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Juergen Jung
Indiana University Bloomington

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The Timing of Redistribution

Juergen Jung
Indiana University - Bloomington
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Abstract

We investigate whether later redistribution programs that can be targeted towards low income families can “dominate” early redistribution programs that cannot be targeted due to information constraints. We use simple two-period OLG models with heterogenous agents under six policy regimes: A model calibrated to the U.S. economy (benchmark), two early redistribution (lump sum) regimes, two (targeted) late redistribution regimes, and finally a model without taxes and redistribution. Redistribution programs are financed by a labor tax on the young and a capital tax on the old generation. We argue that late redistribution, if the programs are small in size, can dominate early redistribution in terms of welfare but not in terms of real output. Better targeting of low income households cannot offset savings distortions. In addition we find that optimal tax policy includes a positive capital tax rate.

JEL Classification: H20, H22

Keywords: Taxation Timing, Transfer Timing, Redistribution, Capital Accumulation, Optimal Taxation, Capital Taxation.

1 Introduction

In recent years some long running government transfer programs have come under attack. The unsuccessful privatization attempt of Social Security by the Bush II administration is one such example. Since government run transfer programs are financed from the same pool of tax revenue, government transfer programs are constantly reevaluated.¹

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¹Technically the social security system is independent of the government budget and therefore independently financed from, say, public education. However, there is no question that should Social Security run out of funds, the government budget would be directly, or at least indirectly, affected.
In this paper, we ask whether it is better to transfer to the young or to the old generation and which generation should pay the bill? Obvious advantages of transfers to the young generation are that human capital can be built up quicker which has important growth effects (e.g. Glomm and Ravikumar (1992)). Transfers to the old might make sense because there is a certain insurance component embedded in Social Security. In his defense of the U.S. Social Security system Diamond (2004) gives a similar justification of why transfers should occur late in an agent’s life. In addition, he claims that annuity markets are not fairly prized and that workers cannot sort out basic portfolio diversification to effectively self insure.\(^2\)

Both, early and late redistribution programs have negative effects as well. Early redistribution is not efficient in insuring income shocks later in an individual’s life, as the above discussion already suggests. Late redistribution programs on the other hand have strong adverse savings distortion effects.

The question of transfers to the old vs. the young has an additional political economy aspect, as the generations compete against each other for government transfers and tax advantages. A recent literature investigates government run transfer programs like social security or public education in terms of their political implementability (e.g. Cooley and Soares (1999) Soares (2003), Conde-Ruiz and Galasso (2004), Gradstein and Kaganovich (2004)).

From data we know that the U.S. spends roughly 38% of all transfers on the adult population on the young generation aged 20 – 50 years old (see table 5). A sizeable amount of these transfers are means tested. Rector, Kim and Watkins (2007) classify these transfers between federal, state and local transfers. We present their summary in table 4.

In this paper we ask the hypothetical question of whether a means tested redistribution program, that transfers to the old generation, is able to “dominate” a transfer program that redistributes to the young generation. We address this question in a highly stylized two period overlapping generations model with early and late redistribution policies financed by alternative tax rules. The idea is the following:

If the policy maker redistributes early in an agent’s life, little is known about the agent. Records on an agent’s ability, educational choices, health or related lifestyle choices, income shocks etc. are either not known or not accessible. It is therefore difficult for the policy maker to target redistribution programs that shift funds early in an agent’s life. Programs that fall into this category are public education (FAFSA education credits depend on parents’ income situation and other demographic factors), and to a lesser extent Medicaid, unemployment payments, foodstamp programs etc.

Programs that redistribute late in an agent’s life-cycle have the advantage of

\(^2\)Compare also Calvet, Campbell and Sodini (2006) who calculate the welfare costs of household investment mistakes using wealth and investment data from Sweden. They identify two main inefficiencies in the financial portfolio of Swedish households: underdiversification of risky assets and nonparticipation in risky asset markets. They find that households with greater financial sophistication tend to invest more efficiently but also more aggressively, so that the welfare cost of portfolio inefficiency is actually greater for these households.
having more information about the agent available. Such programs are therefore more suited for means testing. Information about agents is now available because health and income shocks have been realized and publicly available records are more complete (e.g. surveys and population statistics, divorce statistics etc.). The two major programs that redistribute late in an agent’s lifetime are Social Security and Medicare, to a certain extent also Medicaid.

In this paper we introduce a simple overlapping generations model where agents live for two periods. Agents are heterogeneous with respect to their human capital (ability) endowment. Agents work when young and retire when old. We then postulate two extreme redistribution regimes. In the first regime the government transfers funds to agents when they are young. Since the government does not yet know anything about the income situation of the agents, this will be simple (non-targeted) lump-sum transfers.

In the second regime, the government redistributes to old agents only. The government is then able to observe the income of old agents and can target transfers. The targeting depends on savings and has distortive effects that will partly offset the benefit of targeting. We then compare the two regimes under varying financing options. Some of these regimes will allow for intergenerational transfers, others will exclude them.

We find that late redistribution, although it introduces direct savings distortions into the model, can dominate early redistribution in terms of welfare. This result will depend crucially on the overall size of the redistribution program and on the level of targeting. It turns out that only for very small redistribution programs, late redistribution dominates early redistribution in terms of welfare only. Late redistribution cannot dominate early redistribution in terms of output. If the programs become larger, direct savings distortions offset the efficiency gains from targeting and welfare falls below the early redistribution levels. We then calculate the policy that maximizes aggregate welfare. We find that relative to current U.S. policy, optimal policy suggests to move more funds towards the young generation and to increase capital taxes considerably.

The literature on redistribution is tremendously rich and a lot of emphasis has been placed on efficient redistribution policies, optimal taxation and the public provision of education, unemployment benefits and retirement pensions. There is a large body of literature studying these redistribution programs.

The classic contributions to the optimal tax literature are Mirrlees (1971) and Diamond and Mirrlees (1971). Kocherlakota (2005) surveys the literature on dynamic extensions of the original Mirrlees model. Important contributions to this literature are Chamley (1986), Judd (1985), and Jones, Manuelli and Rossi (1997). One important finding in these papers is that capital should not be taxed. Hubbard and Judd (1986), Aiyagari (1995), and Imrohoroglu (1998) finds that if households face tight borrowing constraints or cannot insure against idiosyncratic income shocks, then a positive capital tax cannot be ruled out in the optimum. Alvarez et al. (1992), Garriga (2000), Erosa and Gervais (2002), and Conesa, Kitao and Krueger (forthcoming) analyze this question in life cycle models and overlapping generations economies and conclude that a positive capital tax is optimal partly
caused by increasing income profiles. A capital tax can then help redistribute from high income cohorts to lower income cohorts in the absence of a progressive labor income tax. Additional papers that address optimal taxation in overlapping generations models together with government commitment and information problems have been analyzed in Brett (1998), Blackorby and Brett (2000), and Pirttila and Tuomola (2001).

We next point to literature that focuses more on the redistribution programs and less on taxation. Seshadri and Yuki (2004) study various forms of redistribution and their effect on the distributions of earnings and consumption. Braeuniger (2004) studies a model that highlights interaction between Social Security, unemployment and growth in a labor search market environment. Bhattacharya and Reed (2003) introduce a search market explanation for how pension programs can increase the efficiency of the labor market. Glomm and Kaganovich (2003) focus on distributional effects of public versus private financing of education and Social Security whereas Gradstein and Kaganovich (2004) analyze the impact of aging on the public funding of education. Corneo and Marquardt (2000) study the interaction between an unfunded pension program and an unemployment insurance program in the presence of a labor market with union wage-setting. Finally, Boldrin and Montes (2004) show how public financing of education and pensions can lead to a complete market allocation. A number of papers is dedicated to risk sharing among generations under pay-as-you-go (PAYG) Social Security systems (e.g. Hassler and Lindbeck (1998)) or the value of information on production economies under uncertainty and its role on income inequality (e.g. Eckwert and Zilcha (2001) and Eckwert and Zilcha (2003)). None of these papers focuses on the timing of public redistribution programs and the theoretical informational advantage of late redistribution programs.

The plan for the rest of the paper is as follows. The next section outlines the model and defines equilibrium. Section 3 describes how we solve the model and presents the numerical solution algorithm. Section 4 presents the calibration of the model to U.S. data. In section 5 we conduct policy analysis by changing the size of the distribution programs as well as the targeting levels of late redistribution programs. Section 6 discusses optimal tax policy. Section 7 concludes the paper. The Appendix contains all tables and figures.

2 The Model

2.1 Demographics and Heterogeneity

We consider a two-period overlapping generations economy with heterogeneous agents. There is no population growth and the size of the population is normalized to one in each period. Agents do not face uncertainty of survival into the second period. Agents differ with respect to their individual ability, $\theta_i$. We assume that $\theta_i$ is an iid random variable that is distributed according to a time invariant distribution function $F$ with support in $\Theta = [\underline{\theta}, \bar{\theta}]$, where $\underline{\theta}, \bar{\theta} \in \mathbb{R}_+$, and $i$ indexes
all agents. We will drop the $i$ superscript in order to not clutter the notation. An agent’s effective unit of labor $z_t$ depends on her ability level $\theta$. For simplicity we assume the identity function to describe the relation between innate ability and effective unit of labor, so that

$$z_t(\theta) = \theta.$$  

Agents are endowed with one unit of time that they can either consume as leisure $l_t$ or supply as labor $(1 - l_t)$ earning wages. The effective human capital per individual that enters the production process is

$$h_t(\theta) = (1 - l_t) z_t(\theta).$$

### 2.2 Preferences and Technology

Preferences of an individual agent in generation $t$ are given by the utility function

$$u(c_t, c_{t+1}) = \left(\frac{(c_t^{y})^{a_1} (1 - l_t)^{1 - \sigma}}{1 - \sigma} + \beta \left(\frac{c_{t+1}^{p}}{1 - \sigma}\right)^{1 - \sigma}\right),$$  

(1)

where $c_t^{y}$ and $c_{t+1}^{p}$ are consumption when young and old, $l_t$ is leisure, $a_1$ is the preference weight on consumption, $a_2$ is the preference weight on leisure, $\sigma$ is then inverse of the intertemporal elasticity of substitution, and $\beta$ is the time preference factor. We assume that $a_1 + a_2 = 1$.

The economy’s aggregate production function is

$$Y_t = AK^\alpha H_t^{1 - \alpha},$$  

(2)

where parameters $A > 0$, $0 < \alpha < 1$, $Y_t$ is total output, $K_t$ is the aggregate capital stock of physical capital, and $H_t$ is the aggregate capital stock of human capital in period $t$. Physical capital $K_t$ will be financed by the aggregate savings of the previous generation $S_{t-1}$ and depreciates each period at rate $\delta$.

### 2.3 Government

The government collects taxes and gives transfers to young and old agents. The government cannot issue debt and has to balance its budget every period. Government consumption is set equal to zero. The government collects a flat rate labor tax $\tau^L$ from the young generation and a flat rate capital tax $\tau^K$ from the old generation. Total tax revenue can be expressed as

$$Tax_t = \left[\tau^L \int_\Theta w_t h_t(\theta) dF(\theta) + \tau^K \int_\Theta q_t s_{t-1}(\theta) dF(\theta)\right].$$  

(3)

In the following we will distinguish two government redistribution programs that are financed with tax revenue. The two programs will differ with respect to when redistribution takes place in an agent’s life. The first program gives lump sum transfers to young agents, whereas the second program is a late redistribution
program that can target transfers to the old generation according to individuals’ wealth levels.

Since we assume that the government cannot observe the ability or skill of an agent at the beginning of her life, early transfers $T_t^e$ have to be lump-sum by assumption. The government uses an exogenous fraction $\lambda$ of its total tax income over the length of a period (approximately thirty years in this OLG setting) to pay for this program. The government budget constraint for the early transfer case is

$$T_t^e = \lambda \times \text{Tax}_t.$$  \hspace{1cm} (4)

The second program gives transfers exclusively to old agents based on the agents’ wealth. Since we assume that the government can observe the wealth levels (and ability) of agents when they are old, it is now possible to target transfers according to the level of wealth. The lower the wealth level, the more transfers an agent will receive from the government. The targeting formula for the late redistribution program is

$$T_t^l (\theta) = \max \left[ a - b R_t s_{t-1} (\theta), 0 \right],$$  \hspace{1cm} (5)

where $R_t s_{t-1} (\theta)$ is wealth (from savings plus interest net of capital taxes) in the second period, $a$ represents the government enforced maximum transfer, and $b$ captures the degree of means-testing of the government transfer program. As wealth $R_t s_{t-1} (\theta)$ increases, transfers decrease at rate $b$. The parameters satisfy $0 \leq a$ and $0 \leq b$.

The government uses the residual tax income, that is fraction $(1 - \lambda)$ of total tax income, to finance this program. The government budget constraint is

$$T_t^l (\theta) = \int_{\Theta} \max \left[ a - b R_t s_{t-1} (\theta), 0 \right] dF(\theta) = (1 - \lambda) \times \text{Tax}_t.$$  \hspace{1cm} (6)

2.4 Households

Agents know the government policy and the late transfer function. In addition they are borrowing constrained. They maximize

$$\max_{\{c_t, l_t, c_{t+1}, s_t\}} \left\{ \frac{(c_{t+1}^1)^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t+1}^2}{1-\sigma} \right\}, \text{s.t.}$$  \hspace{1cm} (7)

$$c_t + s_t = (1 - \tau^L) w_t (1 - l_t) z_t (\theta) + T_t^e,$$  \hspace{1cm} (8)

$$c_{t+1} = (1 + (1 - \tau^K) r_{t+1}) s_t + T_t^l (R_{t+1} s_t),$$  \hspace{1cm} (9)

$$0 < l_t \leq 1, \ s_t \geq 0,$$  \hspace{1cm} (10)

where $c_t$ and $c_{t+1}$ are consumption when young and old, $l_t$ is leisure when young, $s_t$ is savings, $z_t (\theta)$ is the efficiency unit of labor as a function of innate ability $\theta$, so that $(1 - l) z (\theta)$ becomes effectively supplied human capital, which we denote by $h_t (\theta)$. All household choices are functions of the exogenous realization of innate ability $\theta$. 
2.5 Firms

There is a continuum of identical firms that use a standard Cobb-Douglas technology. Firms solve

\[
\max_{\{K_t, H_t\}} \left\{ AK_t^{\alpha} H_t^{1-\alpha} - q_t K_t - w_t H_t \right\},
\]

taking \((q_t, w_t)\) as given.

\[ (11) \]

2.6 Equilibrium

**Definition 1** A competitive equilibrium is a collection of sequences of distributions of individual household decisions \(\{c_t(\theta), c_{t+1}(\theta), s_t(\theta)\}_{t=0}^{\infty}\) for all agents \(\theta \in \Theta\), sequences of aggregate stocks of physical capital \(\{K_t\}_{t=0}^{\infty}\), sequences of factor prices \(\{w_t, q_t, R_t\}_{t=0}^{\infty}\), sequences of government expenditures \(\{T^e_t, T^l_t\}_{t=0}^{\infty}\) and government policy parameters \(\{a, b, \tau^L, \tau^K, \lambda\}\) such that

(i) the sequence \(\{c_t(\theta), c_{t+1}(\theta), s_t(\theta)\}_{t=0}^{\infty}\) solves the maximization problem of the household \((7)\) for each agent \(\theta \in \Theta\),

(ii) factor prices are determined by

\[
q_t = \alpha Y_t / K_t = \alpha AK_t^{\alpha - 1} H_t^{1-\alpha},
\]

\[ (12) \]

\[
w_t = (1 - \alpha) Y^j_t / H_t = (1 - \alpha) AK_t^{\alpha} H_t^{-\alpha},
\]

\[ (13) \]

\[
R_t = 1 + (1 - \tau^K) (q_t - \delta) \equiv 1 + (1 - \tau^K) r_t,
\]

\[ (14) \]

(iii) capital markets clear, so that aggregate capital stocks are given by

\[
K_{t+1} = S_t = \int_{\Theta} s_t(\theta) \, dF(\theta),
\]

\[
H_t = \int_{\Theta} z_t(\theta) (1 - l_t(\theta)) \, dF(\theta),
\]

(iv) commodity markets clear

\[
C_{t-1} + C_t + K_{t+1} = Y_t + (1 - \delta) K_t,
\]

\[ (15) \]

(v) and the respective government budget constraints \((4)\) and \((6)\) hold.

3 Solving the Model

3.1 Households

An individual agent’s maximization problem depends on whether she receives transfers when old or not. This will of course depend on the agent’s initial ability
level which in turn determines her income. We will have to calculate a threshold ability level \( \theta^* \) that determines whether an agent is “poor enough” to receive means tested transfers when old. Whenever the initial ability/income level is below the threshold, \( \theta < \theta^* \), then second period wealth is low enough in order for late transfers to be positive, \( T_l(\theta) > 0 \). Whenever the initial ability is equal or above this threshold, \( \theta \geq \theta^* \), then late transfers will be zero, \( T_l(\theta) = 0 \). Before substituting the budget constraints we first find the optimal relation between consumption and leisure. Deriving preferences with respect to consumption and leisure results in

\[
uc = \left( c_t^{a_1} l_t^{a_2} \right)^{-\sigma} a_1 c_t^{a_1-1} l_t^{a_2}, \quad \text{and}
ul = \left( c_t^{a_1} l_t^{a_2} \right)^{-\sigma} a_2 c_t^{a_1} l_t^{a_2-1}.
\]

The ratio of marginal utilities has to equal the price ratio when solutions for leisure are interior, i.e. for \( l \in (0, 1) \) it holds that

\[
\frac{u_{l,t}}{u_{c,t}} = \frac{a_2 c_t}{a_1 l_t} = (1 - \tau^L) w_t z_t(\theta).
\]

We can now express leisure in terms of consumption as

\[
l_t = \min(\Theta_t c_t, 1), \tag{16}
\]

where \( \Theta_t = \frac{(a_2/a_1)}{(1-\tau^L) w_t z_t(\theta)} \). Using the household budget constraint for the young individual we have

\[
c_t + s_t = (1 - \tau^L) w_t (1 - l_t) z_t(\theta) + T_t^e.
\]

We can then substitute out leisure using (16)

\[
c_t + s_t = (1 - \tau^L) w_t z_t(\theta) \left( 1 - \frac{(a_2/a_1)}{(1-\tau^L) w_t z_t(\theta)} c_t \right) + T_t^e.
\]

Simplifying this becomes

\[
c_t + s_t + (1 - \tau^L) w_t z_t(\theta) \frac{(a_2/a_1)}{(1-\tau^L) w_t z_t(\theta)} c_t = (1 - \tau^L) w_t z_t(\theta) + T_t^e,
\]

or further

\[
c_t \left( 1 + \frac{a_2}{a_1} \right) + s_t = (1 - \tau^L) w_t z_t(\theta) + T_t^e,
\]

which we rewrite as

\[
p_c c_t + s_t = (1 - \tau^L) w_t z_t(\theta) + T_t^e,
\]

where \( p_c = 1 + \frac{a_2}{a_1} \). Finally, we substitute leisure out of the preferences using (16) and get the preferences for the first period as

\[
u(c_t, l_t) = \left( \frac{c_t^{a_1} (\Theta_t c_t)^{a_2}}{1 - \sigma} \right)^{1-\sigma} = \lambda_t \frac{c_t^{(a_1+a_2)(1-\sigma)}}{1 - \sigma},
\]

8
where \( \chi_t = \Theta_t^{a_2(1-\sigma)} \). Now the maximization problem can be written as

\[
\theta < \theta^* : \max_{\{c_t, c_{t+1}, s_t\}} \left\{ \chi_t \frac{c_t^{(1-\sigma)(a_1+a_2)}}{1-\sigma} + \beta \frac{c_{t+1}^{(1-\sigma)}}{1-\sigma} \right\}
\]

\[
\text{s.t.}
\]

\[
p_c c_t + s_t = \left(1 - \tau L\right) w_t z_t + T^e_t,
\]

\[
c_{t+1} = (1-b) R_{t+1} s_t + a.
\]

\[
\theta > \theta^* : \max_{\{c_t, c_{t+1}, s_t\}} \left\{ \chi_t \frac{c_t^{(1-\sigma)(a_1+a_2)}}{1-\sigma} + \beta \frac{c_{t+1}^{(1-\sigma)}}{1-\sigma} \right\}
\]

\[
\text{s.t.}
\]

\[
p_c c_t + s_t = \left(1 - \tau L\right) w_t z_t + T^e_t,
\]

\[
c_{t+1} = R_{t+1} s_t.
\]

After substituting the budget constraints into the respective objective functions we get the following first order conditions

\[
\theta < \theta^* : \quad \frac{\chi_t (a_1 + a_2)}{p_c} \left[ \frac{\left(1 - \tau L\right) w_t z_t + T^e_t - s_t}{p_c} \right]^{(1-\sigma)(a_1+a_2)-1} = \beta R_{t+1} \left[ (1-b) R_{t+1} s_t + a \right]^{-\sigma},
\]

\[
\theta > \theta^* : \quad \frac{\chi_t (a_1 + a_2)}{p_c} \left[ \frac{\left(1 - \tau L\right) w_t z_t + T^e_t - s_t}{p_c} \right]^{(1-\sigma)(a_1+a_2)-1} = \beta R_{t+1} \left[ R_{t+1} s_t \right]^{-\sigma},
\]

Since we assume that the preference parameters on consumption and leisure sum up to one, i.e. \( a_1 + a_2 = 1 \), the powers on the left and right hand side are equal. We can now express savings directly as

\[
\theta < \theta^* : s_t(\theta) = \frac{\left[ \left(1 - \tau L\right) w_t z_t (\theta) + T^e_t \right] \left[ \frac{p_c \beta R_{t+1}}{\chi_t(z_t(\theta))} \right]^{\frac{1}{\sigma}} - p_c a}{p_c (1-b) R_{t+1} + \left[ \frac{p_c \beta R_{t+1}}{\chi_t(z_t(\theta))} \right]^{\frac{1}{\sigma}}},
\]

\[
\theta > \theta^* : s_t(\theta) = \frac{\left[ \left(1 - \tau L\right) w_t z_t (\theta) + T^e_t \right] \left[ \frac{p_c \beta R_{t+1}}{\chi_t(z_t(\theta))} \right]^{\frac{1}{\sigma}}}{p_c R_{t+1} + \left[ \frac{p_c \beta R_{t+1}}{\chi_t(z_t(\theta))} \right]^{\frac{1}{\sigma}}},
\]

Now \( \chi_t \) is a function of the random variable \( \theta \). The threshold \( \theta \), that determines whether the household will receive targeted transfers when old is determined by the payout function (5) and can be expressed as

\[
0 \leq a - b R_{t+1} s_t(\theta).
\]
Using expression (19) this threshold condition becomes
\[
\frac{\left[\left(1 - \tau^L\right) w_t z_t (\theta) + T^e_t \right]}{p_c (1 - b) R_{t+1} + \left[p_c \beta R_{t+1} \right]} - p_c a - \frac{a}{b R_{t+1}} \leq 0,
\]
so that the threshold ability can be expressed as implicit function
\[
F (\hat{\theta}) = \frac{\left[\left(1 - \tau^L\right) w_t z_t (\theta) + T^e_t \right]}{p_c (1 - b) R_{t+1} + \left[p_c \beta R_{t+1} \right]} - p_c a - \frac{a}{b R_{t+1}} = 0.
\]
We use this notation to indicate the threshold ability levels that fall inside the support of \( \theta \), that is \( \hat{\theta} \in [\bar{\theta}, \theta] \). Whenever the threshold ability level would lie outside of the support of \( \hat{\theta} \), we have corner cases that indicate that either all agents will receive late transfers \( \hat{\theta} > \theta \) or none of the agents will receive late transfers \( \hat{\theta} < \theta \). The definition of threshold ability \( \theta^* \) accounts for all of these cases and is written as
\[
\theta^* = \begin{cases} 
\hat{\theta}, & \text{if } \hat{\theta} \in [\bar{\theta}, \theta], \\
\hat{\theta}, & \text{if } \hat{\theta} > \theta, \\
\hat{\theta}, & \text{if } \hat{\theta} < \theta,
\end{cases}
\]
so that savings can be written as
\[
s_t (\theta) = \begin{cases} 
\max \left( \frac{\left[\left(1 - \tau^L\right) w_t z_t (\theta) + T^e_t \right]}{p_c (1 - b) R_{t+1} + \left[p_c \beta R_{t+1} \right]} - p_c a, 0 \right) & \text{if } \theta < \theta^*, \\
\max \left( \frac{\left[\left(1 - \tau^L\right) w_t z_t (\theta) + T^e_t \right]}{p_c (1 - b) R_{t+1} + \left[p_c \beta R_{t+1} \right]} - p_c a, 0 \right) & \text{if } \theta \geq \theta^*.
\end{cases}
\] (21)
We use the maximum notation to indicate that agents are borrowing constrained and that savings can not become negative. Finally, we need to check the corner case for leisure \( l = 1 \). Preferences of the young agent will then reduce to
\[
u (c_t, l_t = 1) = \frac{c_1 (1 - \sigma)}{1 - \sigma},
\]
and the maximization problem reduces to
\[
T^e_t < T^e \max \left\{ c_t, c_{t+1}, s_t \right\} \left\{ \frac{c_1 (1 - \sigma)}{1 - \sigma} + \beta \frac{c_1 (1 - \sigma)}{1 - \sigma} \right\},
\] (22)
\[
s.t.
\]
\[
c_t + s_t = T^e_t, \\
c_{t+1} = (1 - b) R_{t+1} s_t + a.
\]
\footnote{We use a lognormal distribution for our calibration. We truncate the lognormal distribution below and above for computational reasons. The lower and upper bounds are set wide enough, so that the agent mass that falls outside of this region is negligible.}
\[ T_t^e \geq T_t^{es} : \max_{\{c_t, c_{t+1}, s_t\}} \left\{ \frac{c_t (1-\sigma)}{1-\sigma} + \beta \frac{c_{t+1} (1-\sigma)}{1-\sigma} \right\} \]

s.t.
\[ \begin{align*}
  c_t + s_t &= T^e_t, \\
  c_{t+1} &= R_{t+1}s_t,
\end{align*} \]

where \( T_t^e \) is a similar threshold as the one above which determines whether the income (from transfer income only) of the young is large enough, so that they have to much wealth in order to receive transfers when old. Substituting the budget constraints in the objective functions we get again first order conditions of the form
\[ T_t^e < T_t^{es} : a_1 (T_t^e - s_t)^{a_1(1-\sigma)-1} = \beta (1 - b) R_{t+1} ((1 - b) R_{t+1}s_t + a)^{-\sigma}, \]
\[ T_t^e \geq T_t^{es} : a_1 (T_t^e - s_t)^{a_1(1-\sigma)-1} = \beta R_{t+1} (R_{t+1}s_t)^{-\sigma}. \]

Here the powers do not match up anymore. We therefore cannot express savings directly but only as an implicitly function of the following form
\[ T_t^e < T_t^{es} : F (s_t) \equiv a_1 (T_t^e - s_t)^{a_1(1-\sigma)-1} - \frac{\beta (1 - b) R_{t+1}}{(1 - b) R_{t+1}s_t + a)\sigma} = 0, \quad (23) \]
\[ T_t^e \geq T_t^{es} : F (s_t) \equiv a_1 (T_t^e - s_t)^{a_1(1-\sigma)-1} - \beta R_{t+1} (R_{t+1}s_t)^{-\sigma} = 0. \quad (24) \]

In this case savings does not have a common power, unless \( a_1 = 1 \). So that we need to use a nonlinear equation solver to solve for \( s_t (\theta) \). After solving for savings we can calculate the threshold transfer level \( T_t^{es} \) that determines whether the old agent receives a targeted transfer. The criterion is again derived from the payout formula for late transfers, expression (5) and can be written as
\[ 0 \leq a - b R_{t+1}s_t^* \left( T_t^e \right). \]

This is again an implicit function that determines the threshold transfer level
\[ G \left( T_t^{es} \right) \equiv a - b R_{t+1}s_t^* \left( T_t^e \right) = 0. \]

### 3.2 Government

Total tax revenue (3) can be simplified to
\[ Tax = [\tau^L (1 - \alpha) + \tau^K \alpha] AK_t^\alpha H_t^{1-\alpha}. \]

The two government budget constraints can be reduced to
\[ \int_{\Theta} T_t^e dF (\theta_{t-1}) = \lambda Tax, \quad (25) \]
\[ \int_{\theta^*} [a - b R_t s_{t-1} (\theta_{t-1})] dF (\theta_{t-1}) = (1 - \lambda) Tax, \quad (26) \]
where the integral on the left hand side of expression (26) is over the fraction of
the old population that has low enough ability endowment \( \theta \) in order to be entitled
to late transfers. This expression simplifies to

\[
\int_{\theta^*}^{\bar{\theta}} a \times dF(\theta_{t-1}) - bR_tK_{1,t} = (1 - \lambda) \text{Tax}.
\] (27)

The government has to choose a mixture of parameters \( \lambda, a, b, \tau^L \) and \( \tau^K \) such that
equations (25) and (26) hold.

3.3 Algorithm

We cannot get closed form solutions for this problem. We therefore use the follow-
ing algorithm and solve the model numerically on a computer.

**Algorithm 1**

1. 
2. Discretize the space of innate abilities and form a vector \( \tilde{\theta} = [\theta, \ldots, \bar{\theta}] \), so
that an individual ability \( \theta_i \in \tilde{\theta} \)
3. Create a vector of population mass per ability level using the lognormal dis-
tribution: \( \tilde{n} = \log \text{normpdf} (\tilde{\theta}) \)
4. Calculate total population size \( N = \sum_{\Theta} \tilde{n} \)
5. Guess starting value for capital \( K \)
6. Start loop:

   (a) Derive factor prices \( q, w, \) and \( R \) using firm first order conditions

   (b) Solve the household problem for each household \( i \):

      i. solve for savings \( s(\theta) \)

      ii. if \( s < 0 \), set savings \( s = 0 \)

      iii. if \( a - bRs(\theta) < 0 \) solve again for savings using the equation for
      case: \( \theta \geq \theta^* \) in expression (21)

      iv. if \( s < 0 \), set savings \( s = 0 \)

      v. calculate consumption \( c_i \) and leisure \( l_i \)

      vi. if \( l_i > 1 \), set \( l_i = 1 \) and solve again for savings using the maximiz-
      ation problem for \( l = 1 \) in (22) from which we derived the implicit
      function for savings, expression (23)

      vii. if \( s < 0 \), set savings \( s = 0 \)

      viii. if \( a - bRs(T^e) < 0 \), solve again for savings using the expressions
      for case: \( T^e \geq T^{**} \) in expression (24)

      ix. if \( s < 0 \), set savings \( s = 0 \)
x. calculate consumption when young $c_t$
xi. calculate consumption when old $c_{t+1}$

(c) Aggregate savings using population mass vector:

$$K^{\text{new}} = S = \frac{1}{N} \sum_{\Theta} s(\theta) \times \bar{n}$$

(d) Clear government budget constraints (25), (26), and (27) for either: $a, b, \tau^L$, or $\tau^K$.

(e) Calculate error: $\text{err} = \text{abs}(K^{\text{new}} - K)$

(f) Consistency check of aggregate resource constraint (15)

(g) if $\text{err} > \text{tolerance}$, repeat from step (a) with $K = 0.5K^{\text{new}} + 0.5K$

4 Calibration

4.1 Demographics and Heterogeneity

We use data on the lifetime income distribution reported in Fullerton and Rogers (1993) to calibrate the ability or skill distribution $F(\theta)$. Fullerton and Rogers (1993) use data from the Panel Study of Income Dynamics for the years 1970-87 and calculate mean lifetime income before and after taxes/transfers for each decile. We use these deciles of the after tax/transfers figures reported in table 1 to get point estimates for the mean and standard deviation of a lognormal distribution.\footnote{Fernandez and Rogerson (2003) use the same data for calibrating their model of education finance systems. They find that the pretax lifetime income distribution is very similar to the post tax and transfers distribution of lifetime income.}

We use an iterative procedure to estimate the parameters for the mean and the standard deviation of this distribution. We first draw 500,000 lognormally distributed random numbers. We then calculate the deciles and compare them to the deciles reported in column two of table 1. We then minimize the absolute distance of the simulated deciles and the deciles in the table by adjusting the appropriate mean and standard deviation parameters $\mu$ and $\sigma$. Point estimates for parameters $\mu$ and $\sigma$ are reported in table 6. To check the sensitivity of our results with respect to the functional form of our income distribution, we also fit a gamma distribution to the mean income data per decile. We report both estimated lifetime income distributions in the top panel of figure 1. The bottom panel plots the lifetime income figures per decile of our estimates against the estimates from Fullerton and Rogers (1993).

The estimated lognormal distribution represents the lifetime income distribution. However, for our model we need the distribution of innate ability $\theta$. We first normalize wages $w$ to one picking the appropriate total factor productivity $A$. The term $w h(\theta)$ is then equal to $\theta$ and represents wage earnings over the 30 years of active work life, or period one in our model. From the literature on earnings and income distribution (e.g. Lillard (1977)) we know that the earnings distribution
is more concentrated than income or wealth distribution.\textsuperscript{5} We therefore decided to use the lognormal distribution as our benchmark model since it is more concentrated than the gamma distribution. Still our estimated lognormal distribution is the distribution of lifetime income and not of lifetime earnings. We therefore conduct sensitivity analysis on parameters $\mu$ and $\sigma$.

We calibrate the lowest lifetime income individual as $\underline{\theta} = \$1,000$ and the highest lifetime income individual at $\bar{\theta} = \$5,000,000$ which is well above the mean lifetime income of $\$1.7$ million for the 98 – 100 percentile in Fullerton and Rogers (1993).\textsuperscript{6}

\subsection*{4.2 Preferences and Technology}

We pick total factor productivity $A$ to normalize wages to one. The capital share of production $\alpha = 0.36$ as in Kydland and Prescott (1982), the annual depreciation rate is $\delta = 8\%$ which falls well in between the estimates in Nadiri and Prucha (1996) who report numbers between $5.9\% - 12\%$. The time preference rate is $\beta = 0.94$. Parameter $\sigma$ determines the risk aversion of the household and is set $\sigma = 2.5$. Parameter $\beta$ and $\sigma$ together are set to match the capital output ratio $K/Y = 2.98$ and the annual interest rate $r = 4.1\%$.\textsuperscript{7} These are standard values and can be found in the NIPA accounts.

The preference parameters for consumption and leisure are restricted to sum to one, $a_1 + a_2 = 1$. We then chose $a_2 = 0.57$ (the share on leisure) so that average lifetime labor supply equals 0.375 which is close to 0.374 which has been estimated by Gomes, Kotlikoff and Viceira (2007). The coefficient of relative risk aversion in consumption is the given by $-\frac{\sigma a_1}{u} = 1 - a_1 = 1.645$, so that the intertemporal elasticity of substitution is 0.61. We summarize the calibration parameters in table 6.

\subsection*{4.3 Government}

We next calibrate the shares of labor tax income and capital tax income from federal, state and local tax revenues. Table 2 contains data on tax revenue from the U.S. Census and the IRS of fiscal year 2004. Table 3 translates these revenues into labor tax revenue and capital tax revenue in the model. We disregard consumption and sales tax revenues and other government income, since they are not part of our model. We find that 75\% of tax income comes from labor taxes and 25\% comes from capital taxes. Total tax revenue from labor and capital taxes amounts to 21.4\% of GDP. We target this fraction of GDP to be the size of government in our

\textsuperscript{5}Compare also Castaneda, Diaz-Gimenez and Rios-Rull (2003) for new estimates on annual earnings, income and wealth distributions in the U.S. They do not calculate the lifetime earnings distribution.

\textsuperscript{6}The actual distribution that we use here is a truncated lognormal distribution. The truncation is required for computational/numerical reasons and do not affect the results of the paper as the the agent mass outside of the truncation is close to zero.

\textsuperscript{7}It is clear that in a general equilibrium model every parameter affects all equilibrium variables. Here we associate parameters with those equilibrium variables that are the most quantitatively affected.
model. The appropriate labor tax turns out to be \( \tau^L = 27\% \) and the capital tax is \( \tau^K = 15.5\% \).

Table 4 contains data on government spending of fiscal year 2004. We use these data to calibrate the relative share of total tax revenue going into early vs. late transfers as governed by parameter \( \lambda \). In table 5 we classify government transfers to transfers to the young population aged 20 – 50 and transfers to the old population aged 50 – 80. We conclude that 38\% of all government transfers to adults (age 20 – 80) goes to the young population (age 20 – 50), whereas the residual 62\% goes to the old (age 50 – 80). Hence we set \( \lambda = 0.38 \) in the benchmark economy.

The main contributing programs to the old are Social Security (\$496 Billion) and Medicare (\$296 Billion). When splitting Medicaid into transfers to young and old we use information from Kaiser (2005) and allocate 31\% of Medicaid expenditures to the old generation. We then split the category “Health other” in table in a similar fashion. The split for housing assistance is provided in Kochera (2001) (66.6\% to young generation) and the split of the foodstamp program is detailed in Kassner (2001) (91\% to young generation). Transfers to primary and secondary education are excluded because we only model the population from age 20 upwards.

For the benchmark targeting program we choose \( a = 0.22 \) (\$364,800) as the fixed portion of the late transfer. This translates into a maximum monthly transfer of \$1,000. The parameter that governs the targeting rate is set as \( b = 0.2 \). There are multiple combinations of \( a \) and \( b \) that would satisfy the government budget requirement of expression (26). We think this is a good parameterization as people without any savings will receive a \$1,000 monthly benefit. According to Olsen and Hoffmeyer (2002) the special minimum benefit out of Social Security amounts to \$500 on average per month (as of February 2002). If one factors in that this number is about \$2,000 below the annual poverty income level, we think it is reasonable that individual with zero savings get an amount larger than the minimum social security benefit. In addition, our late redistribution program is not only Social Security but includes are transfers to the old, like Medicaid, as well.

5 Policy Analysis of Different Tax Policy Programs

5.1 Comparison of Tax Policy Programs

We present our results for a production economy under six specifications: (i) the U.S. economy denoted \( US \), (ii) early redistribution to the young generation financed by taxes on wage income of the current young generation denoted \( EL \), (iii) early redistribution to the young generation financed by taxes on interest income of the current old generation denoted \( ES \), (iv) late redistribution to the old generation financed by taxes on wage income of the current young generation denoted \( LL \), (v) late redistribution to the old generation financed by taxes on interest income of the current old generation denoted \( LS \), and (vi) no redistribution denoted...
We summarize the parameter settings for these five cases as:

- **United States (US):** \( \tau^L = 27\%, \, \tau^K = 15.5\%, \, \lambda = 0.38 \)

- **Early transfer with labor tax (EL):** \( \tau^L > 0, \, \tau^K = 0, \, \lambda = 1 \)

- **Early transfer with capital tax (ES):** \( \tau^L = 0, \, \tau^K > 0, \, \lambda = 1 \)

- **Late transfer with labor tax (LL):** \( \tau^L > 0, \, \tau^K = 0, \, \lambda = 0 \)

- **Late transfer with capital tax (LS):** \( \tau^L = 0, \, \tau^K > 0, \, \lambda = 0 \)

- **No transfer, no taxes (NR):** \( \tau^L = \tau^K = 0 \)

### 5.2 Size of Redistribution Program

Our first experiment is to find out whether late redistribution programs can ever dominate early redistribution programs in terms of aggregate welfare and output. We therefore compare the different regimes adjusting the tax rates that finance them. In the case of regimes EL and LL we choose labor taxes in the range between 0 – 20% and for regimes ES and LS we use capital tax rates in the same range from 0 – 20%. These tax rates are the only source of funding for the redistribution programs in the respective regimes and therefore directly determine the size of these programs.

Figures 2 to 4 present the results. In order to relate the four distribution regimes EL, LL, ES, and LS to our benchmark calibration we also plot the original calibration of the U.S. economy denoted US as well as the no redistribution regime with zero taxes NR. For the late redistribution regimes we choose targeting parameter \( b = 0.2 \) and let the second targeting parameter \( a \) adjust to clear the government budget constraint as we alter the tax rates. Technology and agent heterogeneity is identical for all regimes. Note that the size of the economies of the different regimes are equal and our experiments are therefore not revenue neutral. Our goal is simply to find ranges for tax rates where late redistribution programs outperform early redistribution programs in terms of welfare and output.

Figure 2 presents the aggregate economy. In panel [1] we see that aggregate output of early redistribution financed with a labor tax, EL is almost identical to the no redistribution case NR. The slight difference between the two cases is explained by the labor distortion caused by the labor tax.\(^8\) Early redistribution financed by a savings tax on the old generation, ES, produces the largest steady state output (red dotted line). We see from panel [2] that in this case aggregate

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\(^8\)It can be shown that with inelastic labor supply the two cases, EL and NR will be identical as tax revenues collected from the young are returned to them immediately in a lump sum fashion so that aggregate savings will be identical. The only difference then is in terms of welfare as the lump sum tax provides some redistribution from the rich to the poor.
savings is the highest and therefore the level of output dominates the other redistribution regimes. This is true for all sizes of government, $\tau = 0$ to $\tau = 20\%$. The only program that grows monotonically with increasing the tax rates is the $ES$ program. Here the additional funds collected via the capital tax are shifted to the young. As labor is not taxed the distortion in labor supply are relatively small and the larger stock in physical capital ensures that output increases. The early redistribution case financed by a labor tax also exhibits an increase in the savings rate of physical capital. However, the increasing labor tax rate causes a strong decrease in labor supply that more than offsets the increase in physical capital. Therefore output for the $EL$ (green dotted line) drops as the redistribution program becomes larger.

The volume of redistribution $VR$, or government size, is presented in panel [3] of figure 2. We define $VR$ as the total amount of funds collected by the government in the steady state which can be written as

$$US: VR = \int \left[ \tau^L \omega h(\theta) + \tau^K q_s(\theta) \right] dF(\theta) = \left[ \tau^L (1 - \alpha) + \tau^K \alpha \right] Y,$$

$$EL, LL: VR = \int \tau^L \omega h(\theta) dF(\theta) = \tau^L (1 - \alpha) Y,$$

$$ES, LS: VR = \int \tau^K q_s(\theta) dF(\theta) = \tau^K \alpha Y,$$

$$NR: VR = 0,$$

for the six cases considered. We see from panel [3] that the early redistribution with taxes on labor, $EL$, generates the largest volume of redistribution in absolute terms. Keep in mind that the size of government as a fraction of GDP is identical to $\tau^L (1 - \alpha)$ for the regimes using the labor tax ($EL, LL$) and equal to $\tau^K \alpha$ for regimes using the capital tax ($ES, LS$). It is now easy to see that for identical tax rates $\tau^L = \tau^K$ the labor tax financed programs ($EL, LL$) will always have a larger government as percentage of GDP since $(1 - \alpha) > \alpha$ given our calibration. This translates into larger $VR$ in levels for $EL$ and $LL$ as well, despite the fact that the early redistribution program with capital taxes, $ES$, produces the largest economy.

The results of aggregate labor presented in panel [4] of figure 2 have a straightforward interpretation. Labor supply is lowest for the regimes that redistribute early as income from early transfers directly rivals income from working. The more income agents receive while they are in their active work period, the less labor they are willing to supply. Panels [5] and [6] plot the interest rate and the wage rates respectively. Late redistribution regimes produce larger interest rates and smaller wage rates. This is directly related to the fact that late redistribution programs are smaller economies which puts them on the steeper area of the production function surface. Since interest rates are equal to the slopes of the production surface at the respective equilibrium points, the the interest rates turn out to be larger for the late redistribution programs. Labor supply in early redistribution programs is lower than in late redistribution programs for reasoned mentioned earlier. In the optimum that means again that at the equilibrium points, the production surface for early redistribution programs is steeper along the labor dimension. This
directly results in higher wages for early redistribution regimes. An alternative interpretation is the increased value for leisure due to the extra income from early redistribution. Companies have to compensate workers more in order to attract labor.

Finally, panels [7] and [8] plot the targeting parameters $a$ and $b$. Parameter $a$ adjusts endogenously to changes in taxes whereas $b$ is held fixed at the original 0.2. We see that as we increase the tax rate in the late redistribution regimes the lump sum transfers $a$ increase as the government budget has to stay balanced. This results in more generous late transfer schemes. Incidentally, as taxes increase the size of early redistribution programs increases as well in order to clear the budget constraint of the government. These results are also depicted in panel [3] and have been discussed earlier.

From the first figure we conclude that late redistribution programs can never dominate early redistribution programs in terms of output. Early transfers work as an "engine of growth" and the larger the early redistribution program is, the larger stock of physical capital becomes. If distortions in the labor market do not completely offset these increases in physical capital accumulation, the entire economy will grow (see regime $ES$, red dotted line). Similar results for OLG models have been reported in Jones and Manuelli (1992). OLG models have the feature that the young generation has to buy the entire capital stock from the old generation. If income of the young generation is too low, then the young cannot afford to buy an ever increasing capital stock and growth cannot happen. Jones and Manuelli (1992) find that income taxes (even taxes on capital) that can be used to finance transfers to the young generation, will allow the young to buy an ever increasing capital stock and economic growth is possible. We abstract from growth in our model, but a similar mechanism ensures that early redistribution programs produce larger economies.

We next turn our attention to welfare analysis and investigate whether late redistribution programs can dominate early redistribution programs in terms of welfare. We first define lifetime utility of an agent type $\theta$ as

$$U(\theta) = u(c^y(\theta), l(\theta)) + \beta u(c^o(\theta)).$$

Aggregate welfare is then defined as the aggregate lifetime utilities of all individuals born directly into the steady state. Aggregate welfare therefore is

$$W = \int_\theta U(\theta) dF(\theta).$$

Figure 3 presents aggregate welfare in panel [1] and lifetime utility of a low income individual in panel [2]. From panel [1] we see that aggregate welfare is an increasing function of the size of the redistribution program for the early transfer regimes $EL$ and $ES$. This again reflects the growth generating effects of early transfers in OLG models. The aggregate welfare of the late redistribution regime is non-monotonic. For very small transfer programs the savings distortions are very low and the
redistributive effect of the targeting late redistribution will dominate. Aggregate welfare levels of late redistribution programs also dominate those of their early redistribution counterparts. However, this is only the case for programs that can be financed with a flat tax rate on labor or capital smaller than 3%. For late redistribution programs larger than that savings distortions become too strong and aggregate welfare levels begin to drop. We see that poor individuals benefit the most from targeted late redistribution if the program is kept very small.

Finally, figure 4 contrasts lifetime utility levels of different income groups by lifetime-income quartiles (panels [1]–[4]) as well as the Gini coefficient of household lifetime income. As one would expect, the low income groups tend to benefit from larger redistribution programs, whereas high income groups lose. The program that does worst in terms of welfare is the late redistribution case financed via a labor tax, $LL$. This regime exhibits a dual distortion. The savings distortion from late transfers are augmented with labor supply distortions from taxing labor. Therefore welfare for almost all income groups is decreasing in the size of the redistribution program.\(^9\) Regime $ES$ (red dotted line) increases welfare of almost all groups as the size of the program becomes larger. Even the highest quartile experiences some welfare improvement over the no tax ($NR$) case. One reason is the high redistributive power of this program as can be induced from the low Gini coefficient in panel [6] of figure 4. Regime $ES$ redistributes strongly without the adverse labor distortions that regime $EL$ incorporates. This explains the relative dominance of regime $ES$ over all measures discussed.

We conclude that late redistribution programs can dominate early redistribution programs in terms of welfare only when the size of the redistribution is kept very small.

5.3 Changing the Targeting Rate of Redistribution

In our next set of experiments we investigate how an increased targeting rate for late redistribution programs (via parameter $b$) can ensure that the late redistribution program stays small. We again compare late redistribution programs and early redistribution programs. In these sets of experiments we fix targeting parameter $a$ at a very small level $a = 0.05$ (~ $197,159$). We need to fix this lump-sum component of the late redistribution program to be small because otherwise the required tax rates to finance the program would be very large and late redistribution programs would always be dominated by early redistribution programs. We do not re-calibrate any of the other parameters.

Figures 5 and 6 report the results of these experiments.

We first look at how aggregate output changes as we increase the targeting rate of late redistribution programs $b$. As we change $b$ we have to think which other government parameter do we want to adjust. Since we already fixed $a$ at a level of 0.05 the only other parameters left are the labor tax rate $\tau^L$ for the regimes financed by labor taxes ($EL, LL$) and the capital tax rate $\tau^K$ for regimes finance by capital taxes ($ES, LS$). We see that as we increase $b$ and the late redistribution programs

\(^9\)The only exception is the very low income group as reported in panel [2] of figure 4.
become more targeted, the necessary taxes to finance them can be reduced (compare panel [7] in figure 5. This endogenous adjustment happens automatically for late redistribution programs due to budget balancing. In order to compare the late redistribution regimes to the early redistribution regimes we will set the tax rates of the early redistribution regime equal to the endogenously adjusting tax rate of the late redistribution regime. So the labor tax rate for \( EL \) is set equal to the endogenously adjust labor tax rate of regime \( LL \), whereas the capital tax rate of regime \( ES \) is set equal to the endogenously adjusting capital tax rate of regime \( LS \) (see also panel [8] in figure 5).

From panel [1] in figure 5 we see that the late redistribution regimes become more targeted and the respective volume of redistribution decreases, output increases. The opposite is true for the early redistribution programs. If the tax rates of early redistribution programs mirror the decrease of their late redistribution counterparts, the "engine of growth" of early transfers begins to stall and output declines. In terms of output we get the same result as before. Late redistribution programs will always be dominated by early redistribution programs, no matter how targeted late redistribution program become.

In terms of welfare the picture changes again. Figure 6 shows that as the targeting of late redistribution increases and the programs become smaller, aggregate welfare in these regimes increases, due to smaller distortions and more aggressive redistribution. On the other hand, as we adjust the size of early redistribution programs at the same rate as the late redistribution programs we see that welfare decreases for regime \( EL \) and \( ES \). There is a threshold targeting parameter around \( b = 0.14 \) after which programs are small enough that the late redistribution regimes start dominating the early redistribution regimes in terms of aggregate welfare. The effects are more pronounced for low income individuals as can be seen in panel [2] of figure 6. For low income groups the targeting threshold after which the dominance switches from early to late redistribution programs is much smaller at \( b = 10 \).

We therefore conclude that more targeted programs, that can therefore be kept small in size are able to dominate early redistribution programs in terms of aggregate welfare but not in terms of output.

### 5.4 Sensitivity Analysis

We conduct the same set of experiments for a model without borrowing constraints and also for a model with inelastic labor supply. Our results are robust to both extensions. However, in the case with inelastic labor supply the late redistribution programs have an even harder time to dominate the early redistribution programs. The reason is that early redistribution programs suffer more in general from distortions in the labor market and once we turn those off there is only a very small region left where late redistribution programs can dominate early redistribution programs.

Our results are also robust to changes in the discount factor. We tried discount factors in the range of \( \beta = [0.94, \ldots, 0.988] \) and found that small sized late redistribu-
bution programs can dominate early redistribution programs in terms of welfare.\textsuperscript{10} With larger discount factors even somewhat larger late redistribution programs can dominate the early ones.

6 Optimal Tax Policy

We define optimal tax policy as a set of government policy parameters \( a, b, \tau^L, \tau^K, \) and \( \lambda \) that maximize total welfare (28) such that consumers still solve their maximization problem and the conditions for competitive equilibrium hold. Total welfare is defined as the equally weighted sum of the lifetime utilities of all individuals who are newly born into the steady state. More formally, the government maximizes the utilitarian welfare function

\[
\max_{\{a,b,\tau^L,\tau^K,\lambda\}} \int_0^\theta \left[ u(c^a(\theta), l(\theta)) + \beta u(c^o(\theta)) \right] dF(\theta)
\]

s.t.

\( (4), (6), (12) - (15), (17), (18), (23), \) and (24).

We report our results in table 8.

Our first finding, shown in table 8 as model \( M1 \), is that using the most general version of our model with heterogeneity, elastic labor supply, and non-separable utilities the optimal government transfer and tax policy for the U.S. economy is: \( a = 0.34 \) (\$436,700), \( b = 1.05, \tau^L = 30.6\%, \tau^K = 100\%, \) and \( \lambda = 0.85. \) Compared to the original calibration of the U.S. economy our model implies that transfers to the young should be increased from currently 38\% of all tax revenue to 85\%. In addition, labor taxes should increase slightly from 27\% to 30.6\% and capital taxes should increase from 15.5\% to 100\%. In addition, the targeting rate increases from \( b = 0.20 \) to \( b = 1.05, \) so that means testing becomes much more aggressive.

This policy increases steady state capital stock by factor three, reduces labor supply from 36.7\% to 29.0\% and increases output by 50\%. We interpret this result as the Jones and Manuelli (1992) finding that transfers to the young generation can increase output significantly. The increase in welfare is a direct result of the higher income and the higher consumption of leisure. Note also that capital taxes are extremely large and positive. We will give the intuition for this result in the next section.

As in Auerbach and Kotlikoff (1987) and Jones and Manuelli (1992) the optimal tax policy described above is not welfare enhancing in a Paretoian sense as the utility of the initial old generation as well as the utility of the high income earners will be lower with increasing tax rates. Also, our concentration on steady states misses the costs imposed on transitional generations who have to build up the capital stock which comes at the expense of their consumption.\textsuperscript{11}

\textsuperscript{10}Annual discount factors \( \beta \) translate into per period discount factors as \( \beta_{\text{period}} = \beta^{30}. \) The range of per period discount factors is accordingly \( \beta_{\text{period}} = [0.1563, ..., 0.6961]. \)

\textsuperscript{11}Apart from computational difficulties that transitions would imply, we would like to compare our results to the existing literature on optimal taxation which is concentrated on steady state analysis.
Positive Capital Tax Rate

Positive capital tax rates are not unusual in overlapping generations economies as has been shown in the literature (e.g. Ordover and Phelps (1975), Atkinson and Sandmo (1980), Hubbard and Judd (1986), Auerbach and Kotlikoff (1987), Alvarez et al. (1992), Aiyagari (1995), Imrohoroglu (1998), Garriga (2000), Erosa and Gervais (2002), Conesa, Kitao and Krueger (forthcoming), and others). Some of the model features that ensure a positive capital tax rate are borrowing constraints, increasing efficiency profiles, and agent income heterogeneity within age cohorts. We can show that none of these features are essential to achieve a positive capital tax rate as the optimal government financing rule.

In order to give more intuition we use a simplified version of our model, where agents supply labor inelastically, they are homogenous within their age cohort, and capital and labor taxes are levied to finance an exogenously given government consumption equal to $\overline{Tax} > 0$. We can derive the steady state capital stock as

$$K = \frac{(1 - \tau_L)(1 - \alpha)AK^\alpha}{1 + \beta^{-\frac{1}{\sigma}}(1 + (1 - \tau^K)(\alpha AK^{\alpha-1} - \delta))^{1-\frac{1}{\sigma}}}$$

and after substituting the government budget constraint we have

$$K = \frac{(1 - \alpha)AK^\alpha - \overline{Tax} + \tau^K(\alpha^2 AK^{\alpha} - \delta K)}{1 + \beta^{-\frac{1}{\sigma}}(1 + (1 - \tau^K)(\alpha AK^{\alpha-1} - \delta))^{1-\frac{1}{\sigma}}}.$$  

We can now show that if $\overline{Tax}$ is small and $\sigma$ is sufficiently large (e.g. $\sigma > 1 - \varepsilon$, with $\varepsilon$ being a small number) then an increase in $\tau^K$ leads to a higher steady state capital stock. The mechanism at work follows the intuition given in Jones and Manuelli (1992).

Even though we abstract from economic growth, we find that young agents with higher incomes are able to buy more capital stock from the old generation. Whenever $\tau^K$ increases and government consumption is held constant, then $\tau_L$ will decrease. This leads to higher income of the young agent (income effect). At the same time the rate of return on capital decreases (substitution effect). It turns out that under certain parameter restrictions (small government program, sufficient curvature on preferences), an increase in capital tax increases the capital stock.

When in addition, the discount factor $\beta$ is small enough and the utility function has enough curvature e.g. $\sigma > 2$, the welfare maximizing capital tax rate is positive. As agents strongly prefer their first period consumption to their second period consumption and the smoothing motive is strong enough (e.g. $\sigma > 2$), it is welfare increasing to tax interest income and shift wealth into the first period.\footnote{In an additional appendix to their paper which is available on their website, Conesa, Kitao and Krueger (forthcoming) show that in a similar model (the only difference is that old agents have labor income as well) with inelastic labor supply and homogenous agents, the optimal capital tax rate is positive. In the absence of Government debt, they then show that the optimal capital tax is not zero and that the first best solution (Planner’s Solution) cannot be achieved. If debt is introduced the optimal capital tax rate becomes zero and the planner’s solution can me achieved by a competitive market.} Again we
abstract from transitions. Accounting for transitional effects the direction of the welfare effect is less clear, since the current old generation would suffer an income loss from a higher capital tax without benefiting from the higher capital stock.

In our more general model, we can now show that if one increases the discount factor \( \beta \), the optimal capital tax rate decreases (compare model \( M3b \) in table 8). When we further increase \( \beta \), the rational for a positive optimal capital tax disappears in this simplified version of our model.\(^{13}\) Since our more general model includes transfer payments to the young – and Jones and Manuelli (1992) have already shown that such transfers are welfare improving – we have a further justification for raising additional tax revenue in the optimum. This is the reason why we cannot find any specification for our more general model that leads to zero capital taxes.

Some more intuition can be gained by noting that in our model the young generation is taxed via a labor tax and the old generation is taxed via a capital tax only. Since it has been shown in the literature that it is optimal to tax income at different ages at different tax rates (e.g. Garriga (2000), and Erosa and Gervais (2002)) we now see that the only way to achieve some sort of intergenerational equality is by taxing the old generation as well. However, in our model we assume the old generation is not working anymore so that the only way to tax the old is via the capital tax. This provides a further justification for a positive capital tax rate.\(^{14}\)

6.2 Labor Elasticity and Income Heterogeneity

Under income heterogeneity (models \( M2 \) and \( M6 \)) the change from elastic labor supply to inelastic labor supply leads to a labor tax rate of 100%. The optimal policy is the attempt to achieve as much agent homogeneity as possible. Since labor taxes become de facto lump sum taxes, it is optimal to tax the entire wage income and redistribute it back equally to all young agents.

If we abstract from heterogeneity and leave labor inelastically supplied (\( M4 \) and \( M8 \)) we find that a negative tax on labor disappears. And depending on the curvature of preferences the capital tax either stays at 100% or drops down to 43.6%. Model \( M4 \) for instance can use a labor tax of 35.7% together with a capital tax of 58.4% and redistribute exclusively to the young (\( \lambda = 1 \)) to achieve maximum aggregate welfare.

\(^{13}\)More detailed results on this toy version of the model are available upon request from the author.

\(^{14}\)If we allowed the old generation to work as well and in addition we would allow an age specific labor tax rate, then it would be possible for the capital tax rate to be zero. In this case the progressivity of the labor tax rate would achieve the optimally feasible intergenerational equality and capital tax would not be needed anymore. This result has been analytically derived in the technical appendix to Conesa, Kitao and Krueger (forthcoming).
6.3 Additively Separable Preferences in Consumption and Leisure

Cases with separable preferences use the form of

\[ a_1 \ln c_t + a_2 \ln l_t + \beta \ln c_{t+1}, \]

where we set parameters \( a_1 = a_2 = \frac{1}{2} \). Results are reported in the lower parts of table 8. Some results in the literature on optimal taxation point to the fact that additive separability in preferences between consumption and leisure can lead to an optimal capital tax rate of zero (e.g. Ordover and Phelps (1979)). We do not find these results in our overlapping generations framework. The earlier results of a nonzero capital tax rate with non-additive Cobb-Douglas preferences still hold under separability. However, we also see that separable preferences exclude late redistribution completely under all specifications of heterogeneity and labor elasticity.

7 Conclusion

We derived simple two-period OLG models with endowment heterogeneity under five policy regimes, (i) no redistribution, (ii) early redistribution to the young generation financed by taxes on wage income of the current young generation, (iii) early redistribution to the young generation financed by taxes on interest income of the current old generation, (iv) late redistribution to the old generation financed by taxes on wage income of the current young generation, and (v) late redistribution to the old generation financed by taxes on interest income of the current old generation.

We furthermore split the population into different income groups and find that high income households fare best under the benchmark model, whereas very low income households can improve welfare under various redistribution regimes. Low income households seem to do better under late redistribution, if tax rates are not too high. Output is highest under the early redistribution regime with intergenerational financing (savings tax on the old). Optimal tax policy points towards a non-zero tax on capital and an emphasis on early redistribution.

References


### Lifetime Income Distribution

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Mean Lifetime Income (Thousands of 1986 Dollars)</th>
<th>Model (lognormal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 2</td>
<td>217</td>
<td>260</td>
</tr>
<tr>
<td>2 – 10</td>
<td>355</td>
<td>358</td>
</tr>
<tr>
<td>10 – 20</td>
<td>433</td>
<td>444</td>
</tr>
<tr>
<td>20 – 30</td>
<td>515</td>
<td>516</td>
</tr>
<tr>
<td>30 – 40</td>
<td>565</td>
<td>582</td>
</tr>
<tr>
<td>40 – 50</td>
<td>665</td>
<td>648</td>
</tr>
<tr>
<td>50 – 60</td>
<td>735</td>
<td>719</td>
</tr>
<tr>
<td>60 – 70</td>
<td>814</td>
<td>801</td>
</tr>
<tr>
<td>70 – 80</td>
<td>911</td>
<td>903</td>
</tr>
<tr>
<td>80 – 90</td>
<td>1,028</td>
<td>1,050</td>
</tr>
<tr>
<td>90 – 98</td>
<td>1,305</td>
<td>1,302</td>
</tr>
<tr>
<td>98 – 100</td>
<td>1,734</td>
<td>1,794</td>
</tr>
</tbody>
</table>

Table 1: Source Column Two: Fullerton and Rogers (1993)

### 8 Appendix: Tables and Figures
### Tax Revenue Fiscal Year 2004: in Billions of $
\hspace{1cm} (3,029 \text{ represents } 3,029,000,000,000)$

<table>
<thead>
<tr>
<th>United States, total:</th>
<th>$3,029</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Tax Income</td>
<td>$2,019</td>
</tr>
<tr>
<td>Individual income tax</td>
<td>$990</td>
</tr>
<tr>
<td>withheld by employers</td>
<td>$747</td>
</tr>
<tr>
<td>Employment tax</td>
<td>$717</td>
</tr>
<tr>
<td>Old-age and disability insurance</td>
<td>$706</td>
</tr>
<tr>
<td>Unemployment insurance</td>
<td>$7</td>
</tr>
<tr>
<td>Railroad retirement</td>
<td>$4</td>
</tr>
<tr>
<td>Corporation income tax</td>
<td>$231</td>
</tr>
<tr>
<td>Estate and gift tax</td>
<td>$26</td>
</tr>
<tr>
<td>Excise tax</td>
<td>$55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State and Local Taxes</th>
<th>$1,010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property</td>
<td>$318</td>
</tr>
<tr>
<td>Individual income tax</td>
<td>$215</td>
</tr>
<tr>
<td>Corporation income</td>
<td>$34</td>
</tr>
<tr>
<td>Sales and gross receipt</td>
<td>$361</td>
</tr>
<tr>
<td>Motor vehicle licenses</td>
<td>$21</td>
</tr>
<tr>
<td>Death and gift</td>
<td>$6</td>
</tr>
<tr>
<td>other</td>
<td>$56</td>
</tr>
</tbody>
</table>

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html

Source for Federal:http://www.irs.gov/taxstats/article/0, id=168610,00.html

Table 2: Tax Revenue 2004
## Tax Split into Labor and Capital Tax: in Billions of $

(\text{2,415 represents $2,415,000,000,000})$

<table>
<thead>
<tr>
<th>Total Tax Revenue, excl. consumption taxes</th>
<th>$2,415</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th><strong>Labor Tax</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal:</td>
<td></td>
</tr>
<tr>
<td>Individual income tax (employer)</td>
<td>$747</td>
</tr>
<tr>
<td>Individual income tax (employee)</td>
<td>$122</td>
</tr>
<tr>
<td>Employment tax</td>
<td>$717</td>
</tr>
<tr>
<td>State:</td>
<td></td>
</tr>
<tr>
<td>Individual income tax (state)</td>
<td>$215</td>
</tr>
<tr>
<td><strong>Wage income tax revenue:</strong></td>
<td>$1,801</td>
</tr>
<tr>
<td>(in %)</td>
<td>75%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Capital Tax</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal:</td>
<td></td>
</tr>
<tr>
<td>Individual income tax (employee)</td>
<td>$122</td>
</tr>
<tr>
<td>Corporation income Tax</td>
<td>$231</td>
</tr>
<tr>
<td>Estate and gift tax</td>
<td>$26</td>
</tr>
<tr>
<td>State:</td>
<td></td>
</tr>
<tr>
<td>Property tax</td>
<td>$318</td>
</tr>
<tr>
<td>Corporation income tax</td>
<td>$34</td>
</tr>
<tr>
<td>Death and gift tax</td>
<td>$6</td>
</tr>
<tr>
<td><strong>Capital income tax revenue:</strong></td>
<td>$615</td>
</tr>
<tr>
<td>(in %)</td>
<td>25%</td>
</tr>
</tbody>
</table>

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html

Table 3: Shares of labor tax revenue and capital tax revenue as percentage of total tax revenue excluding consumption, sales and excise taxes.
**Government Expenditures Fiscal Year 2004: in Billions of $**

(2,293 represents $2,293,000,000,000).

<table>
<thead>
<tr>
<th>United States, total:</th>
<th>$4,127</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Spending</td>
<td>$2,293</td>
</tr>
<tr>
<td>Social Security</td>
<td>$496</td>
</tr>
<tr>
<td>Federal employees retirement</td>
<td>$117</td>
</tr>
<tr>
<td>Unemployment assistance</td>
<td>$43</td>
</tr>
<tr>
<td>Medical Care</td>
<td>$515</td>
</tr>
<tr>
<td>Medicare</td>
<td>$296</td>
</tr>
<tr>
<td>SCHIP</td>
<td>$5</td>
</tr>
<tr>
<td>Medicaid</td>
<td>$176</td>
</tr>
<tr>
<td>Indian Health</td>
<td>$3</td>
</tr>
<tr>
<td>Hospital and medical care for veterans</td>
<td>$22</td>
</tr>
<tr>
<td>Health resources and services</td>
<td>$6</td>
</tr>
<tr>
<td>Substance abuse and mental health services</td>
<td>$3</td>
</tr>
<tr>
<td>Health care tax credit</td>
<td>$0</td>
</tr>
<tr>
<td>Uniformed Services retiree health care fund</td>
<td>$5</td>
</tr>
<tr>
<td>Housing assistance</td>
<td>$31</td>
</tr>
<tr>
<td>Food and nutrition assistance</td>
<td>$46</td>
</tr>
<tr>
<td>Foodstamps</td>
<td>$35</td>
</tr>
<tr>
<td>Child nutrition and special milk programs</td>
<td>$11</td>
</tr>
<tr>
<td>Public assistance</td>
<td>$112</td>
</tr>
<tr>
<td>Earned income tax credit</td>
<td>$33</td>
</tr>
<tr>
<td>Supplemental security income</td>
<td>$31</td>
</tr>
<tr>
<td>Daycare and foster care</td>
<td>$11</td>
</tr>
<tr>
<td>other</td>
<td>$36</td>
</tr>
<tr>
<td>Other payments to individuals</td>
<td>$12</td>
</tr>
<tr>
<td>Education, training, employment and social services</td>
<td>$88</td>
</tr>
<tr>
<td>Elementary and secondary</td>
<td>$34</td>
</tr>
<tr>
<td>Higher education</td>
<td>$25</td>
</tr>
<tr>
<td>Research and general education aid</td>
<td>$3</td>
</tr>
<tr>
<td>Training and employment</td>
<td>$8</td>
</tr>
<tr>
<td>Social services</td>
<td>$16</td>
</tr>
<tr>
<td>others (less transfers to State/Local)</td>
<td>$408</td>
</tr>
</tbody>
</table>

| State and Local Spending (net of federal funds) | $1,834 |
| Education (net of federal funds)               | $584   |
| Elementary and secondary                       | $393   |
| Higher education                               | $154   |
| Other education                                | $37    |
| Public Welfare (net of federal funds)          | $118   |
| Health and Hospitals (net of federal funds)    | $137   |
| Utility & liquore store Expenditure            | $160   |
| Insurance trust expenditure                    | $197   |
| Employee Retirement                           | $138   |
| Unemployment compensation                     | $43    |
| Other infrastructure                           | $638   |

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html
Source for Federal:http://www.irs.gov/taxstats/article/0, id=168610,00.html

Table 4: Government Transfers
Selected Government Spending in 2004: in Billions of $
(1,955 represents \$1,955,000,000,000)$

<table>
<thead>
<tr>
<th>Selected spending, total</th>
<th>$1,955</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early Transfers</td>
<td>$746</td>
</tr>
<tr>
<td>Federal:</td>
<td></td>
</tr>
<tr>
<td>Unemployment assistance</td>
<td>$43</td>
</tr>
<tr>
<td>SCHIP</td>
<td>$5</td>
</tr>
<tr>
<td>Medicaid (69%)</td>
<td>$122</td>
</tr>
<tr>
<td>Health other (69%)</td>
<td>$23</td>
</tr>
<tr>
<td>Housing assistance (66.6%)</td>
<td>$21</td>
</tr>
<tr>
<td>Foodstamps (91%)</td>
<td>$32</td>
</tr>
<tr>
<td>Child nutrition</td>
<td>$11</td>
</tr>
<tr>
<td>Earned income tax credit</td>
<td>$33</td>
</tr>
<tr>
<td>Daycare and foster care</td>
<td>$11</td>
</tr>
<tr>
<td>Higher education</td>
<td>$25</td>
</tr>
<tr>
<td>Research and general education aid</td>
<td>$3</td>
</tr>
<tr>
<td>Training and employment</td>
<td>$8</td>
</tr>
<tr>
<td>State:</td>
<td></td>
</tr>
<tr>
<td>Higher education</td>
<td>$154</td>
</tr>
<tr>
<td>Other education</td>
<td>$37</td>
</tr>
<tr>
<td>Public Welfare/Hospitals/Health (69%)</td>
<td>$176</td>
</tr>
<tr>
<td>Unemployment compensation</td>
<td>$43</td>
</tr>
<tr>
<td>Late Transfers</td>
<td>$1,210</td>
</tr>
<tr>
<td>Federal:</td>
<td></td>
</tr>
<tr>
<td>Social Security+Medicare</td>
<td>$792</td>
</tr>
<tr>
<td>Federal employees retirement</td>
<td>$117</td>
</tr>
<tr>
<td>Medicaid (31%)</td>
<td>$55</td>
</tr>
<tr>
<td>Health other (31%)</td>
<td>$10</td>
</tr>
<tr>
<td>Uniformed services retiree health care fund</td>
<td>$5</td>
</tr>
<tr>
<td>Housing assistance (33.4%)</td>
<td>$10</td>
</tr>
<tr>
<td>Foodstamps (9%)</td>
<td>$3</td>
</tr>
<tr>
<td>State:</td>
<td></td>
</tr>
<tr>
<td>Public welfare/hospitals/health (31%)</td>
<td>$79</td>
</tr>
<tr>
<td>Employee retirement</td>
<td>$138</td>
</tr>
<tr>
<td>Total government expenditures</td>
<td>$4,127</td>
</tr>
<tr>
<td>Size of selected spending as fraction of GDP</td>
<td>17%</td>
</tr>
<tr>
<td>Size of total government spending as fraction of GDP</td>
<td>35%</td>
</tr>
<tr>
<td>Early transfers in percent of total selected spending ($\lambda$)</td>
<td>38%</td>
</tr>
<tr>
<td>Late transfers in percent of total selected spending ($1 - \lambda$)</td>
<td>62%</td>
</tr>
</tbody>
</table>

Source for State and Local:
http://www.census.gov/compendia/statab/cats/state_local_govt_finances_employment.html


Table 5: Government Transfers divided into transfers to the young and the old.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Source/Moment to match</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
</tr>
<tr>
<td>Inv. of elast. of substitution</td>
<td>$\sigma = 2.5$ to match $K/Y$ and $r$</td>
</tr>
<tr>
<td>Time preference</td>
<td>$\beta = 0.94$ to match $K/Y$ and $r$</td>
</tr>
<tr>
<td>Consumption preference</td>
<td>$a_1 = 0.43$ to match lifetime %L</td>
</tr>
<tr>
<td>Leisure preference</td>
<td>$a_2 = 0.57$ to match lifetime %L</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>$A = 3.1834$ to normalize wages</td>
</tr>
<tr>
<td>Capital share in production</td>
<td>$\alpha = 0.36$ are standard values (e.g. Kydland and Prescott (1982))</td>
</tr>
<tr>
<td>Annual discount rate</td>
<td>$\delta = 8%$ 5.9% – 12% in Nadiri and Prucha (1996)</td>
</tr>
<tr>
<td><strong>Ability Distribution</strong></td>
<td></td>
</tr>
<tr>
<td>Log of lifetime income</td>
<td>$\mu = 13.43$ (˜$683,000) Fullerton and Rogers (1993)</td>
</tr>
<tr>
<td>Stand. dev. of lifetime income</td>
<td>$\sigma = 0.415$ and own calculations</td>
</tr>
<tr>
<td>Lowest lifetime income</td>
<td>$\theta = 0.004$ (˜$10,000)</td>
</tr>
<tr>
<td>Highest lifetime income</td>
<td>$\bar{\theta} = 121$ (˜$5,000,000)</td>
</tr>
<tr>
<td><strong>Policy Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Wage income tax rate</td>
<td>$\tau^L = 27.0%$ to match VR and $%\tau^L$</td>
</tr>
<tr>
<td>Capital tax rate</td>
<td>$\tau^K = 15.5%$ to match VR and $%\tau^K$</td>
</tr>
<tr>
<td>Targeting parameter</td>
<td>$b = 0.20$</td>
</tr>
<tr>
<td>Max late transfer</td>
<td>$a = 0.22$ (˜$364,307)</td>
</tr>
</tbody>
</table>

Table 6: Parameters
**Matched Moments**

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual interest rate</td>
<td>$r = 4.1%$ 4% in NIPA accounts</td>
</tr>
<tr>
<td>Capital output ratio</td>
<td>$K/Y = 2.66$ 2.7 – 3 are standard from NIPA accounts</td>
</tr>
<tr>
<td>Average Lifetime Labor Supply</td>
<td>$%L = 0.364$ 0.374 in Gomes, Kotlikoff and Viceira (2007)</td>
</tr>
<tr>
<td>Percent of labor tax revenue</td>
<td>$%\tau^L = 74.5%$ 75% in U.S. Stat.Abstr. 2004 &amp; own calculations</td>
</tr>
</tbody>
</table>

| Table 7: Matched Moments |

<table>
<thead>
<tr>
<th>Categories</th>
<th>Policy Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Income heterog. Elastic labor supply</td>
</tr>
<tr>
<td>U.S.</td>
<td>yes   yes</td>
</tr>
</tbody>
</table>

**Optimal Policy Parameters**

Non-separable utilities:

<table>
<thead>
<tr>
<th>$M1$</th>
<th>yes</th>
<th>yes</th>
<th>0.34</th>
<th>1.05</th>
<th>30.6%</th>
<th>100%</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M1b: \beta = 0.985$</td>
<td>yes</td>
<td>yes</td>
<td>0.61</td>
<td>1.11</td>
<td>35.2%</td>
<td>100%</td>
<td>0.73</td>
</tr>
<tr>
<td>$M2$</td>
<td>yes</td>
<td>no</td>
<td>–</td>
<td>–</td>
<td>100%</td>
<td>100%</td>
<td>1</td>
</tr>
<tr>
<td>$M3$</td>
<td>no</td>
<td>yes</td>
<td>–</td>
<td>–</td>
<td>–100%</td>
<td>73%</td>
<td>1</td>
</tr>
<tr>
<td>$M3a: \tau^L \geq 0$</td>
<td>no</td>
<td>yes</td>
<td>–</td>
<td>–</td>
<td>0%</td>
<td>79.9%</td>
<td>1</td>
</tr>
<tr>
<td>$M3b: \beta = 0.985$</td>
<td>no</td>
<td>yes</td>
<td>–</td>
<td>–</td>
<td>–100%</td>
<td>48%</td>
<td>1</td>
</tr>
<tr>
<td>$M4$</td>
<td>no</td>
<td>no</td>
<td>–</td>
<td>–</td>
<td>85%</td>
<td>100%</td>
<td>1</td>
</tr>
</tbody>
</table>

Additively separable utilities:

<table>
<thead>
<tr>
<th>$M5$</th>
<th>yes</th>
<th>yes</th>
<th>–</th>
<th>–</th>
<th>37.7%</th>
<th>83.0%</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M6$</td>
<td>yes</td>
<td>no</td>
<td>–</td>
<td>–</td>
<td>100%</td>
<td>43.6%</td>
<td>1</td>
</tr>
<tr>
<td>$M7$</td>
<td>no</td>
<td>yes</td>
<td>–</td>
<td>–</td>
<td>–100%</td>
<td>62.9%</td>
<td>1</td>
</tr>
<tr>
<td>$M7a: \tau^L \geq 0$</td>
<td>no</td>
<td>yes</td>
<td>–</td>
<td>–</td>
<td>0%</td>
<td>52.1%</td>
<td>1</td>
</tr>
<tr>
<td>$M8$</td>
<td>no</td>
<td>no</td>
<td>–</td>
<td>–</td>
<td>8.8%</td>
<td>43.6%</td>
<td>1</td>
</tr>
</tbody>
</table>

| Table 8: Optimal Steady State Tax Policies. Model $M5$ is run with only a late redistribution program in place, so that $\lambda$ is fixed exogenously to equal zero. |

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Figure 1: Lifetime Income Distribution. Fitted Log Normal and Gamma distributions. Source of mean lifetime income data per deciles: Fullerton and Rogers (1993).
Figure 2: Steady State Results for Transfer Programs of Variable Size.
Figure 3: Aggregate Welfare and Welfare of the Poorest Household
Figure 4: Welfare per Income Quartile
Figure 5: Steady State Results for Differently Targeted Late Transfer Programs and Size Adjusted Early Transfer Programs.
Figure 6: Aggregate Welfare and Welfare of the Poorest Household
Figure 7: Welfare per Income Quartile