frac{K_{mt}}{H_{mt}} \right) \]

For firms to be profit maximizing, it must be that the marginal products of both human and physical capital are equal in all sectors. This implies the additional condition:

\[ \alpha A \left( \frac{K_{yt}}{H_{yt}} \right)^{\alpha-1} = (1 - \alpha) A \left( \frac{K_{yt}}{H_{yt}} \right)^{\alpha} \]

which can be simplified to:

\[ \frac{\alpha}{1-\alpha} = \left( \frac{K_{yt}}{H_{yt}} \right) \]

And similarly for the medical sector:

\[ \frac{\psi}{1-\psi} = \left( \frac{K_{mt}}{H_{mt}} \right) \]

We can then solve for prices using the no-arbitrage conditions and we find:

\[ p_{mt} = \frac{\alpha}{\psi} \left( \frac{\alpha}{1-\alpha} \right)^{\alpha-1} \left( \frac{\psi}{1-\psi} \right)^{1-\psi} \], which is constant over time.

Using these conditions from the firms’ maximization problems, we can now apply them to the household and market clearing conditions to find the equilibrium.

We begin with the health care market. From the individual’s health care decision rule we know:

\[ d_{j,t+1} = \left[ \frac{\gamma_s}{(1+\eta_s+\rho_s+\rho_{jt})} \right]^{(1+\tau_{t+1})} \frac{(1-\tau_t - \theta_t) w_{t} h_{j,t}}{\left(1-\tau_{t+1}\right) p_{mt}} \]

And from the government health care budget we know:

\[ Z_t = \theta_t w_t \sum_i h_{it} = p_{mt} z_t \sum_i p_{ij}\rho d_{ij-i,t} \]
Imposing the simplifying restriction of homogeneity of households and $N=1$ for simplicity in this base model, we can rearrange the government budget condition above to yield:

$$1 - z_t = \frac{\rho p_{mt} d^o_{j,t} - \theta_t w_t H_{j,t}}{\rho p_{mt} d^o_{j,t}}$$

Plugging this into the decision rule yields:

$$d^o_{j,t+1} = \left[\frac{\psi}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}\right] \rho d^o_{j,t+1} p_{mt+1} - \theta_{t+1} w_{t+1} H_{j+1,t+1} (1 + r_{t+1}) \left((1 - \tau_t - \theta_t) w_t H_{j,t}\right)$$

Rearranging this we get the following:

$$\rho d^o_{j,t+1} p_{mt+1} - \theta_{t+1} w_{t+1} H_{j+1,t+1} = \left[\frac{\psi}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}\right] (1 + r_{t+1}) \left((1 - \tau_t - \theta_t) w_t H_{j,t}\right)$$

Next, we can use the market clearing condition for the medical sector:

$$\rho d^o_{j-1,t} = D(K_{mt})^{\psi}(H_{mt})^{1-\psi}$$

We then plug this into the previous equation to yield:

$$p_{mt+1} D(K_{mt})^{\psi}(H_{mt})^{1-\psi} - \theta_{t+1} w_{t+1} H_{j+1,t+1} =$$

$$\left[\frac{\psi}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}\right] \psi D \left(\frac{\psi}{1-\psi}\right)^{-1} (1 - \tau_t - \theta_t) (1 - \psi) D \left(\frac{\psi}{1-\psi}\right)^{\psi} H_{j,t}$$

Then, imposing the conditions from the firm’s problem and plugging in for wages and interest rates:

$$p_{mt+1} D H_{mt} \left(\frac{\psi}{1-\psi}\right)^{\psi} - \theta_{t+1} D (1 - \psi) \left(\frac{\psi}{1-\psi}\right)^{\psi} H_{j+1,t+1} =$$

$$\left[\frac{\psi}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}\right] \psi D \left(\frac{\psi}{1-\psi}\right)^{-1} (1 - \tau_t - \theta_t) (1 - \psi) D \left(\frac{\psi}{1-\psi}\right)^{\psi} H_{j,t}$$

Simplifying this equation:

$$p_{mt+1} \frac{H_{mt}}{H_{j+1,t+1}} - \theta_{t+1} (1 - \psi) = $$

$$\left[\frac{\psi}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}\right] \psi (1 - \psi) D \left(\frac{\psi}{1-\psi}\right)^{-1} (1 - \tau_t - \theta_t) \frac{H_{j,t}}{H_{j+1,t+1}}$$
Or more simply:

\[
\frac{H_{j,t}}{H_{j+1,t+1}} = 
\left[p_{mt+1} \frac{H_{mt}}{H_{j+1,t+1}} - \theta t+1 (1 - \psi) \left( \left( \frac{\gamma_4}{(1+\gamma_2+\gamma_3+\gamma_4)} \right) \psi (1 - \psi) D \left( \frac{\psi}{\psi - \psi} \right)^{\psi-1} (1 - \tau t - \theta t) \right)^{-1} \right]
\]

Then, note that since we’ve shown \( p_{mt} \) is constant, the growth factor \( \frac{H_{j+1,t+1}}{H_{j,t}} \) will be constant if \( \frac{H_{mt}}{H_{j+1,t+1}} \), the share of human capital employed in the health sector, is constant. To show this, we will utilize the goods market clearing condition:

\[
Y_t = c_t + \rho c_{t-1} + s_t
\]

Plugging in the household decision rules yields:

\[
Y_{t+1} = \left[ \frac{1}{(1+\gamma_2+\rho \gamma_3+\rho \gamma_4)} \right] \left( (1 - \tau_{t+1} - \theta_{t+1}) w_{t+1} H_{j+1,t+1} + \right.
\]

\[
\rho \left[ \frac{\gamma_3}{(1+\gamma_2+\rho \gamma_3+\rho \gamma_4)} \right] \left( 1 + r_{t+1} (1 - \tau_t - \theta_t) w_t H_{j,t} + \right.
\]

\[
\left[ \frac{\rho \gamma_3 + \rho \gamma_4}{(1+\gamma_2+\rho \gamma_3+\rho \gamma_4)} \right] \left( (1 - \tau_{t+1} - \theta_{t+1}) w_{t+1} H_{j+1,t+1} \right)
\]

Then, imposing the firm conditions for prices and production:

\[
A(K_{yt+1})^\alpha (H_{yt+1})^{1-\alpha} = 
\]

\[
\left[ \frac{1}{(1+\gamma_2+\rho \gamma_3+\rho \gamma_4)} \right] \left( (1 - \tau_{t+1} - \theta_{t+1}) (1 - \alpha) A(K_{yt+1})^\alpha (H_{yt+1})^{-\alpha} H_{j+1,t+1} + \right.
\]

\[
\rho \left[ \frac{\gamma_3}{(1+\gamma_2+\rho \gamma_3+\rho \gamma_4)} \right] \alpha A(K_{yt})^{\alpha-1} (H_{yt})^{1-\alpha} \left( (1 - \tau_t - \theta_t) (1 - \alpha) A(K_{yt})^\alpha (H_{yt})^{-\alpha} H_{j,t} + \right.
\]

\[
\left[ \frac{\rho \gamma_3 + \rho \gamma_4}{(1+\gamma_2+\rho \gamma_3+\rho \gamma_4)} \right] \left( (1 - \tau_{t+1} - \theta_{t+1}) (1 - \alpha) A(K_{yt+1})^\alpha (H_{yt+1})^{-\alpha} H_{j+1,t+1} \right)
\]
Simplifying gives the result:

$$1 = \left[ \frac{1}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)^{\frac{H_{j+1,t+1}}{H_{y,t+1}}} +$$

$$\rho \left[ \frac{\gamma_3}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] \alpha (1 - \alpha)^{\frac{(K_{yt})^{2-\alpha}(H_{yt})^{-\alpha}}{(K_{yt})^{2-\alpha}(H_{yt})^{-\alpha}}} (1 - \tau_t - \theta_t)^{\frac{H_{j,t}}{H_{y,t+1}}} +$$

$$\left[ \frac{\rho \gamma_3 + \rho \gamma_4}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)^{\frac{H_{j+1,t+1}}{H_{y,t+1}}}$$

Or further simplifying:

$$\frac{H_{yt+1}}{H_{j+1,t+1}} = \left[ \frac{(1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] +$$

$$\rho \left[ \frac{\gamma_3}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] \alpha (1 - \alpha)^{\frac{\alpha}{1-\alpha}} \left(1 - \tau_t - \theta_t \frac{1}{(1+g)}\right) +$$

$$\left[ \frac{\rho \gamma_3 + \rho \gamma_4}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)$$

Where \((1+g)\) is the growth factor along the balanced growth path. Note that the right hand side of the equation is constant over time, implying that \(\frac{H_{yt+1}}{H_{j+1,t+1}}\), the proportion of human capital used in the goods sector, is also constant. Also note that the market clearing condition for human capital implies:

$$\frac{H_{mt+1}}{H_{j+1,t+1}} = 1 - \frac{H_{yt+1}}{H_{j+1,t+1}}$$

Then, plugging this equality into the previous equation, we get:

$$\frac{H_{mt+1}}{H_{j+1,t+1}} = 1 - \left[ \frac{(1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] +$$

$$\rho \left[ \frac{\gamma_3}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] \alpha (1 - \alpha)^{\frac{\alpha}{1-\alpha}} \left(1 - \tau_t - \theta_t \frac{1}{(1+g)}\right) +$$

$$\left[ \frac{\rho \gamma_3 + \rho \gamma_4}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right] (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)$$
Then, simplifying we can re-write this as:

\[
\frac{H_{mt+1}}{H_{j+1,t+1}} = \frac{1}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \left( (1 + \eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4) - (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha) - \rho \gamma_3 \alpha (1 - \alpha) A \left( \frac{\alpha}{1-\alpha} \right)^{\alpha-1} \left( 1 - \tau_t - \theta_t \right) \frac{1}{(1+g)} \right) - (\rho \gamma_3 + \rho \gamma_4)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha))
\]

Returning to our original equation for the growth factor and plugging in this equation yields:

\[
\frac{H_{j,t}}{H_{j+1,t+1}} = (1 + g)^{-1} = \left( (1 + g)^{-1} \frac{p_{mt+1}}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right) \left( (1 + \eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4) - (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha) - \rho \gamma_3 \alpha (1 - \alpha) A \left( \frac{\alpha}{1-\alpha} \right)^{\alpha-1} \left( 1 - \tau_t - \theta_t \right) \frac{1}{(1+g)} \right) - (\rho \gamma_3 + \rho \gamma_4)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)) \right) - \theta_{t+1}(1 - \psi) \left( \frac{\gamma_4}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right) \psi(1 - \psi) D \left( \frac{\psi}{1-\psi} \right)^{\psi-1} \left( 1 - \tau_t - \theta_t \right)^{-1}
\]

Then, rearranging and solving for the growth factor:

\[
1 = \left( \frac{p_{mt+1}}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right) \left( (1 + \eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4) - (1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha) - \rho \gamma_3 \alpha (1 - \alpha) A \left( \frac{\alpha}{1-\alpha} \right)^{\alpha-1} \left( 1 - \tau_t - \theta_t \right) \frac{1}{(1+g)} \right) - (\rho \gamma_3 + \rho \gamma_4)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)) \right) - \theta_{t+1}(1 - \psi)(1 + g) \left( \frac{\gamma_4}{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)} \right) \psi(1 - \psi) D \left( \frac{\psi}{1-\psi} \right)^{\psi-1} \left( 1 - \tau_t - \theta_t \right)^{-1}
\]

Which we can then re-write as:

\[
(1 - \psi)(1 + g)\theta_{t+1} \left[ \frac{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}{p_{mt+1}} \right] + \left( 1 + \rho \gamma_3 + \rho \gamma_4 \right)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)) + \left( \frac{\gamma_4}{p_{mt+1}} \right) \psi(1 - \psi) D \left( \frac{\psi}{1-\psi} \right)^{\psi-1} \left( 1 - \tau_t - \theta_t \right) - (1 + \eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4) + \rho \gamma_3 \alpha (1 - \alpha) A \left( \frac{\alpha}{1-\alpha} \right)^{\alpha-1} \left( 1 - \tau_t - \theta_t \right) \frac{1}{(1+g)} \right) = 0
\]
This is a quadratic equation in the growth factor, which we can find solutions to using the quadratic formula as follows. Let:

\[ Q_a = (1 - \psi)\theta_{t+1} \left[ \frac{(1+\eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4)}{p_{mt+1}} \right] \]

\[ Q_b = (1 + \rho \gamma_3 + \rho \gamma_4)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)) + \]

\[ \left( \frac{\gamma_4}{p_{mt+1}} \right) \psi(1 - \psi)D \left( \frac{\psi}{1-\psi} \right)^{-1}(1 - \tau_t - \theta_t) \] - \[ (1 + \eta \gamma_2 + \rho \gamma_3 + \rho \gamma_4) \]

\[ Q_c = \rho \gamma_3 \alpha(1 - \alpha)A \left( \frac{\alpha}{1-a} \right)^{\alpha-1}(1 - \tau_t - \theta_t) \]

Then:

\[ (1 + g) = \frac{-Q_b \pm \sqrt{Q_b^2 - 4Q_aQ_c}}{2Q_a} \]

Evaluating this expression, we see that \((1 + g) > 1\) under conditions given in Appendix 1.

Unfortunately, while these conditions can pin down the sign of the growth rate little intuition can be gained from them due to the complexity of the conditions. Also found in Appendix 1 are results for the comparative statics exercises examining the effects of increased taxes and longevity on the growth rate. Conditions for positive effects are found, but little intuition can be gained from examination of these conditions. We therefore next turn to computational methods to better understand these results.
V. Model Simulation and Computational Experiments

In order to answer the questions posed by this model, and examine the effect of Medicare subsidies on growth when longevity increases, we undergo a computational experiment. To simulate this model, parameter values were chosen for the model to roughly correspond with the values in the U.S. economy. These parameter values are found in the table below.

<table>
<thead>
<tr>
<th>Preference Parameters</th>
<th>Technology Parameters</th>
<th>Taxes</th>
<th>Initial Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1 = 1.0$</td>
<td>$\alpha = 0.30$</td>
<td>$\tau = 0.1$</td>
<td>$H_0 = 9.0$</td>
</tr>
<tr>
<td>$\gamma_2 = 0.3$</td>
<td>$\Psi = 0.20$</td>
<td>$\theta = 0.1$</td>
<td>$H_1 = 10.0$</td>
</tr>
<tr>
<td>$\gamma_3 = 1.0$</td>
<td>$\eta = 0.40$</td>
<td></td>
<td>$K_0 = 10.0$</td>
</tr>
<tr>
<td>$\gamma_4 = 0.8$</td>
<td>$A = 3.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_0 = 0.3$</td>
<td>$B = 2.40$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1 = 0.5$</td>
<td>$D = 1.0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These parameter values were chosen to match the following targets from the data: a two percent annual growth rate, a ratio of private to public education investment of roughly 25%, and ratio of private health spending to old age consumption of roughly 0.8.

<table>
<thead>
<tr>
<th>Calibration Parameters</th>
<th>Target from Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual growth rate</td>
<td>2%</td>
</tr>
<tr>
<td>$E/X$ ratio of private to public education investment</td>
<td>0.25* (OECD Education database 2006)</td>
</tr>
<tr>
<td>$D(1-z)P_m/C^*$ ratio of private health spending to old age consumption</td>
<td>(0.66-1.00)* (NCPA 2007 Report on Health Spending of Seniors)</td>
</tr>
</tbody>
</table>
The experiment which was then conducted was to simulate the economy for 25 periods until a balanced growth path was achieved. In the next period, the survival rate (or longevity parameter) underwent a once and for all shock which increased the parameter from 0.3 to 0.5. In this economy, that is equivalent to the average lifespan increasing from 77 to 85, roughly corresponding with the U.S. data over the last 50 years. We then compare the effect of this increased longevity in economies with different levels of Medicare tax rates. The results of this experiment can be seen below:

<table>
<thead>
<tr>
<th>θ = 0.0</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>20</th>
<th>%Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>100.0</td>
<td>123.8</td>
<td>128.2</td>
<td>132.8</td>
<td>137.4</td>
<td>142.0</td>
<td>210.6</td>
<td>+110.6</td>
</tr>
<tr>
<td>Growth factor(annual)</td>
<td>1.0215</td>
<td>1.019498</td>
<td>1.020298</td>
<td>1.019978</td>
<td>1.020106</td>
<td>1.020055</td>
<td>1.02007</td>
<td>-0.140</td>
</tr>
<tr>
<td>E/X</td>
<td>0.2778</td>
<td>0.2283</td>
<td>0.2472</td>
<td>0.2393</td>
<td>0.2423</td>
<td>0.2411</td>
<td>0.2415</td>
<td>-13.067</td>
</tr>
<tr>
<td>D/C(\theta)</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>---</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>θ = 0.05</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>20</th>
<th>%Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>100.0</td>
<td>123.5</td>
<td>127.9</td>
<td>132.5</td>
<td>137.0</td>
<td>141.6</td>
<td>209.6</td>
<td>+109.6</td>
</tr>
<tr>
<td>Growth factor(annual)</td>
<td>1.0211</td>
<td>1.0191</td>
<td>1.0199</td>
<td>1.0196</td>
<td>1.0197</td>
<td>1.0197</td>
<td>1.0197</td>
<td>-0.137</td>
</tr>
<tr>
<td>E/X</td>
<td>0.2667</td>
<td>0.2192</td>
<td>0.2371</td>
<td>0.2297</td>
<td>0.2327</td>
<td>0.2315</td>
<td>0.2318</td>
<td>-13.086</td>
</tr>
<tr>
<td>D/C(\theta)</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>--</td>
</tr>
</tbody>
</table>
These results do not seem to indicate that the presence of a Medicare system plays much role in the growth levels of this economy. Increased longevity has a negative effect on growth under all levels of Medicare subsidy, and this negative effect seems to be slightly stronger when Medicare exists, but is roughly equivalent in all cases.

**VI. Results, Discussion, and Extensions:**

The results of the computational experiment for this economy indicate that under plausible parameter values with homogeneous agents, the direct channel of intergenerational redistribution offered by a Medicare program may be insufficient to generate positive growth effects from increased longevity. This result suggests that in order to find conditions under
which such effects could be positive, the model must be expanded in one or more ways. These
extensions will be undertaken in the next iteration of this study.

Several approaches could be used to highlight additional channels for positive effects for
longevity. The first I would like to explore is to alter the market structure of the firms so that
prices can change as demand changes in the marketplace. In this channel, increased longevity
will create increased demand for health services. The health care market is assumed to be more
human capital intensive, and therefore increased health demand will increase the return to human
capital relative to physical capital, and encourage parents to invest more in their children. (Note
that getting this effect would require a slight alteration of parental preferences such that utility
was gained from children’s consumption or income rather than human capital.) In this setup,
each parent will invest more in their child’s human capital production so that their child can earn
higher wages, however the aggregate effect of this will be to increase the tax base in the next
period and allow higher public investment as well as a spillover effect.

If this alteration to the model is able to produce the desired result of increased growth
when longevity increases, then further extensions can be made by looking at the distributional
implications in a model with heterogeneous agents, possible implications of endogenous
longevity, and a comparison of the relative effectiveness of in-kind transfers to seniors such as
Medicare and cash transfers such as Social Security.
Appendix 1: Proof of positive growth rate and comparative static results

Recall the definitions from the quadratic growth equation:

\[
Q_a = (1 - \psi)\theta_{t+1}\left[\frac{(1+\eta\gamma_2+\rho\gamma_3+\rho\gamma_4)}{P_{mt+1}}\right]
\]
\[
Q_b = (1 + \rho\gamma_3 + \rho\gamma_4)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)) + \left(\left[\frac{\gamma_4}{P_{mt+1}}\right] \psi(1 - \psi)D \left(\frac{\psi}{1-\psi}\right)^{\psi^{-1}}(1 - \tau_t - \theta_t)\right) - (1 + \eta\gamma_2 + \rho\gamma_3 + \rho\gamma_4)
\]
\[
Q_c = \rho\gamma_3 \alpha(1 - \alpha)A \left(\frac{\alpha}{1-\alpha}\right)^{\alpha^{-1}}(1 - \tau_t - \theta_t)
\]

Then:

\[
(1 + g) = \frac{-Q_b \pm \sqrt{Q_b^2 - 4Q_aQ_c}}{2Q_a}
\]

Evaluating this expression, we see that \((1 + g) > 1\) under the following conditions:

**Case 1:** \(1 < \frac{-Q_b + \sqrt{Q_b^2 - 4Q_aQ_c}}{2Q_a}, \ Q_b > 0\)

This implies \(2Q_a < -Q_b + \sqrt{Q_b^2 - 4Q_aQ_c}\)

Note that \(Q_a, Q_c > 0. \ Q_b > 0 \) when:

\[
\left[\left(1 + \rho\gamma_3 + \rho\gamma_4\right)((1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha)) + \left(\left[\frac{\gamma_4}{P_{mt+1}}\right] \psi(1 - \psi)D \left(\frac{\psi}{1-\psi}\right)^{\psi^{-1}}(1 - \tau_t - \theta_t)\right)\right] > (1 + \eta\gamma_2 + \rho\gamma_3 + \rho\gamma_4)
\]

This can be rearranged as:

\[
\left[\left(1 + \rho\gamma_3 + \rho\gamma_4\right)(-\tau_{t+1} - \theta_{t+1})(1 - \alpha) - (1 + \rho\gamma_3 + \rho\gamma_4)\alpha + \left(\left[\frac{\gamma_4}{P_{mt+1}}\right] \psi(1 - \psi)D \left(\frac{\psi}{1-\psi}\right)^{\psi^{-1}}(1 - \tau_t - \theta_t)\right)\right] > \eta\gamma_2
\]

Then for Case 1, two conditions must hold for growth to be positive:

**Condition 1A)** \(4Q_aQ_c < Q_b^2\)

**Condition 1B)** \(2Q_a + Q_b > \sqrt{Q_b^2 - 4Q_aQ_c}\)
Focusing on 1B first, note that if $Q_b > 0$ this can be re-written:

$$4Q_a^2 + 2Q_a Q_b + Q_b^2 > Q_b^2 - 4Q_a Q_c$$

Or more simply:

$$Q_a + \frac{1}{2} Q_b > -Q_c$$

Note that it is sufficient that $Q_b > 0$ for this condition to hold. For condition 1A to hold requires:

$$4(1 - \psi)\theta_{t+1} \left[ \frac{(1 + \eta\gamma_2 + \rho\gamma_3 + \rho\gamma_4)}{\theta_{t+1}} \right] \rho\gamma_3 \alpha(1 - \alpha)A \left( \frac{\alpha}{1 - \alpha} \right)^{\alpha - 1} (1 - \tau_t - \theta_t) < \left[ (1 + \rho\gamma_3 + \rho\gamma_4)(1 - \tau_{t+1} - \theta_{t+1})(1 - \alpha) + \left( \frac{\gamma_4}{\theta_{t+1}} \right) \psi(1 - \psi)D \left( \frac{\psi}{1 - \psi} \right)^{\psi - 1} (1 - \tau_t - \theta_t) \right] - (1 + \eta\gamma_2 + \rho\gamma_3 + \rho\gamma_4)^2$$

**Case 2:** $1 < \frac{-Q_b - \sqrt{Q_b^2 - 4Q_a Q_c}}{2Q_a}, \quad Q_b > 0$

This implies $2Q_a < -Q_b - \sqrt{Q_b^2 - 4Q_a Q_c}$

Then for Case 2, two conditions must hold for growth to be positive:

*Condition 2A*) $4Q_a Q_c < Q_b^2$

*Condition 2B*) $2Q_a + Q_b > -\sqrt{Q_b^2 - 4Q_a Q_c}$

Condition 2A is the same as 1A, therefore, focusing on 1B first, note that if $Q_b > 0$ this can be re-written:

$$4Q_a^2 + 2Q_a Q_b + Q_b^2 > -Q_b^2 + 4Q_a Q_c$$

Or

$$Q_a^2 + \frac{1}{2} Q_a Q_b + \frac{1}{2} Q_b^2 > Q_a Q_c$$

There are also two similarly analogous possible cases when $Q_b < 0$.  

19
Unfortunately, while these conditions can pin down the sign of the growth rate little intuition can be gained from them due to the complexity of the conditions. We must turn to computational methods to better understand this system.

**Results and Comparative Statics:**

The first question of interest to examine is: How does the growth factor vary with the Medicare tax rate? To find the answer we evaluate the following:

\[
\frac{\partial (1+g)}{\partial \theta} = \frac{\frac{\partial Q_a}{\partial \theta} + \frac{1}{2} \left( \frac{Q_b^2}{Q_a} - 4Q_aQ_c \right)^{-1/2} \left( 2Q_b \frac{\partial Q_b}{\partial \theta} - 4 \left( Q_a \frac{\partial Q_a}{\partial \theta} + Q_c \frac{\partial Q_c}{\partial \theta} \right) \right) 2Q_a - 2 \frac{\partial Q_a}{\partial \theta} \left( -Q_b \pm \sqrt{Q_b^2 - 4Q_aQ_c} \right)}{4Q_a^2}
\]

Where:

\[
\frac{\partial Q_a}{\partial \theta} = (1 - \psi) \left[ \frac{(1 + \eta + \rho \gamma_3 + \rho \rho_4)}{\rho_{mt+1}} \right]
\]
\[
\frac{\partial Q_b}{\partial \theta} = -(1 + \rho \gamma_3 + \rho \rho_4)(1 - \alpha) - \left[ \frac{\psi}{\rho_{mt+1}} \right] \psi(1 - \psi)D \left( \frac{\psi}{1 - \psi} \right) \psi^{-1}
\]
\[
\frac{\partial Q_c}{\partial \theta} = -\rho \gamma_3 \alpha(1 - \alpha)A \left( \frac{\alpha}{1 - \alpha} \right) \psi^{-1}
\]

Note that \(\frac{\partial Q_a}{\partial \theta} > 0, \frac{\partial Q_b}{\partial \theta} < 0, \frac{\partial Q_c}{\partial \theta} < 0\), and recall that \(Q_a > 0, Q_c > 0\, and\, Q_b\) can be of either sign.

We can also look at the question: How does the growth factor vary with the education tax rate?

\[
\frac{\partial (1+g)}{\partial \tau} = \frac{\frac{\partial Q_a}{\partial \tau} + \frac{1}{2} \left( \frac{Q_b^2}{Q_a} - 4Q_aQ_c \right)^{-1/2} \left( 2Q_b \frac{\partial Q_b}{\partial \tau} - 4 \left( Q_a \frac{\partial Q_a}{\partial \tau} + Q_c \frac{\partial Q_c}{\partial \tau} \right) \right) 2Q_a - 2 \frac{\partial Q_a}{\partial \tau} \left( -Q_b \pm \sqrt{Q_b^2 - 4Q_aQ_c} \right)}{4Q_a^2}
\]

Where:

\[
\frac{\partial Q_a}{\partial \tau} = 0
\]
\[
\frac{\partial Q_b}{\partial \tau} = -(1 + \rho \gamma_3 + \rho \rho_4)(1 - \alpha) - \left[ \frac{\psi}{\rho_{mt+1}} \right] \psi(1 - \psi)D \left( \frac{\psi}{1 - \psi} \right) \psi^{-1} < 0
\]
\[
\frac{\partial Q_c}{\partial \tau} = -\rho \gamma_3 \alpha(1 - \alpha)A \left( \frac{\alpha}{1 - \alpha} \right) \psi^{-1} < 0
\]

Finally, we examine how the growth factor varies with the survival rate:

\[
\frac{\partial (1+g)}{\partial \rho} = \frac{\frac{\partial Q_a}{\partial \rho} + \frac{1}{2} \left( \frac{Q_b^2}{Q_a} - 4Q_aQ_c \right)^{-1/2} \left( 2Q_b \frac{\partial Q_b}{\partial \rho} - 4 \left( Q_a \frac{\partial Q_a}{\partial \rho} + Q_c \frac{\partial Q_c}{\partial \rho} \right) \right) 2Q_a - 2 \frac{\partial Q_a}{\partial \rho} \left( -Q_b \pm \sqrt{Q_b^2 - 4Q_aQ_c} \right)}{4Q_a^2}
\]
Where:
\[
\frac{\partial Q_a}{\partial \rho} = \frac{(\gamma_3 + \gamma_4)(1-\psi)}{p_{mt+1}} > 0
\]
\[
\frac{\partial Q_b}{\partial \rho} = (\gamma_3 + \gamma_4)((1 - \tau - \theta)(1 - \alpha) - 1) < 0
\]
\[
\frac{\partial Q_c}{\partial \rho} = \gamma_3 \alpha (1 - \alpha) A \left( \frac{a}{1-a} \right)^{a-1} (1 - \tau - \theta) > 0
\]

Again, unfortunately little intuition can be learned by examining these comparative static results, and so we will turn to a computational exercise to determine the effects of the taxes on growth in this economy.
References:


