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First versus Second-Mover Advantage with Information Asymmetry about the Size of New Markets

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Eric Rasmusen* and Young-Ro Yoon **

Abstract

Is it better to move first, or second—to innovate, or to imitate? Suppose one player has superior information about which of two new markets is better. If he enters first, he might be able to secure a natural monopoly. (The less-informed player also has this motive.) If he enters second, he can prevent the other player from imitating him. We find, predictably, that the more accurate the informed player’s information the more he wants to delay in order to prevent the spillover of his information. Also, the less accurate the informed player’s information the more he wants to move first in order to foreclose a market. In addition, the bigger the difference in markets, the more likely the two players will make the same choice. More surprisingly, if the informed player’s information becomes more accurate that can hurt both industry profits and consumer welfare by inducing both players to choose what they hope is the bigger market, leaving the other market not served.

JEL codes: D81, D82, L13.

Keywords: Market Entry, First- and Second Mover Advantage, Payoff Externalities, Informational Externalities, Endogenous Timing

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This draft has step-by-step proofs, for the convenience of readers and referees. We expect to strip out much of the algebra in the published version.

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1 Introduction

Whether it is better to move first or second is an old question in game theory. This has most commonly been seen as a question of whether it is better to commit or to outbid—of whether actions are strategic substitutes or strategic complements. Under uncertainty, however, a new consideration enters: that a second mover may have more information available to him. Even if information is symmetric, observing what happens to the first mover helps the second mover know what action to take. If information is asymmetric, then merely observing the action of the first mover, even without observing the consequences, reveals information.

We look at a setting in which commitment may be valuable, the results of the first move are not immediately observable, and information is asymmetric. Two players, one of whom is better but not perfectly informed about market quality, must decide which of two markets to enter. If the uninformed player moves second, he can imitate the choice of the informed player, but he must balance this against the undesirability of competing head-to-head in the same market, which may turn out not to be the best one after all.

We describe the model in terms of entering new geographic markets, but the model is equally suited to new products, new input markets, or any of the other varieties of innovation described by Joseph Schumpeter [1911]). Should a player make his investment gamble early, or late, when he knows other players are watching? Simple though it is, such a situation contains elements of preemption, coordination, and signal-jamming.

We find that the more accurate the informed player’s information, the more both players want to delay. The uninformed player wants to delay to infer the informed player’s information and make the same choice. The informed player also wants to delay, but for the sake of preventing his information from being revealed. On the other hand, if the informed player’s information is poor, each player is more likely to move first, in which case his purpose is to foreclose the market.

The bigger the difference in market quality, we find, the more likely the players will make the same choice. Also, better information can hurt both industry profits and consumer welfare by inducing both players to choose what they hope is the bigger market, leaving the other market not served. If improved information results in both players choosing the same apparently best market where otherwise they would have chosen to be monopolist in each separate market, the result can actually be a lower probability that the truly best market is served.

2 The Literature

The Stackelberg model [Heinrich von Stackelberg [1934]) is an early model of first-mover advantage, a sequential-move quantity game in which the first mover, by committing to a large quantity, gains
a profit advantage over the second mover compared to in the simultaneous-move (Cournot) model. Articles by Esther Gal-or (1985, 1987), J. Hamilton & S. Slutsky (1990), and Paul Klemperer & Margaret Meyer (1986) study this variety of game. Two articles by Eric Van Damme & S. Hurskens (1999, 2004) look at whether a player would prefer to move first or second in various other games in which both player are already in the market and the only question is price or quantity. They consider a linear quantity setting and a price-setting duopoly game when the timing of commitment is decided endogenously. They show that in the risk-dominant equilibrium, the high cost player will choose to wait and the low cost player will emerge as the endogenous Stackelberg leader and price leader. Although the timing of action is decided endogenously, in both models, the informational externalities do not play a role in characterizing the equilibrium.

The articles mentioned above are about commitment, in a setting without uncertainty. Gal-Or (1987) shows that a player with superior information on demand may prefer ex ante to have to move simultaneously rather than first, because otherwise if demand is strong the player will reveal that to its rival by its choice of high output (though George Mailath [1993] shows that if the well-informed player has the option of moving first, it will always take that option, since to delay reveals that it is trying to hide strong demand). Hans-Theo Normann (1997, 2002) also looks at what happens when one duopolist is better informed when he makes his quantity decision.

In all these models, the players are making decisions about how hard to compete in one market, not whether to compete at all, and whether the informed player’s information is high or low quality is unimportant. In the present paper, two major concerns will be what happens when the well-informed player turns out to be wrong after all (since he will not have perfect information), and whether it is desirable from the point of view of industry profits or social welfare to have entry in two markets rather than just one. Also, the decisions will be binary— to enter a market or not— rather than a continuous quantity or price decision, so we can focus strictly on the order of moves rather than on distortions arising from entry deterrence tactics.

Whether it is best to move first or second has received attention in other fields as well. An example in the marketing is the empirical study by Venkatesh Shankar, Gregory Carpenter and Lakshman Krishnamurthi (1998); examples from management strategy are the articles by Marvin Lieberman and David Montgomery (1988, 1998).

Another possible situation is where information is symmetric but there is uncertainty and the first player’s move creates publicly observable information. Rafael Rob (1991) is about entry with an informational and payoff externality, but the players are not asymmetrically informed, and the market is competitive. The timing of action is given exogenously. Hence, the analysis about the endogenous timing of action is not dealt with. Instead, it focuses on the second mover advantage initiated from the availability of learning. Moreover, what the second mover learns in both papers are different. In Rob (1991), the second mover can learn something about the unknown true state from observing the activity of the first mover. On the other hand, in this paper, what the second mover can get from observing the first mover’s choice is not the direct information about the
unknown true state. If informed player’s information quality is relatively low, the opportunity in which the first mover’s choice can be observed provides the second mover an opportunity to take the different action from that of the first mover. Also, if informed player’s information quality is relatively high, the advantage of preventing information from being revealed by acting as the second mover, which is not derived in Rob (1991) can be derived. Similarly in Midori Hirokawa & Dan Sasaki (2001) and Heidrun Hoppe (2000) (see too Heidrun Hoppe & Ulrich Lehmann-Grube (2001)), look at the first mover’s move reveals the unknown information. In the present paper, on the other hand, market quality is only revealed after both players have moved. There is no “Who will bell the cat? ” problem.

Christopher Chamley and Douglas Gale (1994) and Jianbo Zhang (1997) discuss strategic delay and the endogenous timing of action when only informational externalities are present and there is no negative externality from one player choosing the same action as another, but there is some intrinsic cost to delay. In Chamley and Gale (1994), a player has an incentive to delay his action to observe other players’ decisions for information updating. Zhang (1997) links this result to informational cascades. He concludes that the most-informed player is least willing to wait, because he has the least to learn than other players. He acts as the leader, and other players mimic him immediately. In these models, although the action timing is endogenous, there are not payoff externalities from actions. Each player’s main concern is whether the cost of delay is worth learning other players’ information.

The papers closest to the present one are by Lars Frisell (2003) and Young-Ro Yoon (2006, 2007).

Frisell (2003), like the present paper, asks who will enter first, the less informed or the better informed player. He uses a continuous-time model of a reverse war of attrition: who is willing to wait the longest, when delay means the loss in profits? Fitting his model into the present context, if duopoly profits are higher than monopoly profits (a case we do not consider in this paper) or just a little lower, the informed player enters first. If duopoly profits are enough lower, however, the informed player waits longer. What matters is the ratio of duopoly to monopoly profits, and not the degree of information superiority.

In the present paper, delay will not be costly, but it cannot be indefinitely long either. Players must decide to enter either early, or late. One player cannot simply outwait the other— if he waits, the result will be simultaneous choices. If both players want to move late, they can do so— but then they will be simultaneous movers. As a result, in our model a player who moves late must be concerned about ending up in the same market as his rival by accident, even if he has prevented purposeful imitation. We will find, in contrast to Frisell, that even if industry profits suffer heavily when both players are in the same market, the informed player may decide to move first if he is not very much better informed. There will, in fact, be multiple equilibria in that case. Frisell does not look at industry or social welfare, but we will see that the possibility of simultaneous entry has interesting implications for that too.
In Yoon (2006), the less-informed player wishes to delay in order to learn. The more-informed player wishes delay to prevent that learning. This conflict yields a war of attrition. Although both players can benefit from acting as the follower, the gain from a delay for learning is greater than that for preventing the other’s learning. Thus, the leader is the more-informed player. Yoon (2007) extends the model to the case of three players and shows that the weakly-informed players may voluntarily relinquish the option to wait, although no cost is imposed for a delay of action. When acting without a delay, they reveal their information with the hope that others will imitate them. This type of information spillover is due to their incentive to make use of the relative performance evaluation structure under which a bad reputation can be shared if others are also wrong.

The difference between this model and Yoon (2006, 2007) is in the payoff structure, which in the Yoon papers models reputation rather than entry, because though the best outcome is to be correct alone, the worst outcome is not to be incorrect in company with the other player, but to be incorrect alone. Misery loves company. Whereas in a model of market entry, the worst outcome is to be competing in a small market, in a model of labor competition, the worst outcome is to make the bad choice when the other player has made the good choice. Also different is that in the present paper the summed payoff from the two players making different choices might be bigger than the payoff from one player making the better decision and one making the worse, rather than the summed payoff being the same as if neither played the game at all (zero), as in the Yoon models.

3 The Model

An informed player (I) and an uninformed player (U) each will enter a market, either the North (N) or the South (S). One market is big and one is small, but players do not know which one is big. The players make choices simultaneously in the first period to either enter North, enter South, or wait. If one player waits and the other does not, the waiting player can observe what the other player did in the first period before choosing his own market in the second, though he cannot observe profits, which are received only at the end of the second period. Player i’s action set can thus be represented as \( A = \{a_i, t_i\} \), where \( i \in \{U, I\} \) denotes the player, \( a_i \in L = \{N, S\} \) denotes the market entered, and \( t = \{t_1, t_2\} \) denotes the period of entry. The periods are instantaneous, existing only to model sequentially and simultaneity.

Table 1 shows the payoffs, with \( x < \alpha y \) for \( 0 < \alpha < 1 \). These payoffs are chosen to model the situation in which a monopolist earns \( x > 0 \) or \( y > x \) depending on whether its market is small or large, and each of two duopolists would earn \( \alpha \) times as much as a monopolist would have.
If competition hurts a player’s profits, as it does unless the two players’ products are comple-
ments, then $0 < \alpha < 1$. If a duopoly earns less than a monopoly, as in the Cournot model with
identical products, then $\alpha < 0.5$. If a duopoly industry earns more than a monopoly, as happens
when consumers sufficiently value differentiated products, then $\alpha > 0.5$. We allow for both cases.

The parameter $\alpha$ increases with three things: (1) the degree of product differentiation, (2) the
degree to which the two goods are complements, and (3) the ability of the two players to collude
when they are a duopoly. If the products are identical, then $\alpha \leq \frac{1}{2}$, with perfect collusion having
$\alpha = 0.5$, Bertrand competition having $\alpha = 0$, and Cournot competition having $0 < \alpha < 0.5$. If
there is perfect collusion, then $0.5 \leq \alpha < 1$, depending on the degree of product differentiation and
product complementarity.

In the following, for the first analysis, we assume that $x < \alpha y$ where $0 < \alpha < 1$. That is,
we assume that the duopoly profit in a big market is greater than the monopoly profit in a small
market. This corresponds to the case where $\frac{y}{x} < \alpha < 1$. In the final subsection, we will also
consider the case where the monopoly profit in a small market is greater than the duopoly profit
in a big market, i.e., $x > \alpha y$ where $0 < \alpha < 1$, which corresponds to the case where $0 < \alpha < \frac{y}{x}$.
Hence, we consider all cases where $0 < \alpha < 1$.

The common prior is that both markets are equally likely to be the big market. Before the
first period, the informed player observes his own private signal $\theta \in \Theta = \{N, S\}$ about which is the
big market. The signal $\theta$ is such that:

\[
\begin{align*}
\Pr(\theta = N | w = N) &= \Pr(\theta = S | w = S) = p \quad \text{(1)} \\
\Pr(\theta = S | w = N) &= \Pr(\theta = N | w = S) = 1 - p \quad \text{(2)}
\end{align*}
\]

where $p \in (\frac{1}{2}, 1)$ and $p$ is drawn from the uniform distribution over the interval $(\frac{1}{2}, 1)$.  
Here, $w$ denotes the actual big market. Hence, for example, if we denote $w = N$, it means that North is a big
market. Also, $\Pr(\theta = N | w = N)$ means the probability that observed signal says that a big market
is North when actually North is a big market. Thus, (1) and (2) mean that the informed player’s

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1 a) $w$ denotes the actual big market. Hence, for example, if we denote $w = N$, it means that North is a big
market.

b) The assumption that the signal’s distribution is uniform is relevant only when we come to comparative statistics.
information is correct with probability $p$ and wrong with probability $1 - p$ and the parameter $p$ measures the information’s precision. As $p$ approaches $\frac{1}{2}$, the signal is less informative. As $p$ approaches 1, the signal becomes perfectly informative. The uninformed player does not observe the signal, but he does know the value of $p$.

The informed player’s pure strategy is

$$s_I : \Theta \rightarrow T \times L$$

where $\Theta = \{N, S\}$, $T = \{t_1, t_2\}$, and $L = \{N, S\}$  

Thus, for given $\theta$ and $p$, the informed player decides whether to enter in round 1 or round 2. Also, he should decide whether to make a choice following his signal or not. If $a_I = \theta$, we will say that he “uses the signal”.

The uninformed player’s strategy is

$$s_U : T \times L$$

where $T = \{t_1, t_2\}$ and $L = \{N, S\}$

For the informed player, whether $\theta = a_I$ or $\theta \neq a_I$ cannot be verified. Let $\lambda$ be the uninformed player’s belief as to the probability that the informed player uses the signal in choosing a market. Then, the strategy profile $s = \{s_U, s_I\}$ and $\lambda$ constitute a Perfect Bayesian equilibrium if $E\pi_I(s_I, s_U)$ and $E\pi_U(s_I, s_U)$ are maximized for given $\lambda$ and $s = \{s_U, s_I\}$ and $\lambda$ is consistent with strategies and Bayesian updating.

One particular value of $p$ is critical for determining the equilibrium, so let us define:

$$p = \frac{y - x\alpha}{(y - x)(\alpha + 1)}$$

4 Exogenous Timing of Entry

We will start by assuming that the sequence of entry is exogenous. This is interesting in itself and a necessary prelude for endogenous entry. For simplicity, we will assume that if the uninformed player moves first, he chooses his by flipping a fair coin; that is: $Pr(a_U = N) = Pr(a_U = S) = \frac{1}{2}$.

4.1 Simultaneous Entry

If the players choose simultaneously, there are multiple equilibria. In the simplest equilibrium, the uninformed player chooses randomly between North and South and the informed player uses the signal. This can be checked as follows.
Suppose that $\theta = N$. Then:

$$E_{\pi I} [a_I = \theta] = \frac{1}{2} \sum_{w \in \{N,S\}} \Pr(w | \theta = N) [\pi_I (\cdot, a_U = N) + \pi_I (\cdot, a_U = S)]$$

(6)

$$E_{\pi I} [a_I \neq \theta] = \frac{1}{2} \sum_{w \in \{N,S\}} \Pr(w | \theta = N) [\pi_I (\cdot, a_U = N) + \pi_I (\cdot, a_U = S)]$$

(7)

where $E_{\pi I} [a_I = \theta]$ is the expected payoff when informed player reveals his signal truthfully and $E_{\pi I} [a_I \neq \theta]$ is the expected payoff when he deviates from his signal. Then:

$$E_{\pi I} [a_I = \theta] - E_{\pi I} [a_I \neq \theta] = \frac{1}{2} (y - x) (2p - 1) (\alpha + 1) > 0$$

(8)

Hence, the informed player’s best response is to use the signal and we have an equilibrium.

This equilibrium is at the center of a continuum of mixed-strategy equilibria in which the uninformed player chooses North with probability $\mu \in [0,1]$. The extreme is $\mu = 0$, in which he always chooses South (or $\mu = 1$, always choosing North). The informed player’s best response in that equilibrium is to use his signal.

4.2 Sequential Entry

We now come to Proposition 1, which confirms what we would expect about sequential entry: that if the informed player’s signal is accurate enough, then the uninformed player will imitate him and the informed player will prefer sharing the market he thinks is best to monopolizing the market expected to be inferior.

**Proposition 1.** Whether the informed player uses the signal depends on its accuracy.

1) If the uninformed player chooses first, the informed player uses the signal if it is accurate enough (if $p > \overline{p}$), and otherwise simply makes the opposite choice.

2) If the informed player chooses first, he uses the signal. The uninformed player imitates him if the signal is accurate enough (if $p > \overline{p}$), but otherwise diverges.

**Proof:** In the appendix.

When the informed player is the leader, his best response is to use the signal. When he is the follower, he knows he is better informed about which market is big. It is natural for him to use the
signal, but he must also consider the competition that arises when both players are in the same market. Hence, the degree of his information quality affects his decision on whether to use it or not. If his information quality is relatively low, i.e., \( p \in \left( \frac{1}{2}, \bar{p} \right) \), he has weak confidence in the correctness of the signal. If the uninformed player already accidentally selected the location signalled, using the signal is a bad idea because of the possibility that both players end up in a small market, yielding the lowest payoff, \( x \). The informed player cares more about avoiding competition than being in a big market, so he selects the location opposite to what the signal reveals.

On the other hand, if his information quality is relatively high, i.e., \( p \in (\bar{p}, 1) \), he has relatively strong confidence in the correctness of the signal. Even if the uninformed player already chose the signalled location, it is better to join him there in what is very likely the best market. Hence, regardless of the uninformed player’s choice of location, the informed player uses the signal.

Similar reasoning applies to the uninformed player’s strategy. If the informed player’s information quality is low, the uninformed player thinks mainly of avoiding competition. If the informed player already preempted one market, the uninformed player chooses the other one. On the other hand, if the informed player’s information quality is high, as uninformed player gives much credit to the correctness of \( \theta \), he imitates the informed player’s choice.

Note that from (3), \( \frac{\partial p}{\partial y} = \frac{(\alpha - 1)y}{(\alpha + 1)(y - x)^\alpha} < 0 \) and \( \frac{\partial p}{\partial x} = -\frac{(\alpha - 1)y}{(\alpha + 1)(y - x)^\alpha} > 0 \). As \( y \) increases, the parameter set for which \( p \in (\bar{p}, 1) \) increases. Also, as \( x \) increases, the parameter set for which \( p \in (\bar{p}, 1) \) decreases. Amount \( y \) is the payoff when a player is in a big market. For the informed player, as \( y \) increases, his wish to be in a big market dominates trying to avoid competition. So, as \( y \) increases, he has less incentive to ignore the signal. The uninformed player knows that the informed player observed a signal. Hence, as \( y \) increases, he is biased toward being in a big market and is more likely to imitate the informed player’s choice based on the meaningful information. Also, amount \( x \) is the profit earned when by being a monopolist in a small market. Hence, as \( x \) increases, the loss from being in a small market falls. Each player is more likely to diverge from the choice of the leader.
4.3 The Expected Payoffs

\[ t_1 \quad t_I \quad t_2 \]
\[ t_U \]
\[ t_1 \quad -\frac{(p(y-x)(1-\alpha)-y-x\alpha)}{2}, \frac{(x-px+py)(\alpha+1)}{2} \quad \frac{(x+y)}{2}, \frac{(x+y)}{2} \]
\[ t_2 \quad (y+px-py), (x-px+py) \quad -\frac{(p(y-x)(1-\alpha)-y-x\alpha)}{2}, \frac{(x-px+py)(\alpha+1)}{2} \]

Table 2: Payoffs Depending on Who Moves When if the Signal Is Imprecise, \( \frac{1}{2} < p < \bar{p} \)

\[ t_1 \quad t_I \quad t_2 \]
\[ t_U \]
\[ t_1 \quad -\frac{(p(y-x)(1-\alpha)-y-x\alpha)}{2}, \frac{1}{2}(x-px+py)(\alpha+1) \quad -\frac{(p(y-x)(1-\alpha)-y-x\alpha)}{2}, \frac{1}{2}(x-px+py)(\alpha+1) \]
\[ t_2 \quad \alpha(x-px+py), \alpha(x-px+py) \quad -\frac{(p(y-x)(1-\alpha)-y-x\alpha)}{2}, \frac{1}{2}(x-px+py)(\alpha+1) \]

Table 3: Payoffs Depending on Who Moves When if the Signal Is Precise, \( \bar{p} < p < 1 \)

**Lemma 1:** Tables 2 and 3 show the equilibrium payoffs.

**Proof:** In the appendix.

5 Endogenous Timing of Entry

5.1 Equilibrium

Denotes \( i \)'s ex-ante expected payoff when he acts as the leader and follower by \( \pi^L_i \) and \( \pi^F_i \), and denote his payoff when the players act simultaneously by \( \pi^S_i \). When \( p < \bar{p} \), Table 2 tells us that:

\[ \pi^S_U < \pi^F_U < \pi^L_U \quad \text{and} \quad \pi^S_I < \pi^F_I < \pi^L_I \quad (9) \]

Recall that when the information precision \( p \) is low, each player’s best response as the follower is to choose a different location from the leader. Hence, equation (10) says that the best case for
each player is when he can preempt a market and operate as a monopolist. Even if a player acts in round 2, he can operate as a monopolist in one market. In the case of the informed player, he observes his own informative signal. If he uses the signal, the probability that he can be in a big market is greater than $\frac{1}{2}$ as $p > \frac{1}{2}$. If the uninformed player acts in round 2, the informed player operates as a monopolist in that market. However, if the informed player acts as the follower, he should select a location opposite to what the uninformed player selected in round 1. In this case, if the market which uninformed player preempted is what his signal says, informed player should select the market which is opposite to what his meaningful information says as a big market. Hence, if it is available, that would be better for him to choose a location which the signal reveals. This can be always attained only if he acts as the leader. In the case of the uninformed player, from the similar reasoning, if he acts as the follower, he should choose a location opposite to what the informed player’s signal reveals. So, the probability that he is in a big market is less than $\frac{1}{2}$. Therefore, he can attain the greatest expected payoff when he acts in round 1 because, in this case, he can avoid being in the market of which the probability that it is a big market is less than $\frac{1}{2}$. Also, note that $\pi_i^S < \pi_i^F$ where $i \in \{U, I\}$. This implies that $i$ is biased toward avoiding a competition for his weak confidence in $\theta$. If both players choose a location simultaneously, the might both end up as monopolists, but they might not. Instead, if he acts as the follower, although being in a big market is not guaranteed, at least he can be a monopolist in one market. Hence, the second-best case is the one in which he is the follower and the worst case is the one in which both act simultaneously.

When $p > \bar{p}$, Table 3 tells us that:

$$\pi_U^S = \pi_U^L < \pi_U^F \quad \text{and} \quad \pi_I^L < \pi_I^S = \pi_I^F$$ (10)

Expression (11) says that if information precision $p$ is high, each player attains the greatest expected payoff when he acts as the follower. The informed player knows that, if the uninformed player has a chance to observe the informed player’s choice, the uninformed player will choose the same location. However, for the relatively high $p$, the informed player has a strong confidence in the correctness of the signal. Hence, he believes that the signal reveals a big market correctly with high probability and intends to be in that market as a monopolist to earn $y$. Hence, the informed player wants to prevent $\theta$ from being revealed to the uninformed player, which is why acting as the follower attains the greatest expected payoff. The reasoning that the informed player intends to act as the follower to prevent the uninformed player’s same choice can also be verified from that $\pi_I^S = \pi_I^F$. Although, the informed player does not act as the follower, at least if both players act simultaneously, the uninformed player still has no chance to observe the informed player’s choice. As the uninformed player’s imitation can be prevented under the simultaneous actions, $\pi_I^S = \pi_I^F$ is derived. The reasoning that the uninformed player intends to act as the follower in order to observe the informed player’s choice can be verified easily if we recall his best response as the follower, which is to imitate the informed player’s choice. This can also be verified from $\pi_U^S = \pi_U^I$. Whether the uninformed player is the leader or both players act simultaneously, he has no chance to observe the informed player’s action and infer $\theta$. Hence, $\pi_U^S = \pi_U^I$. 

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If the information precision $p$ is less than $\bar{p}$, from equation (9) there exists a first-mover advantage, but otherwise there is a second-mover advantage. If $p < \bar{p}$, a player would prefer to go first, but he will delay entry if he thinks the other player will enter first, so as to avoid ending up in the same market. Otherwise (that is if $p > \bar{p}$), the uninformed player would like to delay entry in order to observe the informed player’s choice, but if he does, the informed player will also delay entry in order to prevent the uninformed player from learning.

There is always an equilibrium in which the uninformed player moves first. Moreover, if the informed player’s signal is informative but weak, the uninformed player strongly wishes to enter first, despite his complete lack of information about which market is better. That is because entering first gives him at least a 50-50 chance of becoming a monopolist in the better market.

An Important Intuition. Even though each player would rather share the better market than monopolize the worse market, so there would be no first-mover advantage if the game had symmetric information, a first-mover advantage does exist in the game with uncertainty but a somewhat informative signal observed by one player. This first-mover advantage is so strong that even the uninformed player would prefer to go first and choose a market randomly rather than imitate the informed player. If information becomes more accurate, though, the first-mover advantage turns into a second-mover advantage.

Proposition 2. If information is precise enough (if $\bar{p} < p < 1$), the informed player moves in the second period, and the uninformed player is indifferent about when he moves. Otherwise (if $\frac{1}{2} < p < \bar{p}$), there are two pure-strategy equilibria, one for each of two players entering first, and a mixed-strategy equilibrium in which the informed player enters early with probability $z$ and the uninformed player enters early with probability $w$:

\[
(z, w) = \left(\frac{(x - px + py) (\alpha - 1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}, \frac{(x - px + py) (\alpha - 1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}\right)
\]

Proof: In the appendix.

The precision of the informed player’s signal is high if $\bar{p} < p < 1$. The informed player delays his choice and acts in round 2 to conceal his information. Although the best case for the uninformed player is when he can observe the informed player’s choice, as $t_I = t_2$, he must then use only with his prior belief and he attains the same ex-ante expected payoff regardless of when he enters. This results in the equilibrium in which $w = 0$ and $z \in [0, 1]$ where $w = \Pr(t_I = t_2)$ and $z = \Pr(t_U = t_2)$.

Next, consider the equilibrium where the informed player’s signal prevision is low, i.e., $\frac{1}{2} < p < \bar{p}$. There exist two pure-strategy equilibria and one mixed-strategy equilibrium. In the pure equilibria, both players’ choices are sequential always. If we consider each player’s best response as the follower, these sequential choices result in that both players operate as a monopolist in a
different market endogenously. As the informed player’s information quality is relatively low, each player does not give much credit to the correctness of $\theta$. Then, each player is more slanted toward avoiding a competition than being in a big market. For given $t_{-i} = t_1$, $i$ has no incentive to deviate from $t_i = t_2$ because, by acting in round 2, $i$ can observe the choice of $-i$ and he can prevent being in the same market by selecting a different location. By the same reasoning, for given $t_{-i} = t_2$, $i$ has no incentive to deviate from $t_i = t_1$ because acting in round 1 gives $-i$ a chance to select a different location. If each player has an incentive to delay his action, it is to observe the other player’s choice. His delay is not in order to imitate, however, but to avoid ending up in the same market.

In the mixed-strategy equilibrium, comparative statics yield that:

$$\frac{\partial z}{\partial p} = \frac{\partial w}{\partial p} > 0 \quad (12)$$

$$\frac{\partial z}{\partial x} = \frac{\partial w}{\partial x} < 0 \quad (13)$$

$$\frac{\partial z}{\partial y} = \frac{\partial w}{\partial y} > 0 \quad (14)$$

Inequality (12) says that $\frac{\partial z}{\partial p} = \frac{\partial w}{\partial p} > 0$. The increase in $p$ means that the precision of the informed player’s signal increases. Then, it is intuitive for the informed player to have the greater incentive to act in round 1 to preempt the location which his information reveals. Recall that, being in a big market is better than being in a small market whether it operates as a monopolist or competes with the other player in that market. Moreover, when he acts in round 2, if the uninformed player acts in round 1, he should select a location opposite to what signal reveals. Therefore, as $p$ increases, he may have the greater incentive to avoid a situation in which he has to ignore the signal. In the case of the uninformed player, as $p$ increases, delaying his choice becomes less attractive. If the informed player chooses a location in round 1, the uninformed player should take a location opposite to what $\theta$ reveals. It means that, the probability that the uninformed player is in a big market is $p < \frac{1}{2}$ and it decreases as $p$ increases. If the uninformed player acts in round 1, the probability that he can be in a big market is $p = \frac{1}{2}$, which is better than the case in which he acts in round 2. Therefore, as $p$ increases, the uninformed player is also more likely to act in round 1.

Inequality (13) says that $\frac{\partial z}{\partial x} = \frac{\partial w}{\partial x} < 0$. Note that, as $x$ increases, the payoff a player earns when he is in a small market increases. Hence, each player may not have the greater incentive to be in a big market and, instead will be more biased toward avoiding a competition. If he delays a choice and the other player acts in round 1, he can observe the other player’s choice and prevent a competition certainly by selecting a different location. Of course, if he acts in round 1 and the other player acts in round 2, it is same that each player can operate as a monopolist in each market. However, unlike the increase in $p$, the increase in $x$ does not provide informative information about which market is a big one. So, the informed player may not have a strong incentive to preempt a
market which the signal reveals. The uninformed player may also not have a strong incentive to preempt a market to increase the probability that he is in a big market.

Inequality (14) says that \( \frac{\partial z}{\partial y} = \frac{\partial w}{\partial y} > 0 \). As \( y \) increases, the payoff a player earns when he is in a big market increases. Then, each player may have the strong incentive to be in a big market. Then, the informed player may not want to ignore his informative signal. If he acts in round 2, as he has to ignore the signal sometimes, he is more likely to act in round 1. The uninformed player may also have the strong incentive to be in a big market. Then, like the case in which \( p \) increases, he may want to maintain a probability that he can be in a big market as at least \( \frac{1}{2} \), which can be achieved when he acts in round 1.

5.2 Efficiency

5.2.1 Ex-Post Efficiency

For each player, being in a big market is better than being in a small market, and being a monopolist is better than being a duopolist. It is better to be a duopolist in the big market, however, than a monopolist in the small market. And if both players end up in the same market and it is the small one, payoffs are lowest.

Industry payoffs, as opposed to individual ones, are maximized by both players locating in the big market if and only if \( 2\alpha y \geq x + y \); that is, if and only if:

\[
\alpha \geq \frac{x + y}{2y} \tag{15}
\]

Recall Proposition 2. When \( p \in \left( \frac{1}{2}, \bar{p} \right) \), there exist two pure-strategy equilibria such that \((t_U, t_I) = (t_2, t_1)\) and \((t_1, t_2)\). When \( p \in \left( \frac{1}{2}, \bar{p} \right) \), the follower’s best response is to select a different location from that of the leader. Hence, the players choose different locations, and when \( \alpha < \frac{x + y}{2y} \), the outcome is efficient. We state this as Proposition 3.

**Proposition 3.** If duopoly competition is moderate \( \left( \frac{x}{y} < \alpha < \frac{x + y}{2y} \right) \) and information is poor \( \left( \frac{1}{2} < p < \bar{p} \right) \), then both of the two pure-strategy equilibria are ex-post efficient.

When \( p \in \left( \frac{1}{2}, \bar{p} \right) \), if \( \alpha > \frac{x + y}{2y} \), ex post efficiency cannot be attained at all because both players are always in the different markets in pure-strategy equilibrium. On the other hand, if \( p \in (\bar{p}, 1) \), in equilibrium, \( t_I = t_2 \) and \( z \in [0,1] \) where \( z = \Pr (t_U = t_1) \). In this case, the informed player uses the signal in choosing a location. Hence, for \( \alpha \in \left( \frac{x}{y}, 1 \right) \), ex post efficiency is possible but not inevitable.\(^2\)

\(^2\) For example, when \( p \in (\bar{p}, 1) \), if \( \frac{x}{y} < \alpha < \frac{x + y}{2y} \), the ex-post efficiency is attained only if \( a_U \neq \theta \). On the other hand, if \( \frac{x + y}{2y} < \alpha < 1 \), the ex-post efficiency is attained only if \( a_U = \theta = w \).
As an example, in the Cournot model with linear demand, profit per player is less than the half of the monopoly profit, i.e., $2x < y$. Also, note that $\frac{x+y}{2y} > \frac{1}{2}$. As a result, ex post efficiency is guaranteed only if the informed player has imprecise enough information about which market is big. If his information improves, ex post efficiency worsens.\(^3\)

In addition, recall that $\frac{\partial p}{\partial x} = -\frac{(\alpha-1)y}{(\alpha+1)(y-x)^2} > 0$ and $\frac{\partial p}{\partial y} = \frac{(\alpha-1)y}{(\alpha+1)(y-x)^2} < 0$. Hence, for given $p$, as the profit of the small market increases or as the profit of the big market decreases, there is a greater possibility that the players select different locations, and it is easier for the pure strategy equilibrium to be efficient. That is, the smaller the difference in markets, the parameter set of $p$ for which pure strategy equilibrium is efficient increases.

### 5.2.2 Ex-Ante Efficiency

Ex ante, before it is known which market is big, industry profits are maximized by locating both players in the market the signal indicates is big if and only if $2\alpha(py + (1 - p)x) \geq x + y$; that is, if and only if:

$$\alpha \geq \frac{x + y}{2[py + (1 - p)x]}$$  \hspace{1cm} (16)

That is because under uncertainty, locating both players together might result in neither of them serving the big market, so $\alpha$ needs to be bigger than in equation (16) for co-location to be optimal.

**Proposition 4.** (Ex-ante) Efficiency depends on the ratio of duopoly to monopoly profit ($\alpha$) and the quality of information ($p$) as follows:

1) Suppose $\alpha < \frac{x+y}{2y}$

1-1) If $\frac{1}{2} < p < \overline{p}$, both pure strategy equilibria are efficient and the mixed strategy equilibrium is not. (A1 in Figure 1)

1-2) If $\overline{p} < p < 1$, all equilibria are efficient. (A2 in Figure 1)

2) Suppose $\alpha > \frac{x+y}{2y}$.

2-1) If $\frac{1}{2} < p < \overline{p}$, both pure strategy equilibria are ex-ante efficient and the mixed strategy equilibrium is inefficient. (A5 in Figure 1)

2-2) If $\overline{p} < p < \frac{y+x-2x\alpha}{2\alpha(y-x)} < 1$, all equilibria are efficient. (A4 in Figure 1)

2-3) If $\frac{y+x-2x\alpha}{2\alpha(y-x)} < p < 1$, all equilibria are inefficient. (A3 in Figure 1)

---

\(^3\) Here, it can be checked that $\frac{x}{y} - \frac{1}{2} = \frac{2x-y}{2y} < 0$, because in the Cournot model with linear demand, the profit per player is less than the half of the monopoly profit, i.e., $2x < y$. Hence, we can apply the result to the Cournot model with linear demand.
Proof: In the appendix.

Figure 1: Information Quality, Market Size, and Efficiency (see end of paper)

Proposition 4 implies that when duopolists would earn enough more than a monopolist and the markets differ enough in size, better information can reduce industry profits. That means consumers would be hurt more too. The worsening occurs if a bigger \( p \) means that we move from area A4 in Figure 1 to area A3.

6 What if a Monopolist in the Small Market Would Have Higher Profits than a Duopolist in the Big Market? (\( \alpha y < x \))

So far we have assumed that a player would rather be a duopolist in the big market than a monopolist in the small market. Let us now reverse that assumption, i.e., \( \alpha y < x \). Then, each player’s best response in timing can be derived as follows.

Lemma 2 Consider the case where \( \alpha y < x \).

1) Suppose that \( t_I = t_1 \) and \( t_U = t_2 \). Then, the informed player chooses a location following his given signal and the uninformed player deviates from the informed player’s choice for all \( p \in (\frac{1}{2}, 1) \).

2) Suppose that \( t_U = t_1 \) and \( t_I = t_2 \). Then, the informed player chooses a location different from the uninformed player’s choice for all \( p \in (\frac{1}{2}, 1) \). ( That is, if \( \theta_I = a_U \), the informed player chooses a location opposite to what the signal reveals. If \( \theta_B \neq a_A \), he always chooses a location following his given signal.)

3) Suppose that \( t_U = t_1 \). Then, the informed player chooses a location following his given signal.

Proof: In the appendix.

If \( \alpha y < x \), monopoly in a small market is better than duopoly in a big market. Lemma 2 says that if \( i \in \{ U, I \} \) acts as the follower, he will always select a location different from that of \(-i\) who acted as the leader for all \( p \in (\frac{1}{2}, 1) \), which is a quite intuitive result. As this is the same best response as that of the case where \( \alpha y > x \) and \( \frac{1}{2} < p < \frac{y-x}{y-x(\alpha+1)} \), we can use table 1. The computation yields that the equilibrium of our timing game can be characterized as follows.
Proposition 5 Suppose that $\alpha y < x$, so a player prefers being a monopolist in a small market to being a duopolist in a big market. Then, for all $p \in \left(\frac{1}{2}, 1\right)$, there exist two pure-strategy equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ and one mixed-strategy equilibrium $(z, w) = \left(\frac{(x - px + py)(\alpha - 1)}{2x\alpha - y - 2px\alpha + 2py\alpha}, \frac{(x - px + py)(\alpha - 1)}{2x\alpha - y - 2px\alpha + 2py\alpha}\right)$ where $z = \Pr(t_U = t_1)$ and $w = \Pr(t_I = t_1)$.

Proof of Proposition 5: In the appendix.

Compared to the case where $\alpha y > x$, if $\alpha y < x$, for all $p \in \left(\frac{1}{2}, 1\right)$, there exist two pure equilibria in which the sequential timing of actions is derived endogenously and one mixed equilibrium. If we focus on the pure equilibria, the sequential timing of actions which yields the different choices of both players is derived from the following reasoning. Compared to the case where $\alpha y > x$ in which being in the big market attains the greater payoff, regardless of the other player’s choice, than being in the small market, if $\alpha y < x$, operating as a monopolist in a small market is greater than operating as a duopolist in a big market. Hence, being in a big market is not attractive much. More precisely, being in a big market is attractive only if a player can operate as a monopolist. Moreover, being together in the same market always attains the smaller profit than operating as the monopolist in a market as $y > x > \alpha y > \alpha x$. Hence, both players give more weight to avoiding the competition in the same market and instead care more about being a monopolist in one market. In other words, for $i \in \{U, I\}$, if $t_{-i} = t_1$, $i$ has no incentive to act in round 1 because in that case both players can be in the same market. For $t_{-i} = t_1$, if $t_i = t_2$, always both players can operate as a monopolist separately in the different market because both players’ best responses as the follower is to make a different choice. Hence, the sequential timing of actions is derived endogenously in pure equilibria. In addition, in following, it can be checked that both pure equilibria are ex-post and ex-ante efficient.

Proposition 6. Suppose that $\alpha y < x$, so a player prefers being a monopolist in the small market to being a duopolist in the big market.

1) Two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ are ex-post efficient.

2) Two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ are ex-ante efficient, but the mixed strategy equilibrium is not ex-ante efficient.

Proof of Proposition 6: In the appendix.

7 Concluding Remarks

Often, a player must make a choice knowing that the choice may be imitated by another player. This choice might be of a new geographic market, as in our model, or of a new product, which could
be modelled with exactly the same mathematics. Moving first may or may not deter entry into the market by the rival player, but it certainly will reveal information. Hence, in a setting of endogenous timing of entry, the decision on the timing of entry can be interpreted as the decision on the flow of his private information. Of course, how is revealed information used by the other player affects the decision on the timing of entry. If the informed player’s information is not relatively valuable, both players attempt to avoid crowding in one market and this results in the pure equilibrium in which both player operate as a monopolist in each market. On the other hand, if the informed player’s information is relatively valuable, the rival player wants to imitate the choice of informed player. Hence, an informed player may well choose to delay its entry to prevent imitation, which result in the equilibrium in which no learning is available. Then, both players may end up in the same market, and this can reduce both industry profits and consumer welfare.
8 Appendix

In following, "I" denotes the "informed player" and "U" denotes the "uninformed player".

8.1 Proof of Proposition 1

Proposition 1. Whether the informed player uses the signal depends on its accuracy.

1) If the uninformed player chooses first, the informed player uses the signal if it is accurate enough (if \( p > \overline{p} \)), and otherwise simply makes the opposite choice.

2) If the informed player chooses first he uses the signal. The uninformed player imitates him if the signal is accurate enough (if \( p > \overline{p} \)), but otherwise diverges.

Proof of Proposition 1

1) First, we consider the case where U acts in round 1 and I acts in round 2. Then, there can be following two cases, \( \theta = a_U \) and \( \theta \neq a_U \).

Consider the case in which \( \theta = a_U \). Without loss of generality, assume that \( \theta = a_U = N \). Then, under the posterior beliefs \( \Pr(w = N| \theta = N) = p \) and \( \Pr(w = S| \theta = N) = 1 - p \),

\[
E\pi_I(a_I = \theta) = \sum_{w \in \{N,S\}} \Pr(w| \theta = N)\pi_U(a_I = \theta = a_U, w) = p(\alpha y) + (1 - p)(\alpha x) \tag{17}
\]

\[
E\pi_I(a_I \neq \theta) = \sum_{w \in \{N,S\}} \Pr(w| \theta = N)\pi_U(a_I \neq \theta = a_U, w) = p(x) + (1 - p)(y) \tag{18}
\]

From \( E\pi_I(a_I = \theta) - E\pi_I(a_I \neq \theta) = p((y - x)(\alpha + 1)) + x\alpha - y \),

\[
p \geq \frac{(x\alpha - y)}{(\alpha + 1)(x - y)} \implies E\pi_I(a_I = \theta) \geq E\pi_I(a_I \neq \theta) \tag{A3}
\]

where \( \frac{(x\alpha - y)}{(\alpha + 1)(x - y)} \in (\frac{1}{2}, 1) \) from \( \frac{(x\alpha - y)}{(\alpha + 1)(x - y)} - \frac{1}{2} = \frac{(1 - \alpha)(x + y)}{2(\alpha + 1)(y - x)} > 0 \) and \( \frac{(x\alpha - y)}{(\alpha + 1)(x - y)} - 1 = -\frac{(y\alpha - x)}{(\alpha + 1)(y - x)} < 0 \).

Next, consider the case in which \( \theta \neq a_U \). Without loss of generality, assume that \( \theta = N \) and \( a_U = S \). Then, under the posterior beliefs. Then, under the posterior beliefs \( \Pr(w = N| \theta = N) = p \) and \( \Pr(w = S| \theta = N) = 1 - p \),

\[
E\pi_I(a_I = \theta) = \sum_{w \in \{N,S\}} \Pr(w| \theta = N)\pi_U(a_I = \theta, a_U, w) = py + (1 - p)x \tag{19}
\]

\[
E\pi_I(a_I \neq \theta) = \sum_{w \in \{N,S\}} \Pr(w| \theta = N)\pi_U(a_I \neq \theta, a_U, w) = p(\alpha x) + (1 - p)(\alpha y) \tag{20}
\]
Then,
\[ E\pi_I(a_I = \theta) - E\pi_I(a_I \neq \theta) = p((y - x)(\alpha + 1)) + x - y\alpha \]  \hspace{1cm} (A6)

So, if \( p > \frac{(y\alpha - x)}{(\alpha + 1)(y - x)} \), \( E\pi_I(a_I = \theta) > E\pi_I(a_I \neq \theta) \) and if \( p < \frac{(y\alpha - x)}{(\alpha + 1)(y - x)} \), \( E\pi_I(a_I = \theta) < E\pi_I(a_I \neq \theta) \). However, as \( \frac{(y\alpha - x)}{(\alpha + 1)(y - x)} - \frac{1}{2} = \frac{(\alpha - 1)(x + y)}{2(\alpha + 1)(y - x)} < 0 \), for all \( p \in (\frac{1}{2}, 1) \), \( E\pi_I(a_I = \theta) > E\pi_I(a_I \neq \theta) \).

2) Second, we consider the case where \( I \) acts in round 1 and \( U \) acts in round 2. Note that what \( U \) observes is \( a_I \), not \( \theta \). That is, \( \theta \) is private information and \( U \) cannot observe whether \( I \) follows his signal or not in deciding a location. Hence, \( U \) should assign the belief for the possibility that \( a_I = \theta \).

Consider the case in which \( U \) believes that \( a_I = \theta \). Without loss of generality, assume that \( a_I = N \). Then, \( U \)'s posterior beliefs are \( \Pr(w = N|\theta = N) = p \) and \( \Pr(w = S|\theta = N) = 1 - p \). So,
\[
E\pi_U(a_U = a_I) = \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_U(a_I = a_U, w) = p(\alpha y) + (1 - p)(\alpha x) \quad (21)
\]
\[
E\pi_U(a_U \neq a_I) = \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_U(a_U \neq a_I, w) = p(x) + (1 - p)(y) \quad (22)
\]

Consider the case in which \( U \) believes that \( a_I \neq \theta \). Then, for given \( a_I = N \), \( U \)'s posterior beliefs are respectively \( \Pr(w = N|\theta = S) = 1 - p \) and \( \Pr(w = S|\theta = S) = p \) . So,
\[
E\pi_U(a_U = a_I) = \sum_{w \in \{N,S\}} \Pr(w|\theta = S)\pi_U(a_I = a_U, w) = (1 - p)(\alpha y) + p(\alpha x) \quad (23)
\]
\[
E\pi_U(a_U \neq a_I) = \sum_{w \in \{N,S\}} \Pr(w|\theta = S)\pi_U(a_I \neq a_U, w) = (1 - p)(x) + p(y) \quad (24)
\]

Now, suppose that \( U \) assigns a belief \( \lambda \in [0, 1] \) to the node that \( a_I = \theta \). Then:
\[
E\pi_U(a_U = a_I) = \lambda (p(\alpha y) + (1 - p)(\alpha x)) + (1 - \lambda) ((1 - p)(\alpha y) + p(\alpha x)) \quad (25)
\]
\[
E\pi_U(a_U \neq a_I) = \lambda (p(x) + (1 - p)(y)) + (1 - \lambda) ((1 - p)(x) + p(y)) \quad (26)
\]

So
\[
E\pi_U(a_U \neq a_I) - E\pi_U(a_U = a_I) \quad (27)
\]
\[
E\pi_U(a_U = a_I) - E\pi_U(a_U \neq a_I) = \lambda ((y - x)(2p - 1)(\alpha + 1)) - (x - px + py - y\alpha - px\alpha + py\alpha)
\]

Then, the procedure of sequential rationality yields:
\[
\lambda \overset{>}{\leq} \lambda^* \implies E\pi_U(a_U = a_I) \overset{<}{\leq} E\pi_U(a_U \neq a_I) \quad (A14)
\]

where \( \lambda^* = \frac{(x - px + py - y\alpha - px\alpha + py\alpha)}{(y - x)(2p - 1)(\alpha + 1)} \). The computation yields the following result.

\textbf{Lemma A.1.} Suppose that \( I \) acts in round 1.
a) Suppose that \( p \in \left( \frac{1}{2}, \frac{y-x\alpha}{(y-x)(\alpha+1)} \right) \). Then, for all \( \lambda \in [0, 1] \), U diverges from I’s choice.

b) Suppose that \( p \in \left( \frac{y-x\alpha}{(y-x)(\alpha+1)}, 1 \right) \). Then, there exists \( \lambda^* \in (0, 1) \) such that if \( \lambda > \lambda^* \), U imitates I’s choice, if \( \lambda < \lambda^* \), U diverges from I’s choice, and if \( \lambda = \lambda^* \), U is indifferent between imitating and diverging from I’s choice.

**Proof of Lemma A.1.**

Let’s check whether \( \lambda^* \in (0, 1) \) or not. First, for \( \lambda^* \), the numerator term can be written as \( p(y-x-x\alpha + y\alpha) + x - y\alpha \). So if \( p > \frac{y-x}{y-x-x\alpha + y\alpha} \), \( \lambda^* > 0 \) and if \( p < \frac{y-x}{y-x-x\alpha + y\alpha} \), \( \lambda^* < 0 \).

However, \( \frac{y-x}{y-x-x\alpha + y\alpha} - \frac{1}{2} = \frac{(\alpha-1)(x+y)}{2(\alpha+1)(y-x)} < 0 \). So, for \( p \in \left( \frac{1}{2}, 1 \right) \), \( p > \frac{y-x}{y-x-x\alpha + y\alpha} \), which means that \( \lambda^* > 0 \). Also, \( \lambda^* = 1 = \frac{1}{\alpha+1}(\frac{y-x}{y-x-x\alpha + y\alpha}) \). Here, the numerator term can be written as \( p(x-y + x\alpha - y\alpha) + y - x\alpha \). So, if \( p < \frac{y-x\alpha}{x-y+x\alpha-y\alpha} \), \( \lambda^* > 1 \) and if \( p < \frac{y-x\alpha}{x-y+x\alpha-y\alpha} \), \( \lambda^* < 1 \).

Here, as \(-\frac{y-x\alpha}{x-y+x\alpha-y\alpha} - \frac{1}{2} = \frac{(\alpha-1)(x+y)}{2(\alpha+1)(y-x)} > 0 \) and \(-\frac{y-x\alpha}{x-y+x\alpha-y\alpha} - 1 = \frac{(y-x)(\alpha+1)(y-x)}{(\alpha+1)(y-x)} < 0 \), it can be checked that \(-\frac{y-x\alpha}{x-y+x\alpha-y\alpha} \in \left( \frac{1}{2}, 1 \right) \) can be checked. Therefore, if \( p \in \left( \frac{1}{2}, \frac{y-x}{x-y+x\alpha-y\alpha} \right) \), \( \lambda^* < 1 \) and if \( p \in \left( \frac{y-x}{x-y+x\alpha-y\alpha}, 1 \right) \), \( \lambda^* > 1 \).

The above can be summarized as follows: a) Suppose that \( p \in \left( \frac{1}{2}, \frac{y-x}{x-y+x\alpha-y\alpha} \right) \). Then, for all \( \lambda \in [0, 1] \), \( E_{\pi_U}(a_U = a_I) < E_{\pi_U}(a_U \neq a_I) \). b) Suppose that \( p \in \left( \frac{y-x}{x-y+x\alpha-y\alpha}, 1 \right) \). Then, if \( \lambda > \lambda^* \), \( E_{\pi_U}(a_U = a_I) > E_{\pi_U}(a_U \neq a_I) \), if \( \lambda < \lambda^* \), \( E_{\pi_U}(a_U = a_I) < E_{\pi_U}(a_U \neq a_I) \), and if \( \lambda = \lambda^* \), \( E_{\pi_U}(a_U = a_I) = E_{\pi_U}(a_U \neq a_I) \).

a) Suppose that \( p \in \left( \frac{1}{2}, \frac{y-x\alpha}{(y-x)(\alpha+1)} \right) \). Then, for all \( \lambda \in [0, 1] \), U diverges from I’s choice. b) Suppose that \( p \in \left( \frac{y-x\alpha}{(y-x)(\alpha+1)}, 1 \right) \). Then, there exists \( \lambda^* \in (0, 1) \) such that if \( \lambda > \lambda^* \), U imitates I’s choice, if \( \lambda < \lambda^* \), U diverges from I’s choice, and if \( \lambda = \lambda^* \), U is indifferent between imitating and diverging from I’s choice. □

Using Lemma A.1, we derive I’s best response. Without loss of generality, assume that \( \theta = N \). Then, I’s posterior beliefs are \( \Pr(w = N | \theta = N) = p \) and \( \Pr(w = S | \theta = N) = 1 - p \).

First, assume that \( p \in \left( \frac{1}{2}, \frac{y-x\alpha}{(y-x)(\alpha+1)} \right) \). In this case, U always chooses a location different from I’s choice. Then

\[
E_{\pi_I}(a_I = \theta) = \sum_{w \in \{N, S\}} \Pr(w | \theta = N)_{\pi_U}(a_I = \theta \neq a_U, w) = p(y) + (1 - p)(x) \quad (28)
\]

\[
E_{\pi_I}(a_I \neq \theta) = \sum_{w \in \{N, S\}} \Pr(w | \theta = N)_{\pi_U}(a_I \neq \theta = a_U, w) = p(x) + (1 - p)(y) \quad (29)
\]

and

\[
E_{\pi_I}(a_I = \theta) - E_{\pi_I}(a_I \neq \theta) = (y - x)(2p - 1) > 0 \quad (A17)
\]

Thus, I’s best response is to choose the location following the signal.
Next, assume that $p \in \left(\frac{y-x}{(y-x)(\alpha+1)}, 1\right)$. First, consider the case in which $\lambda > \lambda^*$. In this case, $U$ imitates $I$’s choice of location. Then:

\[
E_{\pi_U} (a_I = \theta) = \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_{U}(a_I = \theta = a_U, w) = p(\alpha y) + (1-p)(\alpha x) \quad (30)
\]

and:

\[
E_{\pi_U} (a_I \neq \theta) = \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_{U}(a_I = a_U \neq \theta, w) = p(\alpha x) + (1-p)(\alpha y) \quad (31)
\]

Hence, $I$’s best response is to choose a location following the signal. That is, $\lambda = 1$. However, this is not consistent to $U$’s belief which is that $\lambda < \lambda^*$. Third, consider the case in which $\lambda = \lambda^*$. In this case, $U$ is indifferent between imitating and deviating from $I$’s choice of location. Here, assume that $\sigma_U$ is the probability that $U$ imitates $I$’s choice. Then:

\[
\begin{align*}
E_{\pi_U} [a_I = \theta] &= \sigma \left( \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_{U}(a_I = a_U, w) \right) + (1-\sigma) \left( \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_{U}(a_I = a_U \neq a_U, w) \right) \\
&= \sigma (p(\alpha y) + (1-p)(\alpha x)) + (1-\sigma) (p(y) + (1-p)(x)) \quad (32)
\end{align*}
\]

\[
\begin{align*}
E_{\pi_U} [a_I \neq \theta] &= \sigma \left( \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_{U}(a_I = a_U, w) \right) + (1-\sigma) \left( \sum_{w \in \{N,S\}} \Pr(w|\theta = N)\pi_{U}(a_I = a_U \neq a_U, w) \right) \\
&= \sigma (p(\alpha x) + (1-p)(\alpha y)) + (1-\sigma) (p(x) + (1-p)(y)) \quad (33)
\end{align*}
\]

It can be checked that $E_{\pi_U} [a_I = \theta] = E_{\pi_U} [a_I \neq \theta]$ at $\sigma = \frac{1}{1-\alpha} \notin [0,1]$ for $0 < \alpha < 1$. Hence, there exists no $\sigma \in [0,1]$ which yields $E_{\pi_U} [a_I = \theta] = E_{\pi_U} [a_I \neq \theta]$.

Finally, $I$’s best response in round 1 is to reveal his signal truthfully always. Also, $U$’s best response in round 2 is as follows: a) Suppose that $p \in \left(\frac{y-x}{(y-x)(\alpha+1)}, 1\right)$. Then, $U$ diverges from $I$’s choice. b) Suppose that $p \in \left(\frac{y-x}{(y-x)(\alpha+1)}, 1\right)$. Then, $U$ imitates $I$’s choice. □

8.2 Proof of Lemma 1

**Lemma 1:** Tables 2 and 3 show the equilibrium payoffs.

**Proof of Lemma 1.** First, we derive the uninformed player’s expected payoff from:

\[
\pi_U(t_U, t_I) = \sum_{\theta \in \{N,S\}} \sum_{w \in \{N,S\}} \Pr(w, \theta)\pi_U(a_I, a_U, w) \quad (A23)
\]
where $\Pr(w, \theta = w) = \frac{p}{2}$ and $\Pr(w, \theta \neq w) = \frac{1-p}{2}$. U’s expected payoffs should be calculated based on the belief $\Pr(w, \theta)$ because he has no chance to observe $\theta$ and $w$ is not revealed yet. Consider the case in which $(t_U, t_I) = (t_1, t_2)$. In this case, U picks either $N$ or $S$ and both selections yield the same expected payoff. Without loss of generality, assume that $a_U = N$. If $p \in \left( \frac{1}{2}, \frac{y-x \alpha}{(y-x)(\alpha+1)} \right)$, U knows that, in round 2, I chooses a location different from what U selects regardless of their expected payoff. Without loss of generality, assume that $\pi_U(t_i, t_2) = \Pr(N, \theta = N)\pi_U(a_I = S, \cdot) + \Pr(N, \theta = S)\pi_U(a_I = S, \cdot)$ (34)

$$
\begin{align*}
\pi_U(t_1, t_2) &= \Pr(N, \theta = N)\pi_U(a_I = S, \cdot) + \Pr(N, \theta = S)\pi_U(a_I = S, \cdot) \\
+ \Pr(S, \theta &= N)\pi_U(a_I = S, \cdot) + \Pr(S, \theta = S)\pi_U(a_I = S, \cdot) \\
= p \frac{1}{2} (y) + \frac{1-p}{2} (y) + \frac{1-p}{2} (x) + \frac{p}{2} (x) \\
= \frac{1}{2} (x + y)
\end{align*}
$$

On the other hand, if $p \in \left( \frac{y-x \alpha}{(y-x)(\alpha+1)}, 1 \right)$, U knows that, in round 2, I selects a location following the signal regardless of U’s choice in round 1. Then,

$$
\begin{align*}
\pi_U(t_1, t_2) &= \Pr(N, \theta = N)\pi_U(a_I = N, \cdot) + \Pr(N, \theta = S)\pi_U(a_I = S, \cdot) \\
+ \Pr(S, \theta &= N)\pi_U(a_I = N, \cdot) + \Pr(S, \theta = S)\pi_U(a_I = S, \cdot) \\
= p \frac{1}{2} (\alpha y) + \frac{1-p}{2} (y) + \frac{1-p}{2} (\alpha x) + \frac{p}{2} (x) \\
= \alpha \frac{1}{2} (y - x) (1 - \alpha) - y - x \alpha
\end{align*}
$$

Next, consider the case in which $(t_U, t_I) = (t_2, t_1)$. In this case, U knows that, in round 1, I chooses a location following the signal. Now, if $p \in \left( \frac{y-x \alpha}{(y-x)(\alpha+1)}, 1 \right)$, U’s best response in round 2 is to choose the same location as that of I. Then:

$$
\begin{align*}
\pi_U(t_2, t_1) &= \Pr(N, \theta = N)\pi_U(a_U = a_I = N) + \Pr(N, \theta = S)\pi_U(a_U = a_I = S, \cdot) \\
+ \Pr(S, \theta &= N)\pi_U(a_U = a_I = N, \cdot) + \Pr(S, \theta = S)\pi_U(a_U = a_I = S, \cdot) \\
= p \frac{1}{2} (\alpha y) + \frac{1-p}{2} (\alpha x) + \frac{1-p}{2} (\alpha y) + \frac{p}{2} (\alpha y) \\
= \alpha \frac{1}{2} (x - px + py)
\end{align*}
$$

On the other hand, if $p \in \left( \frac{1}{2}, \frac{y-x \alpha}{(y-x)(\alpha+1)} \right)$, in round 2, U’s best response is to choose a location opposite to that of I. Then:

$$
\begin{align*}
\pi_U(t_2, t_1) &= \Pr(N, \theta = N)\pi_U(a_U \neq a_I = N) + \Pr(N, \theta = S)\pi_U(a_U \neq a_I = S, \cdot) \\
+ \Pr(S, \theta &= N)\pi_U(a_U \neq a_I = N, \cdot) + \Pr(S, \theta = S)\pi_U(a_U \neq a_I = S, \cdot) \\
= p \frac{1}{2} (x) + \frac{1-p}{2} (y) + \frac{1-p}{2} (y) + \frac{p}{2} (x) \\
= \alpha (y + px - py)
\end{align*}
$$

Finally, if $t_U = t_I$, whether U picks either $N$ or $S$, it yields the same expected payoff. Also, I
always chooses a location following the signal. Then, from (A25),

$$\pi_U(t_U = t_I) = -\frac{(p(y-x)(1-\alpha) - y \cdot x\alpha)}{2}$$  \hspace{1cm} (A28)

Second, we derive I’s expected payoff. Here, how I’s expected payoffs are derived depends on whether U has a chance to observe I’s action or not. If U has a chance to observe I’s action, as U has a chance to infer $\theta$ perfectly, U’s choice can be expected certainly. Hence, I’s expected payoffs are derived from:

$$\pi_I(t_U, t_I) = \sum_{w \in \{N,S\}} \Pr(w|\theta)\pi_U(a_I, a_U, w)$$  \hspace{1cm} (A29)

If, however, U has no chance to infer I’s signal, U should make a choice only with his prior belief. Hence, I cannot predict certainly what U’s choice will be. In this case, by assumption, I believes that $\Pr(a_U = N) = \Pr(a_U = S) = \frac{1}{2}$. Hence, I’s expected payoffs are:

$$\pi_I(t_U, t_I) = \frac{1}{2} \left( \sum_{w \in \{N,S\}} \Pr(w|\theta)\pi_U(a_I, a_U = N, w) + \sum_{w \in \{N,S\}} \Pr(w|\theta)\pi_U(a_I, a_U = S, w) \right)$$  \hspace{1cm} (A30)

Here, (A29) corresponds to the case in which $(t_U, t_I) = (t_2, t_1)$ and (A30) corresponds to the cases in which $(t_U, t_I) = (t_1, t_2)$, $(t_2, t_2)$ and $(t_1, t_1)$. In following, without loss of generality, assume that $\theta = N$.

Consider the case in which $(t_U, t_I) = (t_2, t_1)$ where I acts as the leader. In this case, I always chooses a location following the signal. If $p \in \left(\frac{1}{2}, \frac{y-x\alpha}{(y-x)(\alpha+1)}\right)$, I knows that, in round 2, U selects a location opposite to what I selected in round 1. Then:

$$\pi_I(t_2, t_1) = \Pr(N|\theta = N)\pi_U(a_I = N, a_U = S, \cdot) + \Pr(S|\theta = N)\pi_U(a_I = N, a_U = S, \cdot)$$

$$= p(y) + (1-p)(x) = (x - px + py)$$  \hspace{1cm} (38)

On the other hand, if $p \in \left(\frac{y-x\alpha}{(y-x)(\alpha+1)}, 1\right)$, in round 2, U selects the same location as that of I. Then:

$$\pi_I(t_2, t_1) = \Pr(N|\theta = N)\pi_U(a_I = a_U = N, \cdot) + \Pr(S|\theta = N)\pi_U(a_I = a_U = N, \cdot)$$

$$= p(\cdot y) + (1-p)(\cdot x) = \alpha(x - px + py)$$  \hspace{1cm} (39)

Next, consider the case in which $(t_U, t_I) = (t_1, t_2)$ where I acts as the follower. If $p \in$
\(\left(\frac{1}{2}, \frac{y-x^\alpha}{(y-x)(\alpha+1)}\right)\), in round 2, I chooses a location opposite to what U selected in round 1. Then:

\[
\pi_I(t_1, t_2) = \frac{1}{2} \left( \sum_{w \in \{N, S\}} \Pr(w|\theta)\pi_U (a_I \neq a_U = N, w) + \sum_{w \in \{N, S\}} \Pr(w|\theta)\pi_U (a_I \neq a_U = S, w) \right)
\]

\[= \frac{1}{2} \left( p(x) + (1-p)(y) \right) + \frac{1}{2} \left( p(y) + (1-p)(x) \right)
\]

\[= \frac{1}{2} (x + y)
\]

On the other hand, if \(p \in \left(\frac{y-x^\alpha}{(y-x)(\alpha+1)}, 1\right)\), as I chooses a location following the signal regardless of \(a_U\). Then:

\[
\pi_I(t_1, t_2) = \frac{1}{2} \left( \sum_{w \in \{N, S\}} \Pr(w|\theta)\pi_U (a_I = a_U = N, w) + \sum_{w \in \{N, S\}} \Pr(w|\theta)\pi_U (a_I = N, a_U = S, w) \right)
\]

\[= \frac{1}{2} \left( p(\alpha y) + (1-p)(\alpha x) \right) + \frac{1}{2} \left( p(y) + (1-p)(x) \right)
\]

\[= \frac{1}{2} (x - px + py)(\alpha + 1)
\]

Finally, if \(t_U = t_I\), I always chooses a location following the signal. Also, he believes that \(\Pr(a_U = N) = \Pr(a_U = S) = \frac{1}{2}\). Then, from (A34):

\[
\pi_I(t_U = t_I) = \frac{1}{2} (x - px + py)(\alpha + 1)
\]

\[\square\]

8.3 Proof of Proposition 2

**Proposition 2.** If information is precise enough \((p \geq \overline{p})\), the informed player will enter either simultaneously with or after the uninformed player, who chooses a probability \(\mu \in [0, 1]\) of going first in the continuum of equilibria. Otherwise, there are two pure-strategy equilibria, one for each of the player’s entering first, and a mixed-strategy equilibrium in which the informed player enters early with probability \(z\) and the uninformed player enters with probability \(w\):

\[
(z, w) = \left( \frac{(x - px + py)(\alpha - 1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}, \frac{1}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)} \right)
\]

**Proof of Proposition 2.** Denote \(z = \Pr(t_U = t_1)\) and \(w = \Pr(t_I = t_1)\). Also, denote that \(E_i(t_i = t_k)\) is \(i\)'s expected payoff when he acts at round \(k\) where \(i \in \{N, S\}\) and \(k \in \{1, 2\}\).
First, consider the case in which $\frac{1}{2} < p < \frac{y-x\alpha}{(y-x)(\alpha+1)}$. From table 1, in the case of U:

$$E_U [t_U = t_1] = w \left( -\frac{(p(y-x)(1-\alpha) - y - x\alpha)}{2} \right) + (1-w) \left( \frac{1}{2} (x+y) \right)$$

(A38)

$$E_U [t_U = t_2] = w(y + px - py) + (1-w) \left( -\frac{(p(y-x)(1-\alpha) - y - x\alpha)}{2} \right)$$

(A44)

Thus:

$$E_U [t_U = t_1] - E_U [t_U = t_2] = w \left( x\alpha - \frac{1}{2} y - \frac{1}{2} x - px\alpha + py\alpha \right) + \left( \frac{1}{2} \right) \left( x - px + py \right) (1-\alpha)$$

(A39)

Here, if $p > \frac{(x+y-2x\alpha)}{2(y-x)\alpha}$, $x\alpha - \frac{1}{2} y - \frac{1}{2} x - px\alpha + py\alpha > 0$ and if $p < \frac{(x+y-2x\alpha)}{2(y-x)\alpha}$, $x\alpha - \frac{1}{2} y - \frac{1}{2} x - px\alpha + py\alpha < 0$. However, our condition is $\frac{1}{2} < p < \frac{y-x\alpha}{(y-x)(\alpha+1)}$ and $\frac{(x+y-2x\alpha)}{2(y-x)\alpha} - \frac{y-x\alpha}{(y-x)(\alpha+1)} = \frac{(1-\alpha)(x+y)}{2(\alpha+1)(y-x)\alpha} > 0$. Hence, for $p \in \left( \frac{1}{2}, \frac{y-x\alpha}{(y-x)(\alpha+1)} \right)$, $x\alpha - \frac{1}{2} y - \frac{1}{2} x - px\alpha + py\alpha < 0$. Hence:

$$w < w^* \implies z = 1$$

$$w = w^* \implies z \in [0,1]$$

$$w > w^* \implies z = 0$$

Thus, it can be shown that $w^* \in (0,1)$. So, (A39) is the uninformed player’s best response for $w$.

Next, in the case of I, from table 1,

$$E_I [t_I = t_1] = z \left( \frac{1}{2} (x - px + py) (\alpha + 1) \right) + (1-z) (x - px + py)$$

(A45)

$$E_I [t_I = t_2] = z \left( \frac{1}{2} (x + y) \right) + (1-z) \left( \frac{1}{2} (x - px + py) (\alpha + 1) \right)$$

(A46)

Thus:

$$E_I [t_I = t_1] - E_I [t_I = t_2] = z \left( x\alpha - \frac{1}{2} y - \frac{1}{2} x - px\alpha + py\alpha \right) + \frac{1}{2} (x - px + py) (1-\alpha)$$

(A42)

Equation (A42) is identical to (A38). Hence, I’s best response for $z$ is

$$z < z^* \implies w = 1$$

$$z = z^* \implies w \in [0,1]$$

$$z > z^* \implies w = 0$$

(A43)

Finally, the intersections of both players’ best response functions (A39) and (A43) yield that $(z, w) = (0, 1), (1, 0)$ and $(z^*, w^*)$. That is, there exist two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ and one mixed equilibrium $(z, w) = \left( \frac{(x-px+py)(\alpha-1)}{(2x\alpha-y-x-2px\alpha+2py\alpha)}, \frac{(x-px+py)(\alpha-1)}{(2x\alpha-y-x-2px\alpha+2py\alpha)} \right)$.

Second, consider the case in which $\frac{2(y-x)}{3(y-x)} < p < 1$. For U, from table 2,

$$E_U [t_U = t_1] = -\left( \frac{(p(y-x)(1-\alpha) - y - x\alpha)}{2} \right)$$

(A47)

$$E_U [t_U = t_2] = w(\alpha (x - px + py)) + (1-w) \left( -\frac{(p(y-x)(1-\alpha) - y - x\alpha)}{2} \right)$$

(A48)
Thus:
\[ E_U [t_U = t_1] - E_U [t_U = t_2] = \frac{1}{2} w (p (-(y-x)(\alpha+1)) + y-x\alpha) \]  
(A46)

Note that if \( p < \frac{y-x\alpha}{(y-x)(\alpha+1)} \), \( p (-(y-x)(\alpha+1)) + y-x\alpha > (>) 0 \). But, as our condition is that \( \frac{y-x\alpha}{(y-x)(\alpha+1)} < p < 1 \). Hence, \( p (-(y-x)(\alpha+1)) + y-x\alpha < 0 \). So, U’s best response for \( w \) is
\[ w < 0 \implies z = 1, w = 0 \implies z \in [0,1], w > 0 \implies z = 0 \]  
(A47)

Next, in the case of I:
\[ E_I [t_I = t_1] = z \left( \frac{1}{2} (x-px+py)(\alpha+1) \right) + (1-z) (\alpha (x-px+py)) \]  
(49)
\[ E_I [t_I = t_2] = \frac{1}{2} (x-px+py)(\alpha+1) \]  
(50)

Thus:
\[ E_I [t_I = t_1] - E_I [t_I = t_2] = z \left( \left( -\frac{1}{2} \right) (x-px+py)(\alpha-1) \right) + \frac{1}{2} (x-px+py)(\alpha-1) \]  
(A50)

As \( (-\frac{1}{2}) (x-px+py)(\alpha-1) > 0 \), the informed player’s best response for the uninformed player is:
\[ z > 1 \implies w = 1, z = 1 \implies w \in [0,1], z < 1 \implies w = 0 \]  
(A51)

The intersections of both players’ best response functions (A47) and (A51) yield that \( z \in [0,1] \) and \( w = 0 \). □

8.4 Proof of Proposition 4

**Proposition 4.** Efficiency depends on the ratio of duopoly to monopoly profit \((\alpha)\) and the quality of information \((p)\) as follows:

1) Suppose \( \alpha < \frac{x+y}{2y} \)

1-1) If \( \frac{1}{2} < p < \bar{p} \), both pure strategy equilibria are efficient and the mixed strategy equilibrium is not. (A1 in Figure 1)

1-2) If \( \bar{p} < p < 1 \), all equilibria are efficient. (A2 in Figure 1)

2) Suppose \( \alpha > \frac{x+y}{2y} \).

2-1) If \( \frac{1}{2} < p < \bar{p} \), both pure strategy equilibria are ex-ante efficient and the mixed strategy equilibrium is inefficient. (A5 in Figure 1)

2-2) If \( \bar{p} < p < \frac{y+x-2x\alpha}{2\alpha(y-x)} < 1 \), all equilibria are efficient. (A4 in Figure 1)
2.3) If \(\frac{y-x-2\alpha}{2\alpha(y-x)} < p < 1\), all equilibria are inefficient. \((A3 \text{ in Figure 1})\)

**Proof of Proposition 4**

Case 1: When \(\frac{y-x\alpha}{(y-x)(\alpha+1)} < p < 1\)

From table 2:

\[
\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) = \sum_{i \in \{U,I\}} \pi_i(t_1,t_2) = \sum_{i \in \{U,I\}} \pi_i(t_2,t_2) = \frac{1}{2} (x + y + 2x\alpha - 2px\alpha + 2py\alpha) (51)
\]

\[
\sum_{i \in \{U,I\}} \pi_i(t_2,t_1) = 2\alpha (x - px + py) \tag{52}
\]

Then:

\[
\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) - \sum_{i \in \{U,I\}} \pi_i(t_2,t_1) = \left(-\frac{1}{2}\right) \left(p(2y\alpha - 2x\alpha) + 2x\alpha - y - x\right) \tag{A54}
\]

So, if \(p > \frac{2x\alpha-y-x}{2xy-2x\alpha}\), \(\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) < \sum_{i \in \{U,I\}} \pi_i(t_2,t_1)\) and if \(p \leq \frac{2x\alpha-y-x}{(y-x)(\alpha+1)}\), \(\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) > \sum_{i \in \{U,I\}} \pi_i(t_2,t_1)\). Here, \(-\frac{2x\alpha-y-x}{2xy-2x\alpha} < \frac{1}{2} = \frac{(x+y)(1-\alpha)}{2(y-x)} > 0, \frac{2x\alpha-y-x}{2xy-2x\alpha} \leq \frac{2x\alpha-y-x}{(y-x)(\alpha+1)}\). Also, \(-\frac{2x\alpha-y-x}{2xy-2x\alpha} - 1 = \frac{(y+x-y-x)}{(y-x)\alpha} > 0\). So, if \(\alpha > \frac{x+y}{2y}\), \(-\frac{2x\alpha-y-x}{2xy-2x\alpha} < 1\) and if \(\alpha < \frac{x+y}{2y}\), \(-\frac{2x\alpha-y-x}{2xy-2x\alpha} > 1\) where \(\alpha \in \left(\frac{x}{y}, 1\right)\). Then, this can be summarized as follows: A) Suppose that \(\alpha > \frac{x+y}{2y}\). Then, if \(\frac{y-x\alpha}{(y-x)(\alpha+1)} < p < \frac{2x\alpha-y-x}{2xy-2x\alpha}\), \(\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) > \sum_{i \in \{U,I\}} \pi_i(t_2,t_1)\) and if \(-\frac{2x\alpha-y-x}{2xy-2x\alpha} < p \leq 1\), \(\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) < \sum_{i \in \{U,I\}} \pi_i(t_2,t_1)\). B) Suppose that \(\alpha < \frac{x+y}{2y}\). Then, always \(\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) > \sum_{i \in \{U,I\}} \pi_i(t_2,t_1)\).

Recall that if \(\frac{y-x\alpha}{(y-x)(\alpha+1)} < p < 1\), in equilibrium, \(t_I = t_2\) and \(z \in \{0, 1\}\) where \(z = \text{Pr}(t_U = t_1)\). First, assume that \(\alpha > \frac{x+y}{2y}\). If \(\frac{y-x\alpha}{(y-x)(\alpha+1)} < p < \frac{2x\alpha-y-x}{2xy-2x\alpha}\), the efficient case is \((t_U,t_I) = (t_1,t_1)\), \((t_1,t_2)\) or \((t_2,t_2)\). If the uninformed player uses a pure strategy, i.e., \(z \in \{0, 1\}\), the outcome is \((t_1,t_2)\) or \((t_2,t_2)\). So, it is efficient. Also, if the uninformed player uses a mixed strategy, as \(E\pi_U = -(y)(y-x)(1-\alpha-y-x)\) and \(E\pi_I = \frac{(x-y)p+y+2x\alpha-2px\alpha+2py\alpha}{2}\), \(E\pi_U + E\pi_I = \frac{1}{2} (x + y + 2x\alpha - 2px\alpha + 2py\alpha)\). So, it is also ex-ante efficient. Therefore, all equilibria are efficient. Next, if \(-\frac{2x\alpha-y-x}{2xy-2x\alpha} < p < 1\), the efficient case is \((t_U,t_I) = (t_2,t_1)\). As \(t_I\) is fixed as \(t_2\) in equilibrium, the equilibrium cannot be efficient. Second, assume that \(\alpha < \frac{x+y}{2y}\). Then, the efficient case is \((t_U,t_I) = (t_1,t_1), (t_1,t_2)\) or \((t_2,t_2)\). Then, from the case in which \(\alpha > \frac{x+y}{2y}\) and \(\frac{y-x\alpha}{(y-x)(\alpha+1)} < p < \frac{2x\alpha-y-x}{2xy-2x\alpha}\), all equilibria are efficient.

Case 2: When \(\frac{1}{2} < p < \frac{y-x\alpha}{(y-x)(\alpha+1)}\).

From the table 1, it can be checked that:

\[
\sum_{i \in \{U,I\}} \pi_i(t_1,t_1) = \sum_{i \in \{U,I\}} \pi_i(t_1,t_2) = \sum_{i \in \{U,I\}} \pi_i(t_2,t_2) = \frac{1}{2} (x + y + 2x\alpha - 2px\alpha + 2py\alpha) \tag{53}
\]

\[
\sum_{i \in \{U,I\}} \pi_i(t_2,t_1) = \sum_{i \in \{U,I\}} \pi_i(t_2,t_1) = x + y \tag{54}
\]
The comparison yields that if \( p > -\frac{2x\alpha - y - x}{2p\alpha - 2x\alpha} \), \( \frac{1}{2} (x + y + 2x\alpha - 2p\alpha + 2py\alpha) > (x + y) \) and if \( p < -\frac{2x\alpha - y - x}{2y\alpha - 2x\alpha} \), \( \frac{1}{2} (x + y + 2x\alpha - 2p\alpha + 2py\alpha) < (x + y) \). Note that \(-\frac{2x\alpha - y - x}{2y\alpha - 2x\alpha} - \frac{y - x}{y - x}(\alpha + 1) = \frac{(1 - \alpha)(y + x)}{2(\alpha + 1)(y - x)\alpha} > 0 \) where our condition is \( p < \frac{y - x}{y - x}(\alpha + 1) \). Hence, for all \( p \in \left( \frac{1}{2}, \frac{y - x}{y - x}(\alpha + 1) \right) \), \( \frac{1}{2} (x + y + 2x\alpha - 2p\alpha + 2py\alpha) < (x + y) \), which yields:

\[
\sum_{i \in \{U,I\}} \pi_i (t_1, t_1) = \sum_{i \in \{U,I\}} \pi_i (t_2, t_2) < \sum_{i \in \{U,I\}} \pi_i (t_1, t_2) = \sum_{i \in \{U,I\}} \pi_i (t_2, t_1) \tag{A57}
\]

Therefore, the efficient case is the one in which both players act sequentially. Recall that, if \( p \in \left( \frac{1}{2}, \frac{y - x}{y - x}(\alpha + 1) \right) \), the pure equilibria are \((t_U, t_I) = (t_2, t_1), (t_1, t_2)\). So, both pure equilibria are efficient. Also, there exists a mixed strategy equilibrium in which \((z, w) = \left( \frac{(x - px + py)(\alpha - 1)}{2(x - px + y)(\alpha - 1)}, \frac{(x - px + py)(\alpha - 1)}{2(x - px + y)(\alpha - 1)} \right) \) where \( \Pr(t_A = t_1) = z \) and \( \Pr(t_B = t_1) = w \). For the mixed strategy equilibrium, the computation yields that:

\[
(E\pi_U + E\pi_I) - (x + y) = \frac{p^2 \left( 2(y - x)^2 (\alpha^2 + 1) \right) + p \left( 2(y - x) (x - y - x\alpha - y\alpha + 2x^2\alpha) \right) + (x^2 - 2xy\alpha + y^2 - 2x^2\alpha + 2x^2\alpha^2)}{2(2x\alpha - y - x - 2p\alpha + 2py\alpha)} \tag{55}
\]

First, for \( p \in \left( \frac{1}{2}, \frac{y - x}{y - x}(\alpha + 1) \right) \), the sign of denominator is negative. Second, if we denote the numerator as \( f(p) \), it is a convex function and attains the minimum value at \( \hat{p} = \frac{(2(y - x)(x - y - x\alpha - y\alpha + 2x^2\alpha))}{2(2(y - x)(\alpha^2 + 1))} \) where \( f(p = \hat{p}) = \frac{(x + y)^2 (\alpha - 1)^2}{2(\alpha^2 + 1)} > 0 \). Therefore:

\[
(E\pi_U + E\pi_I) < (x + y) \tag{A59}
\]

which means that the mixed strategy equilibrium is inefficient. \( \square \)

8.5 Proof of Lemma 2

Lemma 2 Consider the case where \( \alpha y < x \).

1) Suppose that \( t_I = t_1 \) and \( t_U = t_2 \). Then, the informed player chooses a location following his given signal and the uninformed player deviates from the informed player’s choice for all \( p \in \left( \frac{1}{2}, 1 \right) \).

2) Suppose that \( t_U = t_1 \) and \( t_I = t_2 \). Then, the informed player chooses a location different from the uninformed player’s choice for all \( p \in \left( \frac{1}{2}, 1 \right) \). (That is, if \( \theta_I = a_U \), the informed player chooses a location opposite to what the signal reveals. If \( \theta_B \neq a_A \), he always chooses a location following his given signal.)

3) Suppose that \( t_U = t_I \). Then, the informed player chooses a location following his given signal.

Proof of Lemma 2. We can use the procedure of the proof of Proposition 1. Consider the following three cases.
Case 1) When $t_I = t_1$ and $t_U = t_2$

Recall (A14) which states that $\lambda \uparrow \lambda^* \implies E_{\pi_U}(a_U = a_I) \uparrow E_{\pi_U}(a_U \neq a_I)$ where $\lambda^* = \frac{(x - px + py - y + p\alpha + py\alpha)}{(y - x)(2 + x)(a + 1)}$. Here, $\lambda^* - 1 = \frac{(y + px - px + px\alpha + py\alpha)}{(a + 1)(2 + x)(y - x)}$. So, if $p < -\frac{y - x}{x - y + x\alpha - y\alpha}$, $\lambda^* > 1$ and if $p < -\frac{y - x}{x - y + x\alpha - y\alpha}$, $\lambda^* < 1$. Therefore, $U$ always diverges from $I$’s choice. In this case, it was already checked that $I$’s best response as the leader is then to choose a location following the signal from (A17). Therefore, always $a_U \neq a_I$.

Case 2) When $t_U = t_1$ and $t_I = t_2$

First, if $\theta = a_U$, from (A3), $p \geq \frac{(x - y)}{(a + 1)(x - y)} \implies E_{\pi_I}(a_I = \theta) \geq E_{\pi_I}(a_I \neq \theta)$, Here, $\frac{(x - y)}{(a + 1)(x - y)} - 1 = -\frac{(y - x)}{(a + 1)(y - x)} > 0$ from our assumption that $y\alpha < x$. So, for $p \in (\frac{1}{2}, 1)$, $E_{\pi_I}(a_I = \theta) < E_{\pi_I}(a_I \neq \theta)$. Second, if $\theta \neq a_U$, from (A6), $p \geq \frac{(y - x)}{(a + 1)(y - x)} \implies E_{\pi_I}(a_I = \theta) \geq E_{\pi_I}(a_I \neq \theta)$.

However, $\frac{(y - x)}{(a + 1)(y - x)} < 0$ from our assumption that $y\alpha < x$. Hence, always $E_{\pi_I}(a_I = \theta) > E_{\pi_I}(a_I \neq \theta)$. Finally, if $\theta = a_U$, the informed one deviates from the signal and if $\theta \neq a_U$, he uses the signal, which yields that always $a_U \neq a_I$.

Case 3) When $t_U = t_I$

This is obvious from (8). □

### 8.6 Proof of Proposition 5

**Proposition 5** Suppose that $\alpha y < x$, so a player prefers being a monopolist in the worse market to being a duopolist in the better market. Then, for all $p \in (\frac{1}{2}, 1)$, there exist two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$, both ex-post efficient, and one mixed equilibrium $(z, w) = \left(\frac{(x - px + py)(a - 1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}\right)$, where $z = Pr(t_U = t_1)$ and $w = Pr(t_I = t_1)$.

**Proof of Proposition 5.** Recall the proof of Proposition 2. Regarding $U$’s best response, from (A38),

$$E_U(t_U = t_1) - E_U(t_U = t_2) = w \left( x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha \right) + \left( \frac{1}{2} \right) \left( x - px + py \right)(1 - \alpha)$$  \hspace{1cm} (A60)

First, if $p > \frac{(x + y - 2\alpha)}{2(y - x)\alpha}$, $x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha > 0$ and if $p < \frac{(x + y - 2\alpha)}{2(y - x)\alpha}$, $x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha < 0$. Here, it can be checked that $\frac{(x + y - 2\alpha)}{2(y - x)\alpha} > 1$ because $\frac{(x + y - 2\alpha)}{2(y - x)\alpha} - 1 = -\frac{(2\alpha - y - x)}{2(y - x)\alpha}$.

Hence, always $x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha < 0$. Then, (A60) yields that

$$w < w^* \implies z = 1, \quad w = w^* \implies z \in [0, 1], \quad w > w^* \implies z = 0$$  \hspace{1cm} (A61)

where $w^* = \frac{(x - px + py)(a - 1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}$. Here, it can be shown that $w^* \in (0, 1)$ from following: a) At above, it was already checked that $x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha < 0$. Hence, $2x\alpha - y - x - 2px\alpha + 2py\alpha < 0$. 

30
efficient case is the one in which both players compete in a big market. Our condition is $\alpha > 0$. b) $\frac{(x-px+py)(\alpha-1)}{(2x\alpha-y-x-2px\alpha+2py\alpha)} - 1 = -\frac{(y+px-py-x-2px\alpha+py\alpha)}{x+y-2x\alpha+2px\alpha-2py\alpha}$. Note that the sign of the denominator is positive from $2x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha < 0$. Also, for the numerator, if $p < -\frac{y-x\alpha}{x-y+2x\alpha-ya}$, the sign of the numerator is positive and if $p > -\frac{y-x\alpha}{x-y+2x\alpha-ya}$, it is negative. However, $-\frac{y-x\alpha}{x-y+2x\alpha-ya} - 1 = -\frac{(y\alpha-x)}{(a+1)(y-x)} > 0$ from our assumption that $y\alpha < x$. Hence, the sign of the numerator is positive and it yields that $\frac{(x-px+py)(\alpha-1)}{(2x\alpha-y-x-2px\alpha+2py\alpha)} < 1$. Therefore, $w^* \in (0, 1)$ and U’s best response function for given $w$ is (A61).

Next, regarding I’s best response, from (A42):

$$E_I[t_I = t_1] - E_I[t_I = t_2] = z \left( x\alpha - \frac{1}{2}y - \frac{1}{2}x - px\alpha + py\alpha \right) + \frac{1}{2} \left( -px + py \right) (1 - \alpha) \quad (A62)$$

This is identical to (A60). Hence, the informed player’s best response for $z$ is

$$z < z^* \implies w = 1, \quad z = z^* \implies w \in [0, 1], \quad z > z^* \implies w = 0 \quad (A63)$$

Finally, the intersection of (A61) and (A63) yields that there exist two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ and one mixed equilibrium $(z, w) = \left( \frac{(x-px+py)(\alpha-1)}{(2x\alpha-y-x-2px\alpha+2py\alpha)}, \frac{(x-px+py)(\alpha-1)}{(2x\alpha-y-x-2px\alpha+2py\alpha)} \right)$.

8.7 Proof of Proposition 6

**Proposition 6.** Suppose that $\alpha y < x$, so a player prefers being a monopolist in the worse market to being a duopolist in the better market.

1) Two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ are ex-post efficient.

2) Two pure equilibria $(t_U, t_I) = (t_2, t_1), (t_1, t_2)$ are ex-ante efficient, but the mixed strategy equilibrium is not ex-ante efficient.

**Proof of Proposition 6**

1) Ex-post efficiency

The socially efficient case depends on the value of $\alpha$: If $\alpha < \frac{x+y}{2y}$, it is when each player operates as a monopolist in each market separately. On the other hand, if $\alpha > \frac{x+y}{2y}$, the socially efficient case is the one in which both players compete in a big market. Our condition is $\alpha y < x$ where $\frac{x+y}{2y} - \frac{x+y}{2y} = \frac{(x-y)}{2y} < 0$. Hence, if $\alpha y < x$, the socially efficient case is the one where each player operates as a monopolist in each market separately. Recall that each player’s best response as the follower is to choose a different location from that of the leader. Also, in both pure equilibria, both players’ timings of actions are sequential always. Therefore, both pure equilibria are ex-post efficient always.

2) Ex-ante efficiency
For the sign of denominator, if $p > \frac{-2x\alpha - y - x}{2y\alpha - 2x\alpha}, \frac{1}{2} (x + y + 2x\alpha - 2px\alpha + 2py\alpha) > (x + y)$ and if $p < \frac{-2x\alpha - y - x}{2y\alpha - 2x\alpha}, \frac{1}{2} (x + y + 2x\alpha - 2px\alpha + 2py\alpha) < (x + y)$. Note that $\frac{-2x\alpha - y - x}{2y\alpha - 2x\alpha} - 1 = \frac{(2y\alpha - y - x)}{2(y-x)\alpha} = \frac{y(\alpha-1)+\alpha y - x)}{2(y-x)\alpha} > 0$. Hence, for all $p \in (\frac{1}{2}, 1), \frac{1}{2} (x + y + 2x\alpha - 2px\alpha + 2py\alpha) < (x + y)$, which yields that

$$
\sum_{i \in \{U,I\}} \pi_i(t_1, t_1) = \sum_{i \in \{U,I\}} \pi_i(t_2, t_2) < \sum_{i \in \{U,I\}} \pi_i(t_1, t_2) = \sum_{i \in \{U,I\}} \pi_i(t_2, t_1) \tag{A66}
$$

Therefore, the efficient case is the one in which both players act sequentially. So, both pure equilibria are efficient.

Also, there exists a mixed strategy equilibrium in which $(z, w) = \left(\frac{(x-px+py)(\alpha-1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}, \frac{(x-px+py)(\alpha-1)}{(2x\alpha - y - x - 2px\alpha + 2py\alpha)}\right)$ where Pr($t_A = t_1$) = $z$ and Pr($t_B = t_1$) = $w$. For the mixed strategy equilibrium, the computation yields that:

$$
(E\pi_U + E\pi_I) - (x + y)
$$

$$
= \frac{p^2 \left(2(y-x)^2 (\alpha^2 + 1)\right) + p \left(2(y-x)\left(x - y - x\alpha - y\alpha + 2x\alpha^2\right)\right) + \left(x^2 - 2xy\alpha + y^2 - 2x^2\alpha + 2x^2\alpha^2\right)}{2(2x\alpha - y - x - 2px\alpha + 2py\alpha)} \tag{58}
$$

For the sign of denominator, if $p > \frac{-2x\alpha - y - x}{2y\alpha - 2x\alpha}$, it is positive and if $p < \frac{-2x\alpha - y - x}{2y\alpha - 2x\alpha}$, it is negative. We already checked that $\frac{-2x\alpha - y - x}{2y\alpha - 2x\alpha} > 1$. Hence, the sign of denominator is negative. Second, if we denote the numerator as $f(p)$, it is a convex function and attains the minimum value at $\hat{p} = \frac{(2(y-x)(x-y-x\alpha-y\alpha+2x\alpha^2))}{2(2(y-x)^2(\alpha^2+1))}$ where $f(p = \hat{p}) = \frac{(x+y)^2(\alpha-1)^2}{2(\alpha^2+1)} > 0$. Therefore

$$
(E\pi_U + E\pi_I) < (x + y) \tag{A68}
$$

which means that the mixed strategy equilibrium is inefficient. □
9 References


Figure 1: Information Quality, Market Size, and Efficiency