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We review an ad-hoc analytic framework of Angeletos (2003) that is included in his comments on Benigno and Woodford (2003)’s “Optimal Monetary and Fiscal Policy.”

His analysis is essentially a linear-quadratic approach to the optimal monetary and fiscal policy.

We will see that a linear-quadratic approach can be used to analyze the optimal choice of fiscal policy regimes. My emphasis is given on the possibility that the IS curve shifts with changes in the public debt.

The Ramsey approach to a set of non-linear DSGE models is presented as a micro-foundation of this linear-quadratic approach. We also plan to address the optimal design of fiscal policy regimes when the government can issue long-term bonds.
We want to understand the reason why the government allows fiscal stress to play an important role in the determination of inflation and output when it should adopt the optimal fiscal and monetary policy.

The fiscal stress is defined as the present-value of government spending streams plus optimal subsidy that would have been necessary to implement the first best allocation.

When the government should generate unexpected variations in the aggregate inflation, it is optimal to rely on innovations in the exogenous “fiscal stress” variable, especially when it cannot issue state-contingent bonds.

We discuss the implication of this feature for the optimal design of fiscal policy regimes, which might be relevant for recent fiscal policy issues.
Existing Results

1. Optimal Fiscal Policy in Real Models
   - The optimal income tax rate is essentially invariant in the presence of state-contingent government bonds.
   - The optimal tax rate follows essentially a random walk in the absence of state-contingent government bonds.

2. Role of Optimal Inflation in Models of Only Nominal Bonds
   - Flexible Price Model: Unexpected variations in inflation help replicate the allocation that would have been attained in the presence of state-contingent bonds.
   - Sticky Price Model: The government should take into account the trade-off between the relative price distortion and insurance.
Results of Angeletos (2003)

1. Fiscal policy dominance for output stabilization in the presence of lump-sum taxation or complete insurance
   - When the government is free to make any state-contingent lump-sum transfers to the private sector, the shadow value of government budget constraint is zero. When lump-sum transfers are available but the government can issue state-contingent debt, the shadow value of government budget constraint is a positive constant.
   - When the government has access to either lump-sum taxation or complete insurance, the inflation rate is always zero and the output gap is constant.
   - As a result, fiscal policy is primarily responsible for output stabilization.

2. Monetary policy complements fiscal policy when there is incomplete insurance.
   - When there is incomplete insurance, the output gap has a unit root and optimal tax gap follows a random walk. The shadow value of government budget constraint varies over time.
   - Innovations in inflation and output are driven by innovations in an exogenous “fiscal stress” variable.
The key point of the present analysis replaces the choice of state-contingent lump-sum transfers (for complete insurance) by the choice of fiscal policy regime.

When there is incomplete insurance and the IS curve has a positive debt coefficient, the government should choose a fiscal policy regime that attains a constant shadow value of government budget constraint (for example, a zero value).

When there is incomplete insurance and the IS curve has a negative debt coefficient, the government should choose a fiscal policy regime that create time-varying shadow-values of government budget constraint.
A Linear-Quadratic Example

1. Social Welfare

\[ U = - \sum_{t=0}^{\infty} \beta^t E_0 \left\{ (y_t - y_t^*)^2 + \omega \pi_t^2 \right\} \]

2. Aggregate Supply Curve

\[ y_t = -\psi \tau_t + \chi (\pi_t - \beta E_t[\pi_{t+1}]) + \epsilon_t \]

3. Government Budget Constraint

\[ b_{t-1} = \beta b_t + \tau_t + z_t + \bar{d} (\pi_t - \beta E_t[\pi_{t+1}]) - g_t. \]

Fiscal Policy Instruments and Monetary Policy Behavior

1. Labor income tax rate \( (= \tau_t) \): Income tax rate affects the aggregate output and government budget constraint.

2. Government spending \( (= g_t) \): Government spending affects the aggregate demand and government budget constraint.

3. State-contingent lump-sum transfers \( (= z_t) \): Government can use state-contingent transfers to provide consumption insurance.

4. The monetary policy behavior is specified in terms of the cyclical behavior of inflation.
Equilibrium Conditions

1. Optimality Condition for the Aggregate Output
   \[ y_t - y_t^* = -\frac{1}{\psi} \lambda_t \]

2. Optimality Condition for the Aggregate Inflation
   \[ \pi_t = \frac{1}{\omega} \left( \frac{\chi}{\psi} + \bar{d} \right) (\lambda_t - \lambda_{t-1}) \]

3. Shadow Value for the Government Budget Constraint
   \[ \lambda_t = E_t[\lambda_{t+1}] \]

4. Aggregate Supply Curve
   \[ y_t = -\psi \tau_t + \chi (\pi_t - \beta E_t[\pi_{t+1}]) + \epsilon_t \]

5. Government Budget Constraint
   \[ b_{t-1} = \beta b_t + \tau_t + z_t + \bar{d} (\pi_t - \beta E_t[\pi_{t+1}]) - g_t. \]
The implied optimal relation between inflation and output is

$$\pi_t = -\frac{\chi + \psi \bar{d}}{\omega} (x_t - x_{t-1})$$

where $x_t (= y_t - y_t^*)$ is the output gap at period $t$.

The optimal targeting rule implies that inflation should fall when the current level of the output gap rises relative to its lagged level.

We then use the aggregate demand equation to specify the feed-back rule of the short-term nominal interest rate that implements this optimal prescription for monetary policy.
1. Government has unlimited access to lump-sum taxation so that it can freely choose \( z_t \). In this case, \( \lambda_t = 0 \).

2. Lump-sum taxation is not available, but government can issue state-contingent debt (or otherwise replicate full insurance). In this case, the government chooses subject to the constraint \( E_{t-1}[z_t] = 0 \). In this case, \( \lambda_t = \bar{\lambda}(>0) \).

3. Government cannot issue state-contingent bonds, so that it should set \( z_t = 0 \). In this case, \( \lambda_t \) varies over time.
Equilibrium Conditions with Complete Markets

1. Optimality Condition for the Aggregate Output
\[ y_t - y_t^* = -\frac{1}{\psi} \bar{\lambda} \]

2. Optimality Condition for the Aggregate Inflation
\[ \pi_t = 0 \]

3. Shadow Value for the Government Budget Constraint
\[ \lambda_t = \bar{\lambda} \]

4. Aggregate Supply Curve
\[ y_t^* - \frac{1}{\psi} \bar{\lambda} = -\psi \tau_t + \epsilon_t \]

5. Government Budget Constraint
\[ b_{t-1} = \beta b_t + \tau_t + z_t - g_t. \]
1 Two Intertemporal Equations

\[
\left(\frac{1}{\psi} + \delta\right)\lambda_t = \delta\lambda_{t-1} + \psi\hat{\tau}_t
\]

\[
E_t[\lambda_{t+1}] = \lambda_t
\]

where \(\delta = (\chi/\omega)(\chi/\psi + \bar{d})\).

2 Two Intratemporal Equations

\[
\chi_t = -\psi\tau_t + \chi\pi_t
\]

\[
\chi\pi_t = \frac{1}{\psi}\lambda_t + \psi\hat{\tau}_t
\]

where \(x_t = y_t - y_t^*\) and \(\hat{\tau}_t = \tau_t - \tau_t^*\).
Some Technical Points

1. Four linear equations for four endogenous variables ($\lambda, \pi, \hat{\tau}$).

2. No explicit inclusion of exogenous variables in this characterization of equilibrium conditions with incomplete markets.

3. Existence of an expectation channel through which fiscal stress can affect the private sector and thus the monetary policy behavior.

4. Use of government budget constraint to define a “fiscal stress” variable that is the key variable for the expectation channel.

5. Realized values of output gap, inflation, tax gap, the multiplier of government budget constraint are affected by the expectation error of the fiscal stress, where $f_t$ denotes the fiscal stress at period $t$ and its expectation error at period $t$ is $\xi_t = f_t - E_{t-1}[f_t]$. 
Characterization of Fiscal Stress

1. Difference equation for $b_t$

$$b_{t-1} = \beta b_t + \hat{\tau}_t + \bar{d}\pi_t + \tau^*_t - g_t$$

2. Forward-looking solution

$$b_{t-1} = \sum_{i=0}^{\infty} \beta^i E_0[\hat{\tau}_{t+i} + \bar{d}\pi_{t+i} + \tau^*_{t+i} - g_{t+i}]$$

3. Fiscal stress

$$(1 - \beta)b_{t-1} + f_t = \hat{\tau}_t + (1 - \beta)\bar{d}\pi_t$$

where $f_t$ represents the fiscal stress:

$$f_t = (1 - \beta)\sum_{i=0}^{\infty} \beta^i E_0[g_{t+i} - \tau^*_{t+i}]$$
1. Initial condition for the shadow value of the government budget constraint

\[ \lambda_{t-1} = \psi^2((1 - \beta)b_{t-1} + E_{t-1}[f_t]) \]

2. Government budget constraint

\[ \frac{1}{\psi^2} \lambda_{t-1} + \xi_t = \hat{\tau}_t + (1 - \beta)\bar{d}\pi_t \]

3. Optimal tax rates:

\[ \hat{\tau}_t = \hat{\tau}_{t-1} + \varphi_{\tau} \xi_t \]

4. Optimal output gap:

\[ x_t = x_{t-1} - \varphi_{x} \xi_t \]

5. Optimal inflation rate:

\[ \pi_t = \varphi_{\pi} \xi_t \]
Welfare gain of state-contingent debt: There are welfare gains to be made if the government can trade state-contingent debt.

Potential resolution in the absence of state-contingent debt: If state-contingent debt is not available, the government could obtain insurance by appropriately designing the maturity structure of the public debt or the cyclical properties of consumption taxes.

Relative effectiveness of fiscal and monetary policies as instruments for managing the business cycles

- Fiscal policy can do all if stat-contingent transfers are available.
- Monetary policy complements fiscal policy if stat-contingent transfers are not available.
A Simple New Keynesian Model

1. **Aggregate Supply Curve**

\[ \pi_t = \beta E_t[\pi_{t+1}] + \kappa x_t \]

2. **IS Curve**

\[ x_t = \sigma_x E_t[x_{t+1}] - \sigma_r (r_t - E_t[\pi_{t+1}]) + \sigma_b b_t \]

3. **Government Budget Constraint**

\[ b_t = r_t + \beta^{-1} (b_{t-1} - \pi_t) - \frac{1 - \beta}{\beta} s_t. \]
1. Government minimizes the quadratic loss of output gap and inflation subject to aggregate supply curve and its budget constraint.

2. State-contingent lump-sum transfers for complete insurance are not available.

3. The IS curve includes the real public debt as its argument.
   - $\sigma_b > 0$: Wealth Effect of the Public Debt
   - $\sigma_b < 0$: Non-Keynesian Effect of the Public Debt
   - $\sigma_b = 0$: Standard Specification of the IS Curve

4. Tax distortion is not necessarily required.

5. Primary real surplus is exogenously determined.
Motivation for the IS Curve with the Public Debt

1. Wealth Effect of the Public Debt vs. Non-Keynesian Effect of the Public Debt
   - $\sigma_b > 0$: Wealth Effect of the Public Debt
   - $\sigma_b < 0$: Non-Keynesian Effect of the Public Debt
   - $\sigma_b = 0$: Standard Specification of the IS Curve

2. Endogenous Choice of a Fiscal Policy Regime
   - Ricardian Fiscal Policy Regime: The government budget constraint is irrelevant for the determination of the output gap and inflation.
   - Non-Ricardian Fiscal Policy Regime: The optimal relation between output gap and inflation is affected by fiscal variables.
1 Social Welfare

\[ U = - \sum_{t=0}^{\infty} \beta^t \{ x_t^2 + \omega \pi_t^2 \} \]

2 Aggregate Supply Curve

\[ \pi_t = \beta E_t[\pi_{t+1}] + \kappa_x x_t \]

3 Government Budget Constraint

\[ b_{t-1} = \tilde{\beta} b_t + (1 - \beta) s_t + \pi_t - \beta E_t[\pi_{t+1}] + \frac{\beta}{\sigma_r} (x_t - \sigma_x E_t[x_{t+1}]) \]
Relative size of time discount factor and government budget’s own discount factor matters for the optimal design of fiscal policy regimes.

Government budget constraint’s own discount factor is different from the private agents’ time discount factor.

\[ b_{t-1} = \tilde{\beta} b_t + (1 - \beta) s_t + \pi_t - \beta E_t[\pi_{t+1}] + \frac{\beta}{\sigma_r} (x_t - \sigma_x E_t[x_{t+1}]). \]

The government budget’s own discount factor is denoted by \( \tilde{\beta} = \beta(1 - \sigma_b/\sigma_r) \) and \( \rho = \beta/\tilde{\beta} \).
Optimality Conditions

1. Optimality Condition for the Aggregate Output

\[ x_t + \kappa_x \mu_t - \frac{\beta}{\sigma_r} \lambda_t + \frac{\sigma_x}{\sigma_r} \lambda_{t-1} = 0 \]

2. Optimality Condition for the Aggregate Inflation

\[ \omega \pi_t = \mu_t - \mu_{t-1} + \lambda_t - \lambda_{t-1} \]

3. Shadow Value for the Government Budget Constraint

\[ \lambda_t = \frac{1}{\rho} E_t [\lambda_{t+1}] \]
The key equation for the determination of a fiscal policy regime is
\[ \lambda_t = \rho^{-1} E_t [\lambda_{t+1}] . \]

When \( \rho > 1 \), the government should choose a Ricardian fiscal policy regime in order to guarantee that \( \lambda_t = 0 \).

When \( \rho < 1 \), the government should choose a non-Ricardian fiscal policy regime in order to guarantee that the shadow value of its budget constraint follows an AR(1) process of the form:
\[ \lambda_t = \rho \lambda_{t-1} + \eta \epsilon_t . \]
When $\rho > 1$, the government should choose a Ricardian fiscal policy regime in order to guarantee that $\lambda_t = 0$. The optimal targeting rule for the monetary policy is given by

$$\pi_t = -\left(\omega \kappa_x\right)^{-1}(x_t - x_{t-1}).$$

When $\rho < 1$, the optimal policy behavior of the government is described by the following three equations:

$$\lambda_t = \rho \lambda_{t-1} + \eta \epsilon_t.$$

$$x_t + \kappa_x \mu_t - \frac{\beta}{\sigma_r} \lambda_t + \frac{\sigma_x}{\sigma_r} \lambda_{t-1} = 0.$$

$$\omega \pi_t = \mu_t - \mu_{t-1} + \lambda_t - \lambda_{t-1}.$$
Summary of Results

1. When there is incomplete insurance and the IS curve has a positive debt coefficient, the government should choose a fiscal policy regime that attains a constant shadow value of government budget constraint (for example, a zero value).

2. When there is incomplete insurance and the IS curve has a negative debt coefficient, the government should choose a fiscal policy regime that create time-varying shadow-values of government budget constraint.

3. When the optimal fiscal policy regime creates time-varying shadow-values of government budget constraint, it generates a change in the optimal relation for monetary policy’s target variables, which means the potential impact of fiscal instruments in the optimal targeting rule for monetary policy.