1. Canonical real business cycle models
   - Log-linear approximation to equilibrium conditions
   - Solution to dynamic linear models

2. Real business cycle models for emerging-market economies
   - Two real business cycle models for emerging-market economies
   - Sources of business cycles in EME RBC models
     - Permanent and temporary shocks to productivity
     - Country-premium shocks
     - Preference shocks
     - Spending shocks

3. Extensions of real business cycle models for emerging-market economies
   - Imperfect information regarding technology shocks
   - Labor-market search
   - Introduction of working-capital loans

2. The Kydland and Prescott model is a highly simplified, competitive system, in which a single good is produced by labor and capital with a constant-returns-to-scale technology.

3. All consumers are assumed to be infinitely-lived and identical.

4. The only 'shocks' to the system are exogenous, stochastic shifts in the production technology.

5. Can specific parametric descriptions of technology and preferences be found such that the movements induced in output, consumption, employment and other series in such a model by these exogenous shocks resemble the time series behavior of the observed counterparts to these series in the postwar, US economy?
1. The production technology is \( \exp(a_t)F(K_t, N_t) \), where \( F \) is homogenous of degree one in \((K, N)\).

2. Output is divided into consumption \( C_t \) and gross investment \( I_t \).

\[
Y_t = C_t + I_t
\]

3. Capital evolves according to a linear accumulation technology:

\[
K_{t+1} = I_t + (1 - \delta)K_t
\]

4. Households own all factors of production, renting them to profit-maximizing firms each period at wages and capital rentals \( W_t \) and \( R_t \). Markets for production factors are perfectly competitive.
We begin with the optimization problem of households.

Preferences

$$\sum_{t=0}^{\infty} \beta^t E_0[u(c_t, 1 - n_t)]$$

where $E_0$ denotes the expectation conditional on the information set at period 0.

Budget constraint at period $t$

$$c_t + k_{t+1} \leq W_t n_t + (R_t + 1 - \delta)k_t$$

At period 0, each household chooses a plan on current and future levels of consumption, hours worked, and stocks of capital by maximizing its welfare subject to a sequence of budget constraints, taking as given $\{W_t, R_t\}_{t=0}^{\infty}$ and $k_0$. 
A deterministic plan permits only one sequence of numbers for each choice variable such as consumption, hours worked, and capital stock.

There is only one path for current and future levels of total factor productivity.

Rational expectation requires perfect foresight for the time path of the total factor productivity.

A stochastic plan allows for the possibility that the future value of the total factor productivity can be realized randomly within a set of distinctly different values.

We assume that all households and firms know the true stochastic law of motion for the total factor productivity.
The state facing each household at each date is described by its own holdings of capital ($= k$), the capital stock in the economy as a whole ($= K$), and the current technology shock ($= a$).

The definition of state vector for each household: $s_t = \{k_t, K_t, a_t\}$.

The definition of the aggregate state vector: $S_t = \{K_t, a_t\}$.

The history of states: $s^t = (s_0, s_1, \ldots, s_t)$.

The representative household at period 0 chooses its stochastic plan by solving the following problem:

$$
\max \{\{c(s_t), n(s_t), k(s_t)\}\}_{t=0}^{\infty} \sum_{t=0}^{\infty} \sum_{s_t} \beta^t \text{prob}(s^t|s_0)\{u(c(s_t), 1 - n(s_t)) + \lambda(s_t)(W(S_t)n(s_t) + (R(S_t) + 1 - \delta)k(s_{t-1}) - c(s_t) - k(s_t))\}
$$
Optimization Conditions of Household’s Decision Problem

1. Consumption

\[ u_c(c_t, 1 - n_t) = \lambda_t \]

2. Labor

\[ u_l(c_t, 1 - n_t) = \lambda_t W_t \]

3. Capital Stock

\[ \lambda_t = \beta E_t[\lambda_{t+1}(R_{t+1} + 1 - \delta)] \]
Equilibrium Conditions

1. Symmetry of Individual Households: \( K_t = k_t; \ N_t = n_t; \ C_t = c_t. \)

2. Labor Market Equilibrium

\[
u_l(C_t, 1 - N_t) = u_c(C_t, 1 - N_t) \exp(a_t)F_n(K_t, N_t)\]

3. Capital Stock Evolution

\[
1 = \beta E_t \left[ \frac{u_c(C_{t+1}, 1 - N_{t+1})}{u_c(C_t, 1 - N_t)} (\exp(a_{t+1})F_k(K_{t+1}, N_{t+1}) + 1 - \delta) \right]
\]

4. Goods Market Equilibrium

\[
C_t + K_{t+1} - (1 - \delta)K_t = \exp(a_t)F(K_t, N_t)
\]

5. Solutions

\[
C_t = C(K_t, a_t); \quad N_t = N(K_t, a_t); \quad K_{t+1} = K(K_t, a_t)
\]
Log-Linear Approximation

1. Original (nonlinear) equilibrium conditions are replaced by a system of linear equations, which are rewritten in terms of each variable’s logarithmic deviation from its (deterministic) steady-state value.

2. Definition of logarithmic deviation: \( \hat{C}_t = \log C_t - \log C \) where \( C \) is the value of \( C_t \) at the deterministic steady state.

3. Relation between \( C_t \) and \( \hat{C}_t \):
   \[
   C_t = \exp(\ln C_t) = C \exp(\ln C_t - \ln C) = C \exp(\hat{C}_t)
   \]

4. First-order Taylor expansion of \( \exp(\hat{C}_t) \):
   \[
   \exp(\hat{C}_t) \approx 1 + \exp(0)\hat{C}_t
   \]
Equilibrium Conditions under Logarithmic Utility and Cobb-Douglas Production Functions

1. Utility Function and Production Function:

\[ u(C_t, 1 - N_t) = \ln C_t + b_n(1 - N_t); \quad Y_t = \exp(a_t)K_t^\alpha N_t^{1-\alpha} \]

2. Labor Market Equilibrium

\[ b_n \frac{C_t}{1 - N_t} = (1 - \alpha) \exp(a_t) \left( \frac{K_t}{N_t} \right)^\alpha \]

3. Capital Stock Evolution

\[ 1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} (\alpha \exp(a_{t+1}) \left( \frac{N_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta) \right] \]

4. Goods Market Equilibrium

\[ C_t + K_{t+1} - (1 - \delta)K_t = \exp(a_t)K_t^\alpha N_t^{1-\alpha} \]
Example of Log-Linear Approximation to the Holdings of Capital Stock

1. Express the original equilibrium condition in terms of logarithmic deviations

\[ \exp(-\hat{C}_t) = \beta E_t[\exp(-\hat{C}_{t+1})(R \exp(\hat{R}_{t+1}) + 1 - \delta)] \]

2. Find a term-by-term expression of the resulting equation

\[ \exp(-\hat{C}_t) = E_t[\beta R \exp(-\hat{C}_{t+1}) \exp(\hat{R}_{t+1}) + \beta(1 - \delta) \exp(-\hat{C}_{t+1})] \]

3. Apply the first-order approximation to each term

\[ -\hat{C}_t = E_t[\beta R(-\hat{C}_{t+1} + \hat{R}_{t+1}) - \beta(1 - \delta)\hat{C}_{t+1}] \]

4. Simplify the approximated equation by using the corresponding steady-state equilibrium condition

\[ -\hat{C}_t = E_t[-\hat{C}_{t+1} + \beta R\hat{R}_{t+1}] \]
Example of Log-Linear Approximation to the Holdings of Capital Stock

1. Express the original equilibrium condition in terms of logarithmic deviations

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4. Simplify the approximated equation by using the corresponding steady-state equilibrium condition

   \[ -\hat{C}_t = E_t[-\hat{C}_{t+1} + \beta R \hat{R}_{t+1}] \]
Example of Log-Linear Approximation to Labor-Market Equilibrium Conditions

1. Express the original equilibrium condition in terms of logarithmic deviations

\[
\exp(\hat{C}_t) = (1 - N)^{-1} \exp(\hat{W}_t)(1 - N \exp(\hat{N}_t))
\]

2. Find a term-by-term expression of the resulting equation

\[
\exp(\hat{C}_t) = \frac{\exp(\hat{W}_t)}{1 - N} - \frac{N \exp(\hat{W}_t + \hat{N}_t)}{1 - N}
\]

3. Apply the first-order approximation to each term

\[
\hat{C}_t = \frac{1}{1 - N} \hat{W}_t - \frac{N}{1 - N}(\hat{W}_t + \hat{N}_t)
\]

4. Simplify the approximated equation by using the corresponding steady-state equilibrium condition

\[
\frac{N}{1 - N} \hat{N}_t = \hat{W}_t - \hat{C}_t; \quad \hat{W}_t = a_t + \alpha(\hat{K}_t - \hat{N}_t)
\]
Labor Market Equilibrium

\[(\alpha + \frac{N}{1-N})\hat{N}_t = -\hat{C}_t + \alpha\hat{K}_t + a_t\]

Capital Stock

\[-\hat{C}_t = E_t[-\hat{C}_{t+1} + \beta R(a_{t+1} + (1 - \alpha)(\hat{N}_{t+1} - \hat{K}_{t+1}))]\]

Goods Market Equilibrium

\[\frac{C}{Y}\hat{C}_t + \frac{K}{Y}(\hat{K}_{t+1} - (1 - \delta)\hat{K}_t) = a_t + \alpha\hat{K}_t + (1 - \alpha)\hat{N}_t\]

\[ \dot{K}_{t+1} = a_{kk} \dot{K}_t - a_{kc} \dot{C}_t + a_{ka} a_t \]

2. Capital Stock + Goods Market Equilibrium

\[ E_t[\dot{C}_{t+1}] = a_{ck} \dot{C}_t + a_{cc} \dot{C}_t + a_{ca} a_t \]

3. Logarithmic level of the aggregate productivity

\[ a_t = \rho_a a_{t-1} + \epsilon_{a,t} \]
Method of Undetermined Coefficients

1. Guess linear decision rules for consumption and capital stock.

\[ \hat{C}_t = \phi_k \hat{K}_t + \phi_k a_t; \quad \hat{K}_{t+1} = \varphi_k \hat{K}_t + \varphi_k a_t \]

2. Verify that your guess is correct. We do this by substituting these two rules into equilibrium conditions and obtaining equations for coefficients of decision rules. We then solve the resulting equations for coefficients.

3. We have a quadratic equation for \( \phi_k \) and a linear equation for \( \phi_a \) given a value of \( \phi_k \):

\[ a_{kc} \phi_k^2 - (a_{kk} - a_{cc}) \phi_k + a_{ck} = 0; \quad \phi_a = \frac{a_{ca} + \phi_k (a_{kc} - a_{ka})}{\rho_a - a_{cc} - \phi_k a_{ka}} \]
1. **Capital Stock**

\[ \varphi_k = a_{kk} - a_{kc} \phi_k; \quad \varphi_a = a_{ka} - a_{kc} \phi_a \]

2. **Employment**

\[ \hat{N}_t = \frac{\alpha - \phi_k}{\alpha + N/(1 - N)} \hat{K}_t + \frac{\alpha - \phi_a}{\alpha + N/(1 - N)} a_t \]

3. **Output**

\[ \hat{Y}_t = (\alpha + \frac{(1 - \alpha)(\alpha - \phi_k)}{\alpha + N/(1 - N)}) \hat{K}_t + (1 + \frac{(1 - \alpha)(\alpha - \phi_a)}{\alpha + N/(1 - N)}) a_t \]
Construction of State-Space Representation

1. Write the law of motion for the state vector

\[
\begin{pmatrix}
\hat{K}_t \\
\hat{a}_t
\end{pmatrix} = \begin{pmatrix}
\varphi_k & \varphi_a \\
0 & \rho_a
\end{pmatrix} \begin{pmatrix}
\hat{K}_{t-1} \\
\hat{a}_{t-1}
\end{pmatrix} + \begin{pmatrix}
0 \\
1
\end{pmatrix} \epsilon_{a,t}
\]

where \( \varphi_k \) and \( \varphi_a \) are defined as

\[
\varphi_k = a_{kk} - a_{kc} \phi_k; \quad \varphi_a = a_{ka} - a_{kc} \phi_a
\]

2. Express endogenous variables in terms of state variables

\[
\begin{pmatrix}
\hat{Y}_t \\
\hat{C}_t \\
\hat{N}_t
\end{pmatrix} = \begin{pmatrix}
\alpha + \frac{(1-\alpha)(\alpha-\phi_k)}{\alpha+N/(1-N)} & 1 + \frac{(1-\alpha)(\alpha-\phi_a)}{\alpha+N/(1-N)} \\
\phi_k & \frac{1}{\alpha+N/(1-N)} \\
\frac{\alpha-\phi_k}{\alpha+N/(1-N)} & \frac{\alpha-\phi_a}{\alpha+N/(1-N)}
\end{pmatrix} \begin{pmatrix}
\hat{K}_t \\
\hat{a}_t
\end{pmatrix}
\]
The Mendoza model (1991) is a highly simplified, competitive system, in which a single good is produced by labor and capital with a constant-returns-to-scale technology.

All risk-averse consumers are assumed to be infinitely-lived and identical.

The only ‘shocks’ to the system are exogenous, stochastic shifts in the production technology.

There is non-contingent international one-period bond and the asset market is incomplete.
1. The production technology is \( \exp(a_t)K_t^{1-\alpha}N_t^\alpha \), where \( a_t \) is given by
   \[ a_t = \rho_a a_{t-1} + \epsilon_{a,t}. \]

2. The social resource constraint is
   \[ Y_t = C_t + I_t + NX_t. \]

3. Accumulation of international debt
   \[ q_t B_{t+1} = B_t - NX_t \]

4. Capital evolves according to a linear accumulation technology with a quadratic adjustment costs:
   \[ K_{t+1} = I_t + (1 - \delta)K_t + \frac{\phi}{2}(K_{t+1}/K_t - \exp(\mu_g)) \]

5. There is no role of permanent shocks to productivity.
## Empirical Regularities of Emerging-Market Business Cycles

<table>
<thead>
<tr>
<th></th>
<th>Emerging-Market Economies</th>
<th>Advanced Economies</th>
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<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>2.74</td>
<td>1.34</td>
</tr>
<tr>
<td>$\sigma(\Delta Y)$</td>
<td>1.87</td>
<td>0.95</td>
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<tr>
<td>$\rho(Y)$</td>
<td>0.76</td>
<td>0.75</td>
</tr>
<tr>
<td>$\rho(\Delta Y)$</td>
<td>0.23</td>
<td>0.09</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>1.45</td>
<td>0.94</td>
</tr>
<tr>
<td>$\sigma(TB/Y)$</td>
<td>3.22</td>
<td>1.02</td>
</tr>
<tr>
<td>$\rho(TB/Y, Y)$</td>
<td>-0.51</td>
<td>-0.17</td>
</tr>
</tbody>
</table>
1. Canonical RBC models of small open economies are not consistent with empirical stylized facts for emerging-market economies.

2. For example, canonical RBC models of small open economies generate pro-cyclical movements of the trade balance and smooth fluctuations of consumption.
   - A positive temporary shock to TFP increases consumption but less than output, thus leading to a surplus in the trade balance.
   - The standard deviation of consumption is less than that of output.
   - The trade balance is procyclical (or acyclical) in models with Cobb-Douglas preferences or weakly counter-cyclical in models with GHH preferences.
Key Features of Aguiar and Gopinath Model

1. The production technology is \( \exp(a_t)F(K_t, \Gamma_t N_t) \), where \( F \) is homogenous of degree one in \((K, N)\).

2. Output is divided into consumption \( C_t \) and gross investment \( I_t \).

\[
Y_t = C_t + I_t + NX_t
\]

3. Accumulation of international debt

\[
q_t B_{t+1} = B_t - NX_t
\]

4. Capital evolves according to a linear accumulation technology with a quadratic adjustment costs:

\[
K_{t+1} = I_t + (1 - \delta)K_t + \frac{\phi}{2}(K_{t+1}/K_t - \exp(\mu_g))
\]

5. Elasticity of the interest rate to changes in indebtedness

\[
r_t = r^* + \psi(\exp(B_{t+1}/\Gamma_t - b) - 1)
\]
Solow Residuals

1. Specification of production function
   - Cobb-Douglas production function
     \[ Y_t = \exp(a_t) K_t^{1-\alpha} (\Gamma_t N_t)^\alpha. \]
   - Permanent technology shocks
     \[ \Gamma_t = \exp(g_t) \Gamma_{t-1}, \quad g_t = (1 - \rho_g) \mu_g + \rho_g g_{t-1} + \epsilon_{g,t} \]
   - Temporary technology shocks
     \[ a_t = \rho_a a_{t-1} + \epsilon_{a,t} \]

2. Solow residuals
   \[ sr_t = a_t + \alpha \log \Gamma_t \]

3. Random-walk component and transitory component of Solow residuals
   \[ sr_t = \tau_t + s_t \]
   \[ \tau_t = \tau_{t-1} + \alpha \mu_g + \frac{\alpha \rho_g \epsilon_{g,t}}{1 - \rho_g} \]
1. Cobb-Douglas Utility Function

\[ u_t = \left( C_t^{1-\gamma}(1 - L_t)^{1-\gamma} \right)^{1-\sigma} / (1 - \sigma) \quad 0 < \gamma < 1 \]

Labor Supply Curve

\[ \frac{1 - \gamma}{\gamma} \frac{C_t}{1 - N_t} = w_t \]

2. Greenwood, Hercowitz and Huffman Preferences

\[ u_t = \left( C_t - \tau \Gamma_{t-1} L_t^\nu \right)^{1-\sigma} / (1 - \sigma) \]

Labor Supply Curve

\[ \tau \nu \Gamma_{t-1} N_t^{\nu-1} = w_t \]
Permanent Shocks to Productivity and Relative Consumption Volatility

1. Euler Equation

\[ MU_{c,t} = \beta(1 + r_t)E_t[MU_{c,t+1}] \]

2. The log-linearization of the Euler Equation around the deterministic steady state leads to the following equation for the ratio of the aggregate consumption to output

\[ \hat{C}_t - \hat{Y}_t = -\frac{1}{\gamma} \sum_{k=0}^{\infty} E_t[\hat{R}_{t+k}] + \sum_{k=0}^{\infty} E_t[\Delta \hat{Q}_{t+k}] \]

where \( \hat{Q}_t \) is defined as

\[ \hat{Q}_t = \frac{1}{\gamma} \{ (\sigma - 1)(1 - \gamma)\hat{L}_t - (\sigma(1 - \gamma) - 1)\hat{Y}_t \} \]

3. \( \hat{Q}_t \) depends on the calibration of parameters:
   - \( \hat{Q}_t = \hat{Y}_t \) when \( \gamma = 1 \)
   - \( \hat{Q}_t = \hat{L}_t \) when \( \gamma = 1/2 \) and \( \sigma = 2. \)
Long-Sample versus Short-Sample for Argentina and Mexico

<table>
<thead>
<tr>
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<th>Argentian</th>
<th>Mexico</th>
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</thead>
<tbody>
<tr>
<td>$\sigma(\Delta Y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>5.3</td>
<td>4.1</td>
</tr>
<tr>
<td>S</td>
<td>2.3</td>
<td>1.5</td>
</tr>
<tr>
<td>$\rho(\Delta Y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>S</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma(TB/Y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>5.2</td>
<td>4.3</td>
</tr>
<tr>
<td>S</td>
<td>2.6</td>
<td>2.2</td>
</tr>
<tr>
<td>$\rho(TB/Y)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>0.58</td>
<td>0.72</td>
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<tr>
<td>S</td>
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</table>
The autocorrelations of the trade balance predicted by the AG-type RBC model are close to unity, indicating that the model’s trade balance-to-output ratio behaves as a near random walk.

By contrast, the empirical autocorrelation function takes a value slightly below 0.6 at order one and then declines quickly toward zero.

The “near random-walk” behavior of the trade balance-to-output ratio in the AG-type RBC model is not a consequence of the presence of permanent productivity shocks.

Garcia-Cicco, Pancrazi, and Uribe (2010) argue that one way to eliminate the “near random-walk” behavior of the trade balance-to-output ratio is to introduce financial imperfections as in the sovereign debt literature.
Key Features of GPU Model

1. Greenwood, Hercowitz and Huffman Preferences

\[ u_t = \nu_t \{(C_t - \tau \Gamma_{t-1} L_t^\nu / \nu)^{1-\sigma} - 1\} / (1 - \sigma) \]

2. The social resource constraint is

\[ \exp(a_t) F(K_t, \Gamma_t N_t) = C_t + I_t + NX_t + S_t \]

3. Accumulation of international debt

\[ q_t B_{t+1} = B_t - NX_t \]

4. Capital evolves according to a linear accumulation technology with a quadratic adjustment costs:

\[ K_{t+1} = I_t + (1 - \delta) K_t + \frac{\phi}{2} (K_{t+1} / K_t - \exp(\mu g)) \]

5. Elasticity of the interest rate to changes in indebtedness

\[ r_t = r^* + \psi \left( \exp\left( B_{t+1} / \Gamma_t - b \right) - 1 \right) + \exp(\mu_t - 1) - 1 \]
Calibration of AG and GPU Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AG model</th>
<th>GPU model</th>
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<tr>
<td>$\beta$</td>
<td>0.98</td>
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<td>$\psi$</td>
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<td>$\sigma$</td>
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<td>$\phi$</td>
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<tr>
<td>$\delta$</td>
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<td>0.03</td>
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</table>
1. Trade deficits raise the net foreign debt position.
2. An increase in the net foreign debt position magnifies the country premium.
3. Larger country premia tend to increase domestic savings and discourage private investment, thus dampening the increase in the trade deficits.
4. As a result, a high elasticity of the interest rate with changes in the foreign debt position helps eliminate the “near random-walk” behavior of the trade balance-to-output ratio.