Transmission Mechanisms of the Public Debt with Heterogenous Agents and Incomplete Markets

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We will have a brief discussion of Heathcote, Storesletten, and Violante (2009), which reviews the quantitative macroeconomic literature that focuses on household heterogeneity, with a special emphasis on the “standard” incomplete markets model.

In their review, they organize the vast literature according to three themes that are central to understanding how inequality matters for macroeconomics.

- What are the most important sources of individual risk and cross-sectional heterogeneity?
- What are individuals’ key channels of insurance?
- How does idiosyncratic risk interact with aggregate risk?

My discussion is focused on the Keynesian and non-Keynesian effect of fiscal policy in models with heterogenous agents and incomplete financial markets. My analysis is based on a simple example of the “standard” incomplete markets model that permits analytic closed-form solutions.
The Key Points

- The effectiveness of fiscal policy depends on the income and wealth effect of non-financial income and wealth in “standard” incomplete markets model as well as its effect on the inter-temporal substitution between current and future consumptions.

- In this talk, I try to describe fiscal policy implications of heterogenous agents and incomplete markets models in terms of the following aspects:
  - Income Effect of Government Spending
  - Wealth Effect of the Lagged Public Debt
  - Discount-Rate Channel of the Public Debt
  - Importance of the Time Profile of Forward Income Tax Rates
  - Potential Possibility of Expectations-Based Channel of Fiscal Policy

- These channels can arise naturally in “standard” incomplete markets model without having any additional frictions (even with abstracting from the private sector’s accumulation of physical capital).
Important Assumptions in Standard-Incomplete-Markets (SIM) Models

1. Time is discrete and indexed by $t = 0, 1, \cdots, \infty$. An infinitely lived household with discount factor $\beta < 1$ and time-separable preferences derives utility from streams of consumption $\{c_t\}_{t=0}^{\infty}$.

2. Period utility, $u(c_t)$, is strictly concave, strictly increasing, and differentiable.

3. The household faces a stochastic income endowment, $y_t$, with bounded support.

4. There are no state-contingent securities to insure idiosyncratic endowment risk, only a risk-free asset, $a_t$, which yields a constant gross interest rate $R$.

5. The household can save and borrow up to some exogenous limit (which could be zero), but no default is allowed.
1. Self-Insurance and Permanent Income Hypothesis:
   - In order to smooth consumption, the household “self-insures” by accumulating and selling assets.
   - As in Friedman’s permanent income hypothesis, consumption responds strongly to permanent earnings shocks but very little to transitory ones.

2. Permanent Hypothesis versus Precautionary Savings:
   - The notion of precautionary savings distinguishes this class of models from the strict version of the permanent income hypothesis, where agents have quadratic utility and face no debt limit, except for a no-Ponzi game condition.
   - Precautionary saving is defined as the increase in agents’ accumulated wealth that would obtain when switching from a deterministic income path to a stochastic income process.
   - The precautionary saving arises even without a positive third derivative, as long as households are risk-averse and face a borrowing limit that can bind due to risk.
Economic Environment: Modified Version of Standard-Incomplete-Markets (SIM) Models

1. Source of Heterogeneity: In each period, individual households face idiosyncratic preference shocks to the relative utilities between consumption and hours worked at the labor market. These shocks are independent over time and across households, which can be interpreted as productivity shocks to home production functions of individual households.

2. Incomplete Financial Markets: The only financial asset available to individual households is short-term (real) government bonds whose payoffs are not state-contingent. For the simplicity of the analysis, no capital accumulation is permitted.

3. Linear Production Technology: Firms employ labor services to produce their outputs by using the same linear production technology.

4. Fiscal Policy: The government controls labor income tax rates and the amount of the public debt to finance its spending.
A Simple Example Economy

1. Households
   - Preferences
     \[
     \sum_{t=0}^{\infty} \beta^t E_0 [\log c^h_t + (\gamma + \epsilon^h_t) \log x^h_t]
     \]
   - Budget Constraint
     \[
     c^h_t + b^h_t / (1 + r_t) = b^h_{t-1} + w_t (1 - x^h_t) (1 - \tau_t)
     \]

2. Social Resource Constraint
   \[
   c_t + g_t + \chi_t = 1
   \]

3. Government Budget Constraint
   \[
   b_t = (1 + r_t) (b_{t-1} + g_t - \tau_t (1 - \chi_t))
   \]
1 Labor-Supply Decisions

\[(1 - \tau_t)w_t x_t^h = (\gamma + \epsilon_t^h) c_t^h\]

2 Euler Equation

\[\frac{1}{c_t^h} = \beta(1 + r_t) E_t \left[ \frac{1}{c_{t+1}^h} \right].\]
1. We characterize an individual’s private consumption function by deriving present-value budget constraint of individual households.

2. We then aggregate individual private consumption functions linearly, leading to an expression of the aggregate consumption in terms of financial and human wealths.

3. We also use the aggregate labor-market wedge to derive the aggregate consumption function.

4. Logarithmic functions for consumption and leisure are used to facilitate this exact aggregation.
Present Value Budget Constraint

1. Period Budget Constraint

\[
\frac{b_t^h}{1 + r_t} = b_{t-1}^h + (1 - \tau_t)w_t - (1 + \gamma + \epsilon_t^h) c_t^h
\]

2. Present Value Budget Constraint

\[
c_t^h\left(\frac{1 + \gamma}{1 - \beta} + \epsilon_t^h\right) = b_{t-1}^h + (1 - \tau_t)w_t + f_t
\]

3. Stochastic Discount Factor for the Present Value Budget Constraint

\[
\beta_{t,t+k} = \prod_{i=1}^{k}(1 + r_{t+k})^{-1}
\]

where \(k = 1, 2, \ldots, \infty\) and \(\beta_{t,t} = 1\).
Expected Present Value of Disposable Full Incomes and Consumption Function

1. \( f_t \) represents the conditional expected present value of disposable full incomes period \( t+1 \) onward:

\[
f_t = \sum_{k=1}^{\infty} E_t[\beta_{t,t+k}(1 - \tau_{t+k})w_{t+k}]
\]

2. \( f_t \) is the same across households because every household has an identical time endowment.

3. Each household’s consumption depends on its holdings of government bonds and the realization of the idiosyncratic shock and the aggregate state as well:

\[
c_t(b^h_{t-1}, \epsilon^h_t) = \frac{b^h_{t-1} + (1 - \tau_t)w_t + f_t}{(1 + \gamma)/(1 - \beta) + \epsilon^h_t}
\]
Some Properties of Personal Consumption Functions

1. The asset holdings of an individual household’s consumption follows a Markov process conditional on aggregate state variables.

2. An individual household’s personal consumption function depends on its predetermined asset holdings and the realization of the idiosyncratic shock as well as aggregate state variables.

3. An individual household’s personal consumption function is linear in its asset holdings conditional on the realization of the idiosyncratic shock.

4. An individual household’s personal consumption function has a time-varying average propensity to consume, which depends on the realization of the idiosyncratic shock.
Exact Linear Aggregation of Personal Consumption Functions

1. Individual Households’ Policy Functions
   \[ c_t^h = c_t(b_{t-1}^h, \epsilon_t^h), \quad x_t^h = x_t(b_{t-1}^h, \epsilon_t^h), \quad b_t^h = b_t(b_{t-1}^h, \epsilon_t^h) \]

2. In each period, there is an equilibrium density function of wealth distribution (that is determined at the beginning of period \( t \)). The density function of bond holdings is denoted by \( \mu_t(b_{t-1}^h) \). Each household’s real holdings of government bonds has the same support denoted by \([b \quad \bar{b}]\) in each period.

3. Definition of Aggregate Consumption Function:
   \[ c_t = \int \int c_t(b_{t-1}^h, \epsilon)g(\epsilon)\mu_t(b_{t-1}^h)db_{t-1}^h d\epsilon \]

4. Linear Aggregation of Personal Consumption Functions
   \[ \frac{1 + \gamma}{1 - \beta} c_t + \Delta_{c,t} = b_{t-1} + (1 - \tau_t)w_t + f_t \]
### Endogenous Labor-Market Wedge

1. **Optimization Condition of an Individual’s Labor Supply Decision**

   \[(1 - \tau_t)w_t x(b_{t-1}^h, \epsilon) = (\gamma + \epsilon)c(b_{t-1}^h, \epsilon)\]

2. **Linear Aggregation**

   \[(1 - \tau_t)w_t x_t = \gamma c_t + \Delta_{c,t}\]

   where \(\Delta_{c,t}\) is defined as

   \[
   \Delta_{c,t} = \int_{\epsilon_{t}}^{\bar{\epsilon}} c_t(b_{t-1}^h, \epsilon)\epsilon\mu_t(b_{t-1}^h)g(\epsilon)db_{t-1}^h d\epsilon.
   \]

3. **Labor-Market Wedge**

   \[
   \frac{\Delta_{c,t}}{x_t} = (1 - \tau_t)w_t - \gamma \frac{c_t}{x_t}
   \]
Aggregate Consumption Function

1. Aggregate Consumption Function

\[ c_t = \alpha_t (b_{t-1} + f_t + (1 - \tau_t)g_t) \]

2. Present Value of Disposable Full Income Stream

\[ f_t = \frac{1}{1 + r_t} (E_t[(1 - \tau_{t+1})w_{t+1}] + E_t[f_{t+1}]) \]

3. Marginal (Average) Propensity to Consume

\[ \alpha_t = \frac{1 - \beta}{(1 + \beta)\gamma - (1 - \tau_t)(1 - \beta)w_t} \]
Wealth Dynamics of Individual Households

1. Use the period budget constraint of each household to derive its wealth dynamics:

\[ b^h_t = (1 + r_t)(\gamma^h_{b,t}(b^h_{t-1} + (1 - \tau_t)w_t) - \gamma^f_{f,t}f_t) \]

where time-varying coefficients \( \gamma^h_{b,t} \) and \( \gamma^h_{f,t} \) are defined as

\[ \gamma^h_{b,t} = \frac{(1 + \gamma)\beta}{1 + \gamma + (1 - \beta)\epsilon^h_t}, \quad \gamma^h_{f,t} = \frac{(1 - \beta)(1 + \gamma + \epsilon^h_t)}{1 + \gamma + (1 - \beta)\epsilon^h_t} \]

2. Equilibrium Dynamics of Household’s Wealth

\[ b^h_t = (1 + r_t)(\gamma^h_{b,t}(b^h_{t-1} + 1 - \tau_t) - \gamma^h_{f,t}f_t) \]
An Individual Household’s Euler Equation

\[
\frac{1}{c_t^h} = \beta(1 + r_t)E_t[\frac{1}{c_{t+1}^h}] 
\]

Definition of the Euler Equation Wedge

\[
\omega_{c,t+1} = \int_{\bar{\epsilon}}^{\bar{\epsilon}} \int_{\bar{b}}^{\bar{b}} \frac{c_t^h}{c_t} \left[ \int_{\bar{\epsilon}}^{\bar{\epsilon}} \frac{c_{t+1}^h g(\epsilon_{t+1})}{c_{t+1}^h} d\epsilon_{t+1} \right] \mu(b_{t-1}^h)g(\epsilon_t) db_{t-1}^h d\epsilon_t 
\]

Aggregate Euler Equation

\[
1 = (1 + r_t)E_t[\beta_{t,t+1}] 
\]

Aggregate Stochastic Discount Factor

\[
\beta_{t,t+1} = \beta \frac{c_t}{c_{t+1}^h} \omega_{c,t+1} 
\]
Aggregate Equilibrium Conditions

Given a sequence of \( \{\tau_t, g_t\}_{t=0}^{\infty} \) and an initial value of the public debt \( b_{-1} \) at period 0, a set of aggregate variables \( (c_t, y_t, \alpha_t, f_t, x_t, b_t, r_t) \) satisfies the following set of aggregate equilibrium equations:

\[
\begin{align*}
    c_t &= \alpha_t (b_{t-1} + f_t + (1 - \tau_t)g_t) \\
    y_t &= \alpha_t (b_{t-1} + f_t) + g_t (\alpha_t (1 - \tau_t) + 1) \\
    \alpha_t &= \frac{1 - \beta}{(1 + \beta)\gamma - (1 - \tau)(1 - \beta)} \\
    f_t &= \frac{1}{1 + r_t} (E_t[1 - \tau_{t+1}] + E_t[f_{t+1}]) \\
    x_t &= 1 - c_t - g_t \\
    b_t &= (1 + r_t)(b_{t-1} + g_t - (1 - x_t)\tau_t) \\
    1 &= \beta(1 + r_t)E_t[\frac{c_t}{C_{t+1}}\omega_{c,t+1}]
\end{align*}
\]
Fiscal Policy Implications of Heterogenous Agents and Incomplete Markets Models

- Income Effect of Government Spending
- Wealth Effect of the Lagged Public Debt
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Equilibrium Solutions in the Representative Agents Model

1. Consumption
   \[ c_t = \frac{(1 - \tau_t)(1 - g_t)}{1 + \gamma - \tau_t} \]

2. Output
   \[ y_t = \frac{1 - \tau_t + \gamma g_t}{1 + \gamma - \tau_t} \]

3. Leisure
   \[ x_t = \frac{\gamma (1 - g_t)}{1 + \gamma - \tau_t} \]

4. Real Interest Rate
   \[ 1 = \beta (1 + r_t) E_t \left[ \frac{c_t}{c_{t+1}} \right] \]
“Near Perfect” Aggregation Result by Krusell and Smith (1997): A representative agent model is a good approximation of the heterogenous-agents economy with incomplete financial markets.

In SIM models, consumption decision rules are generally concave in wealth, so aggregate consumption will depend on the distribution of wealth, and perfect aggregation will fail.

However, Krusell and Smith (1997) and subsequent users of their methodology have found that even though SIM economies do not perfectly aggregate in theory, these models often deliver “approximate aggregation” in practice.

Krusell and Smith coined this term to label their key result that “in equilibrium all aggregate variables [...] can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution and the aggregate productivity shock.”

Reference: Heathcote, Storesletten, and Violante (2009)
Fiscal Policy with Heterogenous Agents and Incomplete Markets (Heathcoate, 2005)

Changes in the timing of taxes that would be neutral in a representative agent model (Ricardian equivalence) turn out to have large real effects in a model with heterogeneous agents and incomplete markets.

Fiscal Policy in an Incomplete Markets Economy (Gomes, Michaelides, and Polkovnichenko, 2008)

For a given level of government expenditures, a 10% increase in government debt to GDP decreases the capital stock between 1.0% and 1.5%, depending on how the new debt is financed, while the cost of government debt increases by between 6 to 12 basis points, inducing households to hold the extra bonds.

Given the crowding out of investment, the return on capital also rises.
The income effect of the current period’s government spending can arise in models of heterogenous agents and incomplete markets because of current period’s labor income effect.

The magnitude of the current government spending’s income effect depends on the its impact on agents’ expectations about future disposable incomes, which are affected by the time profile of future income tax rates.

The income effect of the current period’s government spending tends to be significant to the extent which a change in the current period’s government spending does not induce a large offsetting impact of the expected present-value of future disposable incomes.

This income effect of government spending does not exist in the representative agent model.
A rise in the level of the public debt leads to an increased rate of real interest.

The increased rate of real interest raises the discount rate of future disposable incomes, leading to a fall in the expected present value of future disposable incomes even without substantial rises of future income taxes.

The aggregate consumption and output drop with decreases in the expected present value of future disposable incomes.

This channel arises from the income effect of future disposable earnings, not from the inter-temporal substitution between current and future consumptions.

This channel also arises without relying on the presence of the private sector’s assets such as the physical capital.
Effect of Government Spending Shocks: Constant Tax Rates

![Graphs showing the effect of government spending shocks on output, consumption, interest rate, and public debt.](image)
Effect of Unexpected Deficit Reduction: Anticipated Future Tax Decreases

- **Output**
- **Consumption**
- **Interest Rate**
- **Public Debt**
- **Tax Rate**
- **Spending**
1 In DSGE models with heterogenous agents and incomplete markets, an increase in government spending can increase output and consumption when agents expect constant tax rates.

2 However, the same increase in government spending can lower consumption in the corresponding representative agent model.

3 In DSGE models with heterogenous agents and incomplete markets, output and consumption can increase when unexpected deficit reduction is initiated by an unexpected fall in government spending with anticipated future tax decreases. However, the same deficit reduction plan does not increase output in the corresponding representative agent model.

4 This channel also arises without relying on the presence of the private sector’s assets such as the physical capital.