Transmission Mechanisms of the Public Debt in Open Economies

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We will have a brief discussion of Arellano (2008), Mendoza and Yue (2010) as well as Kim and Zhang (2011).

In these papers, the sovereign debt model of Eton and Gersovitz (1981) is used to characterize business cycles of small open economies.

In this class, my aim is to present modified versions of these models that lead to potentially significant impacts of monetary policy on output and inflation.

In addition, these modified models can be used to address the issue of over-borrowing that can arise when supply decisions of sovereign bonds (made by small open economies) are not correctly internalized in their bond prices.
Key Technical Points

- Non-Keynesian Effect of the Public Debt: The aggregate demand curve that is implied by these models might have a negative coefficient of the public debt.

- Endogenous Determination of “Rule of Thumb” Consumers: When the government chooses to default, households lose access to the world credit market and thus consume their labor incomes.
The government trades bonds with risk neutral competitive foreign creditors.

Debt contracts are not enforceable and the government can choose to default on its debt at any time.

If the government defaults, it is assumed to be temporarily excluded from international inter-temporal trading and to incur direct output costs.

The price of each bond available to the government reflects the likelihood of default events.

Households are identical and risk-averse. Their preferences at period 0 are represented by

$$E_0\left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

where $0 < \beta < 1$ is the time discount factor and $c_t$ is the consumption at period $t$.

When the government chooses to repay its debt, the resource constraint for the small open economy is

$$c = y + B - q(B', y)B'.$$

When the government chooses to default, consumption equals output:

$$c = y^{def}.$$
Foreign creditors have access to an international credit market in which they can borrow or lend as much as needed at a constant international rate \( r > 0 \).

In each period, lenders choose loans \( B' \) to maximize expected profits \( \phi \), taking prices given:

\[
\phi = qB' - \frac{1 - \delta}{1 + r} B'
\]

where \( \delta \) is the probability of default.

For positive levels of foreign asset holdings, \( B' \geq 0 \), the probability of default is zero and thus the price of a discounted bond is equal to zero. For negative holdings, \( B' < 0 \), the equilibrium price accounts for the risk of default:

\[
q = \frac{1 - \delta}{1 + r}.
\]

The country spread is defined as the country’s interest rate and the risk-free rate: \( r^e - r \) where \( 1 + r^e = q^{-1} \).
Government Decision on Default

- \( v^o(B, y) \) represents the value function for the government that has the option to default and that starts the current period with assets \( B \) and endowment \( y \).

- When the government defaults, the economy is in temporary financial autarky and income falls and equals consumption. The value of default is given by the following.

\[
v^d(y) = u(y^{\text{def}}) + \beta \int_{y'} (\theta v^o(0, y') + (1 - \theta)v^d(y')) f(y', y) dy'.
\]

- When the government chooses to remain in the credit relation, the value conditional on not defining is the following.

\[
v^c(B, y) = \max_{B'} \{u(y - q(B', y)B' + B) + \beta \int_{y'} v^o(B', y') f(y', y) dy'\}
\]
Endogenous “Laffer Curve” for Sovereign Government Debt

- $B^*$ represents the maximum level of borrowing from foreign creditors.
- The borrower would never choose optimally a bond contract with $B < B^*$ because he can find an alternative contract that increases consumption today by the same amount while incurring a smaller liability for next period.
- For the region $B' \in (B^*, \bar{B})$ to be non-empty, the bond price function needs to decrease slowly enough such that lower asset levels are associated with larger capital inflows.

\[
\frac{\partial (q(B')B')}{\partial B'} = q(B')(1 + \epsilon_q)
\]

where $\epsilon_q = q'(B')B' / q(B')$. 
Default causes an endogenous efficiency loss in production and bears an endogenous output cost.

1. The model links default with private economic activity via the financing cost of working capital used to pay for a subset of imported inputs.
2. Domestic and imported inputs are imperfect substitutes.

Output cost of default is an increasing and convex function of the total factor productivity in the final-goods sector.

1. The efficiency loss is larger when TFP is higher.
2. The cost is higher and becomes a steeper function of TFP at lower elasticities of substitution across inputs, because the inputs become less similar.
Households

1. Households choose consumption and labor supply so as to maximize a standard time-separable utility function.

2. The wealth effect on labor supply is eliminated by specifying period utility as a function of consumption net of the disutility of labor $g(L_t)$, where function $g(x)$ is increasing, continuously differentiable, and convex in its argument.

$$u(c_t, L_t) = u(c_t - g(L_t))$$

3. Households do not borrow directly from abroad, but the government borrows, pays transfers, and make default internalizing their utility.

4. The optimization problem of households is

$$\max_{c_t, L_t} E_0 \left[ \sum_{t=0}^{\infty} u(c_t, L_t) \right]$$

$$c_t = w_t L_t + \Pi_t^f + \Pi_t^m + T_t.$$
Firms in the final goods sector produce their outputs using labor and intermediate goods and a time-invariant capital goods \((=k)\). The production function is Cobb-Douglas:

\[
Y_{F,t} = A_t k^{\alpha_k} M_t^{\alpha_M} (L_t^f)^{\alpha_f}
\]

where \(A_t\) denotes the aggregate productivity shock at period \(t\), \(0 < \alpha_L, \alpha_M, \alpha_k < 1\) and \(\alpha_L + \alpha_M + \alpha_k = 1\).

The mix of intermediate goods is determined by a standard CES Arminton aggregator that combines domestic inputs \((= m^d_t)\) and imported inputs \((= m^*_t)\).

\[
M_t = (\lambda (m^d_t)^\mu + (1 - \lambda)(m^*_t)^\mu)^{\frac{1}{\mu}}
\]

The elasticity of substitution between imported and domestic intermediate goods is \(|1/(\mu - 1)|\).
1 Imported inputs are sold in the world markets at exogenous time-invariant prices $p_{jt}^{*}$ for $j \in [0, 1]$

2 A subset $\Omega$ of the imported input varieties defined by the interval $[0, \theta]$, for $0 < \theta < 1$, needs to be paid in advance using working capital financing.

3 Each firm’s purchases of variety of imported inputs are denoted by $m_{jt}^{*}$. The CES aggregator of imported inputs is given by

$$m_{t}^{*} = \int_{i=0}^{1} ((m_{it}^{*})^{\nu} di)^{1/\nu}$$

where the “within” elasticity of substitution across all varieties is given by $|1/(\nu - 1)|$. 

Working Capital Loans for Intermediate Goods

1. Working capital loans $\kappa_t$ are within-period loans provided by foreign creditors.

2. These loans are contracted and repaid after the uncertainty about the government’s repayment of its current debt service resolved.

3. Under this assumption, working capital loans are contracted at the risk-free world real interest rate denoted by $r^*_t$.

4. If government repays, firms borrow at $r^*_t$ and if it does not, they are excluded from world credit markets.

5. Pay-in-advance constraint condition for a subset of imported intermediate goods:

$$\frac{\kappa_t}{1 + r^*_t} \geq \int_0^\theta p^*_{j,t} m^*_{j,t} dj$$
A subset of imported inputs is not used when a country defaults because both firms and governments are excluded from world credit markets, and thus firms cannot obtain working capital financing to import inputs in the $\Omega$ set.

The price index of imported intermediate goods at a symmetric equilibrium becomes

$$P(r^*_t) = p^*/(1 - \theta + \theta(1 + r^*_t)^{-\frac{1-\nu}{\nu}})^{\frac{1-\nu}{\nu}}$$

where $0 < \nu < 1$.

When the country is in default and thus final goods producers cannot access working capital financing, the relative price of imported goods (as a whole) rises as $r^*_t \to \infty$. 

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Equilibrium Conditions for Production Factors and Their Prices

1. Total Demand for Intermediate Inputs: \( M_t = m_t^d(\lambda + (1 - \lambda)(\frac{P^*(r^*)}{p^m_t})^{\mu_{\mu-1}}) \)

2. Demand for Domestic Intermediate Input: \( p_t^m = \alpha_M k^{\alpha_k} \lambda A_t M^{\alpha_M-1} (L^f_t)^{\alpha_f} (m_t^d)^{\mu-1} \)

3. Labor Demand of Domestic Final Goods: \( w_t = \alpha_L k^{\alpha_k} A_t M^{\alpha_M} (L^f_t)^{\alpha_f-1} \)

4. Labor Demand of Domestic Intermediate Goods: \( L_t^m = (\frac{w_t}{\gamma \tilde{A} p^m_t})^{\frac{1}{\gamma-1}} \)

5. Labor Supply Curve: \( L_t^{\omega-1} = w_t \)

6. Market Clearing Condition for Labor: \( L_t^f + L_t^m = L_t \)

7. Production Function of Domestic Intermediate Goods: \( m_t^d = \tilde{A}(L_t^m)^{\gamma} \)
1 The sovereign government issue one-period, non-state-contingent discount bonds, so markets of contingent claims are incomplete.

2 The sovereign government cannot commit to repay its debt. When the country defaults it does not repay at date $t$ and the punishment is exclusion from world credit markets in the same period.

3 The range of bond face values is $B = [b_{\text{min}} \ b_{\text{max}}]$, where $b_{\text{min}} \leq 0 \leq b_{\text{max}}$. The lower bound is set to be $b_{\text{min}} > -\bar{y}/r$ (the largest debt that the country could repay with full commitment).

4 The sovereign government chooses a debt policy along with private consumption and factor allocations so as to solve a recursive social planner’s problem.
The payoff of the government is

\[ \nu(b_t, A_t) = \max \{ \nu^{nb}(b_t, A_t), \nu(A_t) \} \]

The value of “no default” is

\[ \nu^{nd}(b_t, A_t) = \max u(c, L_t) + \beta E_t[\nu(b_{t+1}, A_{t+1})] \]

subject to

\[ c_t + q_t(b_{t+1}, A_t)b_{t+1} - b_t \geq A_t f(M(m^d_t, m^*_t), L^f_t, k) - m^*_t P^*(r^*) \]

\[ L_t = L^f_t + L^m_t, \quad m^d_t = \tilde{A}(L^m_t)\gamma \]

The value of default is

\[ \nu^d(A_t) = \max u(c, L_t) + \beta E_t[(1 - \phi)\nu^d(A_{t+1}) + \phi \nu(0, A_{t+1})] \]

\[ c_t = A_t f(M(m^d_t, m^*_t), L^f_t, k) - m^*_t P^*(r^*) \]
1. The planner faces the same allocation of output and factors as the private sector.

2. The planner internalizes the household’s desire to smooth consumption and hence transfers to them an amount equal to the negative of the balance of trade.
Foreign Lenders

1. International creditors are risk-neutral and have complete information. They invest in one-period sovereign bonds and in within-period private working capital loans.

2. Foreign lenders behave competitively and face an opportunity cost of funds equal to $r^*$.  

3. Under the assumption of the zero profit condition of investors and the absence of arbitrage between returns on sovereign debt and the world’s risk-free asset, the bond pricing function is given by

\[
q_t(b_{t+1}, A_t) = \begin{cases} 
1 & \text{if } b_{t+1} \geq 0 \\
\frac{1-r^*}{1-r^*} & \text{if } b_{t+1} < 0 
\end{cases}
\]
The model’s recursive equilibrium is given by a decision rule for $b_{t+1}(b_t, A_t)$ for the sovereign government with associated with function $V(b_t, A_t)$, consumption and transfers rules $c(b_t, A_t)$ and $T(b_t, A_t)$, default set $D(b_t)$ and default probabilities $p^*(b_{t+1}, A_t)$ and an equilibrium pricing function for sovereign bonds $q^*(b_{t+1}, A_t)$ such that

1. Given the bond pricing function $q^*(b_{t+1}, A_t)$, the decision rule $b_{t+1}(b_t, \epsilon_t)$ solves the social planner’s recursive maximization problem.

2. The consumption plan $c(b_t, A_t)$ satisfies the resource constraint of the economy.

3. The transfer policy $T(b_t, A_t)$ satisfies the government budget constraint $T(b_t, A_t) = q(b_{t+1}, A_t)b_{t+1} - b_t$.

4. Given $D(b_t)$ and $p^*(b_{t+1}, A_t)$, the bond pricing function $q^*(b_{t+1}, A_t)$ satisfies the arbitrage condition of foreign lenders.
The efficiency loss in final goods production caused by sovereign default is the main driver of the endogenous output cost and hence of the model’s financial amplification mechanism.

There are three main effects that result from the loss of access to credit markets.

- The aggregate demand for $m^*$ always falls.
- There are indirect effects that lower the demands for total intermediate goods and labor in the final goods sector, because of the Cobb-Douglas structure of the production function of final goods sector.
- The loss of credit market access has effects on the output and labor allocations of the intermediate goods sector, but the direction of these effect changes with the value of $\mu$ (within the $(0, 1)$ range).
Empirical Motivation: In the past, foreign borrowing by developing countries was comprised almost entirely of government borrowing. Recently, private firms and individuals in developing countries borrow substantially from foreign lenders.

Modelling Strategy: Private agents decide how much to borrow, and the government decides whether to default.

Decentralized borrowing generates over-borrowing incentives of private agents because they fail to internalize the impact of their individual borrowing on aggregate borrowing costs.

In equilibrium, decentralized borrowing increases aggregate credit costs and sovereign default risk, and reduces aggregate welfare, relative to centralized borrowing.
If government decides to repay, households can trade one-period non-contingent bonds, taking as given the aggregate borrowing level and associated bond price.

An individual household with bond holding $b$ and income shock $y$ solves:

$$v^R(s) = \max_{b'}\{u(c) + \beta \int_{y'} [(1 - D')v^R(s') + D'v^D(y')]f(y', y)dy'\}$$

where $s = (b, y, B')$, $c = y + b - q(B', y)b'$, and $D' = D(B', y')$.

The default welfare is

$$v^D(y) = u(y^{\text{def}}) + \beta \int_{y'} [\theta v^R(\tilde{s}') + (1 - \theta)v^D(y')]f(y', y)dy'$$

where $\tilde{s}' = (0, y', \tilde{B}'')$ and $\tilde{B}'' = \Gamma(0, y')$. 
Consumption Euler Equation

- Government Default Decision:

\[ D(B, y) = \arg \max_{d \in \{0,1\}} \{(1 - d) v^R(B, y, \Gamma(B, y)) + d v^D(y)\} \]

where \(d = 1\) indicates default and \(d = 0\) indicates repayment.

- Consumption Euler Equations under Centralized and Decentralized Borrowing
  
  - Consumption Euler Equation with Centralized Borrowing

  \[ u'(c)[q(B', y) + \frac{\partial q(B', y)}{\partial B'} B'] = \beta \int_{y'} (1 - D(B', y')) u'(c') f(y', y) dy' \]

  - Consumption Euler Equation with Decentralized Borrowing

  \[ u'(c) = \beta \int_{y'} (1 - D(B', y')) u'(c') f(y', y) dy' \]
Extension of Baseline Models

- There is a continuum of small open economies indexed by a point in the unit interval $[0, 1]$.
- There is a random fixed (utility) cost if a country chooses to default at period $t$. The cumulative distribution of this random fixed cost is denoted by $F(\xi)$.
- Each country takes bond prices as given because it makes an insignificant contribution to the aggregate bonds market.
- Firms in the final goods sector fix their prices until their next re-optimizations of prices. In each period, the probability of re-optimization facing each firm is constant and exogenously determined.
Default Decision

- Default decision:

  \[
  \max\{v^{nd}(b_t, s_t), v^d(s_t) + \xi_t\}
  \]

- There is a threshold value of random fixed (utility) cost that makes the country indifferent between default and non-default:

  \[
  \bar{\xi}_t = v^{nd}(b_t, s_t) - v^d(s_t)
  \]

- The fraction of countries that choose to default at each period is

  \[
  F_t = F(v^{nd}(b_t, s_t) - v^d(s_t)).
  \]

  - The country chooses to default if \(\xi \leq \xi_t = v^{nd}(b_t, s_t) - v^d(s_t)\).
  - The country chooses to repay if \(\xi > \xi_t = v^{nd}(b_t, s_t) - v^d(s_t)\).

- The number of countries that are in the non-default state is

  \[
  N_t = (1 - F_t)N_{t-1} + \theta(1 - N_{t-1})
  \]

  where \(N_t\) denotes the number of countries that are in non-default state.
The value of “no default” is

\[ v^{nd}(b_t, s_t) = u(c, L_t) + \beta E_t[(1 - F_{t+1})v^{nd}(b_{t+1}, s_{t+1}) + F_{t+1}v^d(s_{t+1})] \]

The value of default is

\[ v^d(s_t) = u(c, L_t) + \beta E_t[\theta v^{nd}(0, s_{t+1}) + (1 - \theta)v^d(s_{t+1})] \]
Households

1. Period Utility Function

\[ u_t = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \psi L_t \]

2. Labor Supply Curve

\[ \psi c_t^\sigma = w_t \]

3. Euler Equation

\[ \frac{q_t}{c_t^\sigma} = \beta E_t \left[ \frac{1 - F_{t+1}}{c_t^\sigma \Pi_{t+1}} \right] \]
1 Social Resource Constraint

\[ Y_t = C_{T,t} + q_t b_{T,t+1} - (1 - F_t) b_{T,t} / \Pi_t \]

2 Aggregate Consumption

\[ C_{T,t} = N_t c_t + (1 - N_t)(w_t / \psi)^{1/\sigma} \]

3 Aggregate Real Debt

\[ b_{T,t} = N_{t-1} b_t \]

4 Monetary Policy

\[ R_t^* = R^*(\frac{\Pi_t}{\Pi})^{\phi_\pi} (\frac{Y_t}{Y})^{\phi_y} \]

where \( R_t^* \) is defined as

\[ R_t^* = E_t[1 - F_{t+1}] / q_t \]
Equilibrium Conditions for Firms

1. The first-order conditions of final-goods producers

\[ F_t = \frac{y_t}{c_t^\sigma} + \alpha \beta E_t [\prod_{t+1}^{\epsilon-1} F_{t+1}] \]

\[ K_t = \frac{w_t y_t}{c_t^\sigma} + \alpha \beta E_t [\prod_{t+1}^{\epsilon} K_{t+1}] \]

2. The evolution of the aggregate price level is

\[ 1 = (1 - \alpha) \left( \frac{\epsilon K_t}{(\epsilon - 1) F_t} \right)^{1-\epsilon} + \alpha \prod_{t}^{\epsilon-1} \]

3. The aggregate production function is

\[ y_t = \left( \frac{A_t}{\Delta_t} \right) L_t \]

4. The relative price distortion is

\[ \Delta_t = (1 - \alpha) \left( \frac{\epsilon K_t}{(\epsilon - 1) F_t} \right)^{-\epsilon} + \alpha \prod_{t}^{\epsilon} \Delta_{t-1} \]
1. In each period, a fraction of households (within the whole system) lose access to the credit market and thus become “rule of thumb” consumers.

2. The aggregate demand curve that is implied by consumption Euler equations shows a negative coefficient of the public debt.